Your Name: ______________________

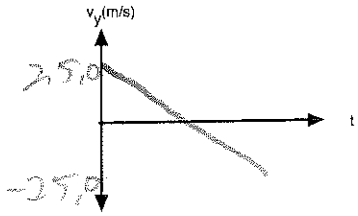
PHY203
Final Exam
Dec. 13, 2017

- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use $g = 9.81\ m/s^2$

Solutions

1. A ball is thrown straight up from ground level with an initial speed of 25.0 m/s. (Ignore air resistance.) Take the +y direction as the "up" direction. **Add numbers to the vertical axes.**

a. Make a sketch of v_y vs. time while the ball is in the air.



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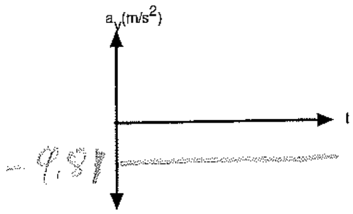
b. Make a sketch of a_y vs. time while the ball is in the air.



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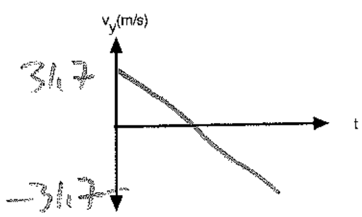
A ball is thrown at an angle of 65.0° with respect to the horizontal from ground level with an initial speed of 35.0 m/s. (Ignore air resistance.) Take the +y direction as the "up" direction; +x in direction of horizontal motion. **Add numbers to the vertical axes.**

c. Make a sketch of v_y vs. time while the ball is in the air.

$(35.0)\sin(65.0) = 31.7$



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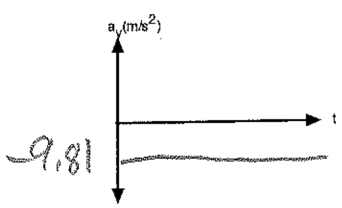
d. Make a sketch of a_y vs. time while the ball is in the air.



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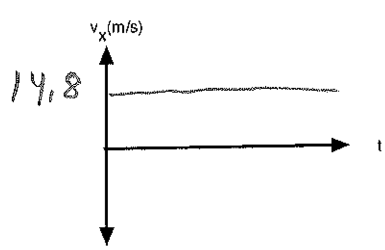
e. Make a sketch of v_x vs. time while the ball is in the air.

$(35.0)\cos(65.0) = 14.8$



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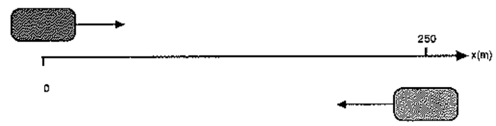


2. Two trains approach each other on parallel tracks. At a time of t=0 the train on the left is at rest and begins to accelerate at 2.50 m/s^2 to the right. The train accelerates for 5.00 s then continues at a constant speed. At t=0 the train on the right is at rest and is separated from the train on the left by 250 m and starts to travel with a constant acceleration of 3.50 m/s^2 to the left.

a. Using the coordinate system depicted above, write equations of motion (x vs. t) for the train on the left for t<5.00 s and t≥5.00s:

t<5.00 s $x_l =$ $\frac{1}{2}(2.50)t^2 = 1.25t^2$ 5

t≥5.00 s $x_l =$ $31.2\text{ m} + 12.5(t-5)\text{ m/s}$ 15

at 5.00 s, $x = 1.25(5)^2 = 31.2\text{ m}$

$V = 2.50(5.00) = 12.5\text{ m/s}$

b. Using the coordinate system depicted above, write an equation of motion (x vs. t) for the train on the right:

$x_r =$ $250 - \frac{1}{2}(3.50)t^2 = 250 - 1.75t^2$ 10

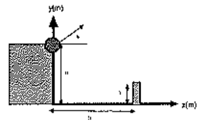
c. Find the time at which the centers of the trains are side-by-side.

$31.2 + 12.5(t-5) = 250 - 1.75t^2$

$1.75t^2 + 12.5t - 281.3 = 0$ 10

$t = \dfrac{-12.5 \pm \sqrt{12.5^2 - 4(1.75)(-281.3)}}{3.50}$

$= 9.60\text{ s}$



3. A cannonball is shot from a cliff of height H = 225 m at a castle wall. The initial velocity of the ball is v_o at an angle of θ with respect to the horizontal direction. The castle wall is Y = 65.0 m high and a horizontal distance of D = 850 m from the base of the wall. Assume the initial horizontal speed is 65.5 m/s and that the cannonball just grazes the top of the wall.

a. Find the time it takes the ball to reach the wall.

$$850 = 65.5t$$

$$t = 13.0\ s$$

5

b. Find the initial velocity of the cannonball and <u>write it in vector notation</u> using the coordinate system above.

$$65.0 = 225 + v_{yo}(13.0) - \frac{1}{2}g(13.0)^2$$

10

$$v_{yo} = 51.5\ m/s$$

$$\vec{v_o} = (65.5\hat{i} + 51.5\hat{j})\ m/s$$

5

c. Find the y position of the cannonball at its highest point.

$$0 = (51.5)^2 - 2g\Delta y$$

$$\Delta y = 135\ m$$

10

$$y = 135\ m + 225\ m = 360\ m$$

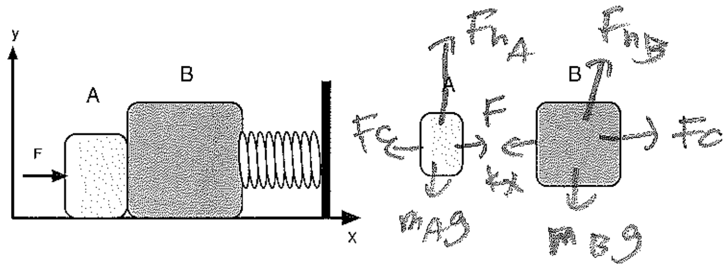
d. Find the velocity in vector notation of the cannonball just before it hits the ground.

$$v_y^2 = 51.5^2 - 2g(0 - 225)$$

$$v_y = -84.1\ m/s$$

10

$$\vec{v_y} = (65.5\hat{i} - 84.1\hat{j})\ m/s$$



4. A horizontal force F is applied to block A above. Block B is pushed against a spring which has a spring constant k. The masses of the blocks are M_A and M_B. Assume a frictionless surface under the blocks.

a. Draw free body diagrams on the blocks shown above and on the right. 10

b. Write out Newton's 2nd Law for both blocks in the x-direction. Assume both blocks are accelerating and that the spring is compressed by d.

A: $F - F_c = m_A a$ 10

B: $F_c - kd = m_B a$ 10

c. Assuming that M_A = 5.50 kg, M_B = 8.50 kg, F = 250 N, d = 15.0 cm, and k = 850 N/m, find the magnitude of the acceleration of the blocks and the magnitude of the contact force.

Add left + right sides above:

$F - kd = (m_A + m_B)a$ 5

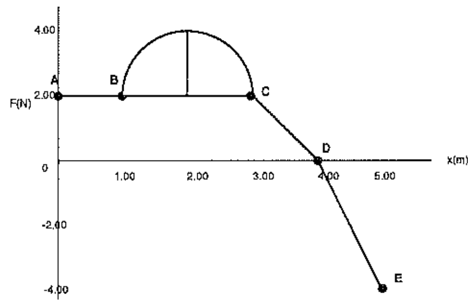
$a = \dfrac{250 - 850(0.15)}{14.0} = 8.75 \text{ m/s}^2$

$F_c = 250 - (5.5)(8.75) = 202 \text{ N}$

d. Under the same pushing force, the blocks keep moving until they stop. Find the maximum compression of the spring.

$250 = 850x$

$x = 0.294 \text{ m}$ 5



5. Given the force vs. distance curve above (assume a semicircular curve from B to C), find the work done between the following points:

a. A to B $(2.00\ N)(1.00 m) = 2.00\ J$ 5

b. B to C $(2.00\ N)(2.00 m) + \frac{1}{2}\pi(2.00)^2 = 10.3\ J$ 5

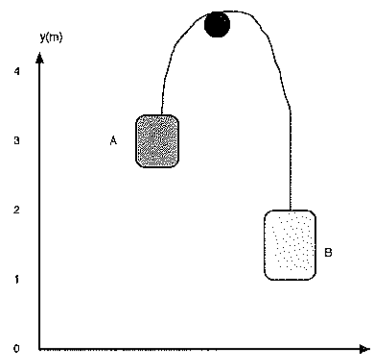
c. C to D $\frac{1}{2}(2.00\ N)(1.00\ m) = 1.00\ J$ 5

d. D to E $-\frac{1}{2}(4.00\ N)(1.00\ J) = -2.00\ J$ 5

e. Assuming a toy car with a mass of 0.500 kg had a speed of 5.50 m/s in the +x-direction at x = 0, find the speed of the car at x = 4.00 m.

$\frac{1}{2} m v^2 = \frac{1}{2} m (5.50)^2 + 11.3\ J$ 10

$v = 8.64\ m/s$



6. Two masses are suspended over a massless, frictionless pulley as shown above. Block A is 2.50 kg and is initially at a y position of 3.00 m. Block B is 4.50 kg and is initially at a y position of 1.50 m.

a. Find the initial total energy of the blocks.

$$U = (2.50)g(3.00) + (4.50)g(1.50)$$

10

$$= 140\ J$$

b. If the masses are then released, use work and energy to find the speed of the masses when block B is at a position of y=0.250 m.

$$140\ J = \frac{1}{2}(2.50 + 4.50)v^2$$

$$+ g\left[(2.5)(4.25) + 4.5(0.250)\right]$$

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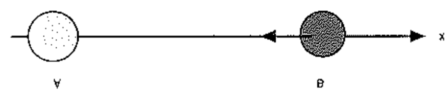
$$= 3.50\, v^2 + 115\ J$$

$$v = 2.67\ m/s$$



7. Two disks traveling on a one-dimensional track (x-direction) collide into each other with an elastic collision. The masses and speeds before the collision of disk A are 2.50 kg and 0 m/s (disk A is initially at rest) and of disk B are 3.50 kg and 4.00 m/s.

a. Which of the following are conserved in the collision? (Circle all that apply.)

momentum   kinetic energy   total energy   6

b. Find the momentum of disk B before the collision in vector notation.

$\vec{p_B} = (3.50)(-4.00)\hat{i}$

$= -14.0 \text{ kg m/s } \hat{i}$ 5

c. Find the kinetic energy of disk B before the collision.

$K_B = \frac{1}{2}(3.50)(4.00)^2$ 4

$= 28.0 \text{ J}$

d. Find the final velocities of the disks after the collision in vector notation.

$-14.0 = (2.50)v_A + (3.50)v_B$

$28.0 = \frac{1}{2}(2.50)v_A^2 + \frac{1}{2}(3.50)v_B^2$

$= 1.25 v_A^2 + 1.75 v_B^2$

$\rightarrow v_A = -1.40 v_B - 5.6$ 20

$28.0 = 1.25(1.4 v_B + 5.6)^2 + 1.75 v_B^2$

$= 2.45 v_B^2 + 19.6 v_B + 39.2 + 1.75 v_B^2$

$0 = 4.2 v_B^2 + 19.6 v_B + 11.2$

quad. eq.: $v_B = -0.667$

$\rightarrow v_A = -1.4(-.667) - 5.6 = -4.67$

$\vec{v_A} = -4.67 \text{ kg m/s } \hat{i}$ $\vec{v_B} = -0.667 \text{ kg m/s } \hat{i}$

8. A wheel is rolling without slipping down a ramp which makes an angle of 55.0° with respect to the horizontal. Take +x as the direction down the ramp. The wheel consists of a thin rim with mass 1.50 kg and radius of 1.25 m and 6 spokes of length 1.25 m and mass 0.250 kg apiece.

a. Find the moment of inertia of the wheel about its axis.

$$I = (1.50)(1.25)^2 + 6\left(\tfrac{1}{3}\right)(.25)(1.25)^2$$

$$= 3.12 \text{ kg m}^2$$

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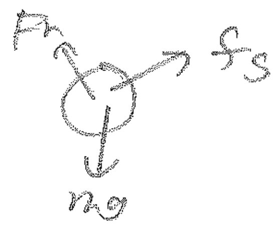
b. Sketch a free body diagram of the wheel as it rolls without slipping down the ramp.



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c. Write our Newton's 2nd Law for the wheel in the x-direction.

$$mg\sin\theta - f_s = ma$$

10

d. Write out Newton's 2nd Law version of the torque equation of the wheel.

$$f_s R = I\alpha = \frac{Ia}{R}$$

10

e. Find the magnitude of the linear acceleration of the wheel on the ramp.

$$f_s = \frac{(3.12)\,a}{1.25^2} = 2.00a$$

$$mg\sin\theta = (2.00 + m)a$$

$$a = \frac{3g\sin(55.0°)}{(2.00 + 3.00)} = 4.82 \text{ m/s}^2$$

10

9. A block of mass 1.75 kg is attached to a spring. The block is stretched and released. The equation of motion of the block is given by:

$$x = (0.550\,m)\cos(3.50t)$$

a. Find the spring constant of the spring.

$$\omega = \sqrt{\frac{k}{m}}$$

5

$$k = m\omega^2 = (1.75)(3.50)^2$$

$$= 21.4 \text{ N/m}$$

b. Find the total energy of the system

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(21.4)(.550)^2$$

5

$$= 3.24 \text{ J}$$

c. Find an expression for the acceleration of the block as a function of time.

$$v = -(3.50)(.550)\sin(3.50t)$$

$$a = -(3.50)^2(.550)\cos(3.50t)$$

10

$$= -6.74\cos(3.50t) \text{ m/s}^2$$

d. Find the maximum acceleration of the block.

$$a_{max} = 6.74 \text{ m/s}^2$$

5