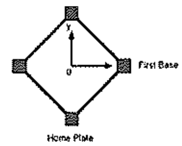
Your Name: ______________________

PHY203
Final Exam
5/1/20

- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use $g = 9.81\ m/s^2$

Solutions

Final1S20



1. The base path in a baseball field (the “baseball diamond”) is basically a square of 90.0 feet on a side (90 ft. = 27.4 m). Assume a baseball player is starting from first base and runs counterclockwise on the base path to home plate starting at t=0 once the ball is hit. He starts from rest and accelerates at a constant acceleration of 3.00 m/s^2 (in magnitude) until he reaches 2nd base. From there he continues at constant speed until he reaches home plate.

a. Find the time it takes to get to 2nd base from 1st base. (**10**)

$$27.4 = \frac{1}{2}at^2 = \frac{1}{2}(3.00)\,t^2$$

$$t = 4.27\,s$$

b. Find the (constant) speed after rounding 2nd base. (**5**)

$$v = v_0 + at = 0 + 3.00\,(4.27) = 12.8\ m/s$$

d. Find the total time to home plate from 1st base. (**10**)

$$\frac{27.4 \times 2}{12.8} = 4.28\,s$$

$$4.27\,s + 4.28\,s = 8.55\,s$$

d. Find the average speed for the entire run from 1st base to home plate. (**10**)

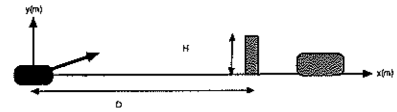
$$\frac{3 \cdot 27.4}{8.55} = 9.61\,\frac{m}{s}$$

e. Find the average velocity (in vector notation-use the coordinate system given above). (**15**)

$$\vec{r_i} = \frac{27.4}{\sqrt{2}}\,\hat{i} = 19.4\,\hat{i}$$

$$\vec{r_f} = -19.4\,\hat{j}$$

$$\vec{v}_{avg} = \frac{-19.4\hat{j} - 19.4\hat{i}}{8.55} = -2.27(\hat{i}+\hat{j})\,\frac{m}{s}$$



2. A cannonball is shot from ground level at a castle wall at an angle of 35.0° with respect to the horizontal direction. The initial vertical component of velocity of the ball is 45.0 m/s in magnitude. The castle wall has height H = 65.0 m and horizontal distance D from the cannon.

a. Write the initial velocity of the cannonball in vector notation using the coordinate system above. (**10**)

$v_{y_0} = 45.0 \text{ m/s}$

$v_{x_0} = \dfrac{45.0}{\tan 35^0} = 64.3 \text{ m/s}$

$\vec{v}_0 = (64.3\hat{i} + 45.0\hat{j})\,\frac{m}{s}$

b. Write the velocity and acceleration of the cannonball at its highest point in vector notation. (**10**)

$\vec{a} = -9.81 \text{ m/s}^2\,\hat{j}$

$\vec{v} = 64.3\hat{i} \text{ m/s}$

c. Find the height of the cannonball at its highest point. (**10**)

$0 = 45.0^2 - 2g\,\Delta y$

$H = \Delta y = 103 \text{ m}$

Assume the cannonball just grazes the top of the wall (<u>after passing its highest point</u>).

d. Find the time it takes the ball to reach the wall. (**20**)

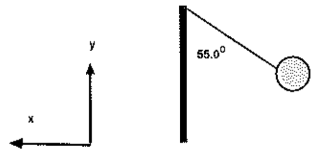
$65.0 = 0 + 45.0t - \frac{1}{2}gt^2$

$4.905t^2 - 45.0t + 65.0 = 0$

$t = \dfrac{45.0 \pm \sqrt{45.0^2 - 4 \cdot 4.905 \cdot 65.0}}{9.81}$

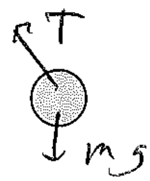
$= \cancel{1.80\,s},\ 7.38\text{ s}$

Final2S20



3. A ball of mass m is attached to a vertical pole with a length of string L and spun with a constant speed v in a horizontal circle.

a. Draw a free body diagram of the ball below: (**5**)



b. Write expressions for Newton's 2nd Law in the x- and y-directions. (**20**)

$$x: \quad T\sin 55^\circ = ma = \frac{mv^2}{r}$$

$$y: \quad T\cos 55^\circ - mg = 0$$

Assume that m = 3.50 kg, and the speed of the ball is 6.50 m/s.

c. Find the magnitude of the tension in the string. (**5**)

$$T = \frac{mg}{\cos 55^\circ} = \frac{(3.50)g}{\cos 55^\circ} = 59.9\ N$$

d. Find the length of the string. (**10**)

$$r = \frac{mv^2}{T\sin 55^\circ} = \frac{(3.50)(6.50)^2}{59.9 \sin 55^\circ}$$

$$= 3.01\ m$$

$$\sin 55 = \frac{r}{L}, \quad L = \frac{r}{\sin 55^\circ}$$

$$L = \frac{3.01}{\sin 55^\circ} = 3.68\ m$$

4. A block with mass of 3.50 kg slides along a frictionless, horizontal surface with a constant speed of 17.5 m/s. It encounters a rough patch and after sliding a distance of 20.0 m, its speed is reduced to 15.0 m/s.

a. Use Conservation of Energy to find the coefficient of kinetic friction of the rough patch. (**30**)

$$\frac{1}{2}m(17.5)^2 = \frac{1}{2}m(15.0)^2 + f_k \underset{20}{\Delta s}$$

$$f_k = \mu_k F_n = \mu_k mg$$

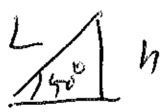
$$\cancel{m}(17.5)^2 = \cancel{m}(15.0)^2 + 2\mu_k \cancel{m}g(20)$$

$$81.25 = 2\mu_k g(20)$$

$$\mu_k = 0.207$$

b. The block then slides up a frictionless ramp that makes an angle of 50.0° with respect to the horizontal. After sliding on the ramp for 2.50 m, the block hits a spring. The block compresses the spring by 0.500 m until the block stops (momentarily). Find the spring constant of the spring. (**30**)

$$\frac{1}{2}m(15.0)^2 = mgh + \frac{1}{2}k(.50)^2$$



$$\sin 50^0 = \frac{h}{L}, \quad h = L\sin 50^0$$

$$h = (2.50 + .5)\sin 50^0 = 2.30\ m$$

$$m(15.0)^2 = 2mg(2.30) + k(.50)^2$$

$$629 = k(.5)^2$$

$$k = 2.52\times10^3\ \frac{N}{m}$$

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5. A block (A) of mass 7.50 kg is traveling on a frictionless surface in the +x-direction with a speed of 15.5 m/s.

a. Write the linear momentum of block A in vector notation. (**10**)

$$\vec{p} = (7.50)(15.5)\,\hat{i}$$
$$= 116\,\hat{i}\ \mathrm{kg}\,\frac{m}{s}$$

b. Block A suddenly explodes into 3 pieces: B (mass 2.00 kg); C (mass 3.00 kg), and D. Piece B travels in the -x direction with a speed of 11.5 m/s; piece C travels in the +y direction with a speed of 16.5 m/s. Find the velocity of piece D and write it in vector notation. (**40**)

$$\vec{p}_B = -(2.00)(11.5)\,\hat{i} = -23.0\,\hat{i}\ \mathrm{kg}\,\frac{m}{s}$$

$$\vec{p}_C = (3.00)(16.5)\,\hat{j} = 49.5\,\hat{j}\ \mathrm{kg}\,\frac{m}{s}$$

$$\vec{p}_D = (2.50)(v_{Dx} + v_{Dy})$$

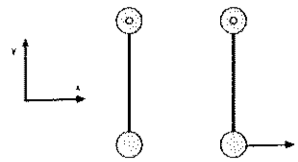
$$x: \quad 116 = -23.0 + 2.50\,v_{Dx}$$

$$v_{Dx} = 55.6\ m/s$$

$$y: \quad 0 = 49.5 + (2.50)\,v_{Dy}$$

$$v_{Dy} = -19.8\ m/s$$

$$\vec{v}_D = (55.6\,\hat{i} - 19.8\,\hat{j})\ \frac{m}{s}$$



6. A barbell consists of a rod of length 2.50 m and mass 1.50 kg and two balls of mass 0.750 kg at the ends of the rod (treat the balls as point masses). The barbell is pinned through one ball as shown above. Initially the barbell is hanging vertically and is at rest. Take the z-direction as out of the paper.

a. Find the moment of inertia of the barbell/ball system about the pivot point. (**10**)

$$I = \frac{1}{3}ML^2 + mL^2 = \left(\frac{1}{3}(1.50) + 0.750\right)(2.50)^2$$

$$= 7.81 \text{ kg m}^2$$

At time t=0 the barbell explodes such that the ball at the end flies off in the horizontal, +x-direction with an initial speed of 4.00 m/s.

b. Find the angular momentum of the ball/barbell system just after the explosion. (**10**)

$$0$$

c. Find the angular momentum <u>in vector notation</u> of the flying ball just after the explosion. (**15**)

$$\vec{L} = \vec{r} \times \vec{p} = rmv\hat{k}$$

$$= (2.50)(0.750)(4.00)\hat{k}$$

$$= 7.50\hat{k} \text{ kg} \frac{m^2}{s}$$

d. Find the angular velocity (<u>in vector notation</u>) of the remaining pieces of the barbell right after the explosion. (**15**)

$$0 = 7.50 + L_{rest}$$

$$L = -7.50 = -I'\omega$$

$$I' = \frac{1}{3}ML^2 = \frac{1}{3}(1.50)(2.50)^2 = 3.125$$

$$\omega = \frac{7.50}{3.125} = 2.40 \frac{rad}{s}$$

$$\vec{\omega} = -2.40\hat{k} \frac{rad}{s}$$