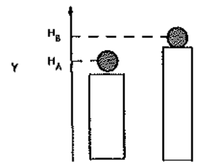
Your Name: ______________________

PHY203
Final Exam
Chapters 1-11,15
Fri., 12/12, 3-5 pm

- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use $g = 9.81\ m/s^2$

Solutions

Final1S23Makeup F25



1. Two stones are initially at rest on cliffs of heights H_A=95.0m (stone A) and H_B=150 (stone B). At t=0, stone A is dropped straight down. At t=0, stone B is thrown down with an initial speed of 15.0 m/s. Take the y-axis as positive up and 0 at ground level.

a. Fill out the tables of known quantities for the two stones:

Stone A: Stone B:

Parameter	Known Value	Parameter	Known Value
x_0	95.0 m	x_0	150 m
x_f		x_f	
v_0	0	v_0	−15.0 m/s
v_f		v_f	
a	−9.81 m/s²	a	−9.81 m/s²
t		t	

b. Write the equation for the position of stone A as a function of time: **15**

$y_A = \quad 95.0 - \frac{1}{2} g t^2$

c. Write the equation for the position of stone B as a function of time: **15**

$y_B = \quad 150 - 15.0t - \frac{1}{2} g t^2$

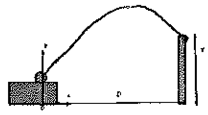
d. Find the time + position when the stones are side-by-side. **20**

$$95 - \cancel{\frac{1}{2} g t^2} = 150 - 15t - \cancel{\frac{1}{2} g t^2}$$

$$15t = 55$$

$$t = \frac{55}{15} = 3.47 s$$

$$y_A = 95.0 - \frac{1}{2} g (3.47)^2 = 28.9 \text{ m}$$



2. A cannonball is shot from a cliff of height H = 115 m at a castle wall. The castle wall is Y = 195 m high and a horizontal distance D = 205 m from the cannon. Take y = 0 at ground level. Assume the ball just misses the top of the wall on the way down. Assume the takeoff angle is 45.0° with respect to the horizontal direction.

a. Make a sketch of the trajectory of the ball from start until it grazes the wall. **5**

b. Fill out the tables of known values. (Take the "final" position as when the ball grazes the top of the wall.)

Parameter	Known Value
x_0	0
x_f	205 m
v_{x0}	$V_0 \cos 45^\circ$
v_{xf}	u
a_x	0
t	

Parameter	Known Value
y_0	115 m
y_f	195 m
v_{y0}	$V_0 \sin 45^\circ$
v_{yf}	
a_y	$-9.81\ m/s^2$
t	

c. Find the initial velocity of the cannonball in vector notation using the coordinate system above. **15**

x: $205 = V_0 \cos 45^\circ t$ } same

y: $195 = 115 + V_0 \sin 45^\circ t - \frac{1}{2} g t^2$

$195 = 115 + 205 - \frac{1}{2} g t^2$ $\boxed{\vec{V_0} = 40.6(\hat{i} + \hat{j})\ m/s}$

$\frac{1}{2} g t^2 = 125, \quad t = 5.05\,s$

x: $205 = V_0 \cos 45^\circ (5.05), \quad V_{0x} = V_{0y} = V_0 \cos 45^\circ = 40.6$

d. Find the acceleration, velocity, and position of the cannonball in vector notation at its highest point. **15** $\vec{a} = -9.81\,\hat{j}\ m/s^2, \quad \vec{v} = +40.6\,\hat{i}\ m/s$

y: $0 = 40.6 - g t, \quad t = 4.14\,s$

$x = 40.6(4.14) = 168$

y: $0 = 40.6^2 - 2g\Delta y, \quad \Delta y = 84.0$

$+115$

199

$\vec{r} = (168\,\hat{i} + 199\,\hat{j})\ m$

e. Find the 2 times the ball is at the height of the castle wall. **15**

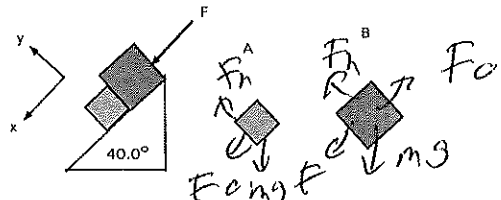
y: $195 = 115 + 40.6t - \frac{1}{2} g t^2$

$4.905t^2 - 40.6t + 80 = 0$ → 8.87

$t = \dfrac{40.6 \pm \sqrt{40.6^2 - 4 \cdot 80 \cdot 4.905}}{9.81}$

$= 3.23\,s, \ 5.04\,s$

Final2S25Makeup



3. Blocks A and B are being held at rest on a smooth ramp and then released and they slide down the ramp in contact with each other and with a force pushing on block B parallel to the ramp. The ramp makes an angle of 40.0° with respect to the horizontal.

a. Draw free body diagrams on the blocks shown above and on the right. **5**

b. Write out Newton's 2nd Law for both blocks in the x- and y-directions. **30**

A: x: $F_c + m_A g \sin 40^\circ = m_A a$

y: $F_{nA} - m_A g \cos 40^\circ = 0$

B: x: $F - F_c + m_B g \sin 40^\circ = m_B a$

y: $F_{nB} - m_B g \cos 40^\circ = 0$

c. Assuming that M_A = 5.50 kg, M_B = 8.50 kg, and F = 55.0 N, find the magnitude of the acceleration of the blocks and the magnitude of the contact force between the blocks. **15**

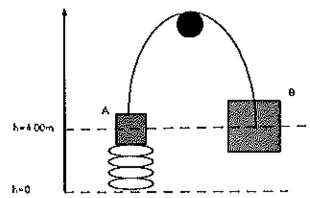
Add: $F + (m_A + m_B) g \sin 40^\circ = (m_A + m_B) a$

$$a = \frac{F}{m_A + m_B} + g \sin 40^\circ$$

$$= \frac{55}{14} + g \sin 40^\circ = 10.2\ m/s^2$$

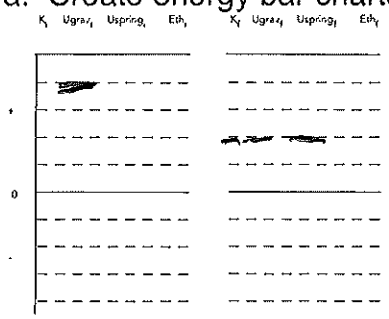
A x: $F_c = m_A(a - g \sin 40^\circ)$

$= 5.5(10.2 - g \sin 40^\circ) = 21.4\ N$



4. Two blocks with masses m_A = 5.00 kg and m_B = 9.50 kg are connected by a light string and suspended over a massless pulley. Assume the initial height of both blocks is h= 4.00 m. Block A is attached to a spring with spring constant k = 20.0 N/m. At the start of the problem the spring is unstretched. The blocks are released. We want to find the speed of the blocks after they have traveled 3.00 m.

a. Create energy bar charts: **10**



b. Find the initial energy of the blocks before they start moving. **10**

$$U_g = (5 + 9.5)(4)g = 569 J$$

c. Find the speed of the blocks after they have traveled 3.00 m. **30**

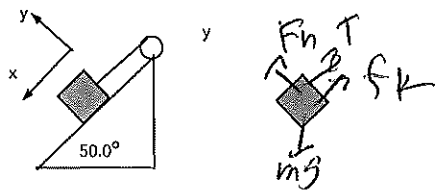
$$569 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kx^2 + g(m_A \cdot 7 + m_B \cdot 1)$$

$$569 = \frac{1}{2}(14.5)v^2 + \frac{1}{2}(20)(3)^2 + g(35 + 9.5)$$

$$= 7.25v^2 + 90 + 436.5$$

$$v = 2.42 m/s$$

Final3MakeupS25



5. A block on a rough ramp is attached to a light string wrapped around a pulley. Given m for the block and M and R for the pulley (a solid disk). Assume the block starts from rest at a height of 5.00 m. We want to find the acceleration of the block on the ramp.

a. Draw a free body diagram for the block above and to the right. **5**

b. Write out Newton's 2nd Law the block in all directions and the torque equation for the pulley. **30**

x: $mg\sin 50^0 - T - \mu_k F_n = ma$

y: $F_n - mg\cos 50^0 = 0$

τ: $TR = I\alpha = I\frac{a}{R} = \frac{1}{2}M\cancel{R^2}\frac{a}{\cancel{R}}$

$T = \frac{1}{2}Ma$

Given m=5.50 kg, M_{Pulley}=4.00 kg, R=0.500 m, and μ_k= 0.350.
c. Find the magnitude of the acceleration of the block. **15**

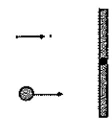
$mg\sin 50^0 - T - \mu_k mg\cos 50^0 = ma$

Sub. for T: $mg\sin 50^0 - \frac{1}{2}Ma - \mu_k mg\cos 50^0 = ma$

$mg(\sin 50^0 - \mu_k\cos 50^0) = (m + \frac{1}{2}M)a$

$a = \frac{1}{7.5}[5.5g(\sin 50^0 - .35\cos 50^0)]$

$= 3.89\ m/s^2$



6. A thin rod of mass 1.50 kg and length 2.00 m is initially at rest on a horizontal frictionless surface and is pinned through its center by a frictionless shaft. (A top view is shown above.) A small ball of mass 0.500 kg is initially traveling with a speed of 12.5 m/s. It hits the rod and sticks to it at a distance of 0.800 m from the rod's center.

a. Find the moment of inertia of the rod about its pivot point point. **5**

$$I = \frac{1}{12} ML^2 = \frac{1}{12}(1.50)(2.00)^2 = 0.500 \text{ kg m}^2$$

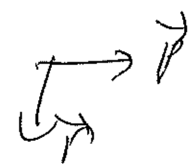
b. Find the linear momentum of the ball before the collision. **5**

$$\vec{p} = (0.500)(12.5) = 6.25 \text{ kg m/s } \hat{i}$$

c. Find the angular momentum of the ball about the rod's pivot point before the collision. Take +z as the direction out of the paper. **10**

$$L = rp\sin\theta = (.8)(6.25)(1) = 5.00$$

rt hand rule $\vec{L} = 5.00 \frac{\text{kg m}^2}{\text{s}} \hat{k}$



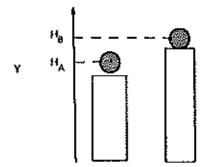
d. Find the angular speed of the rod/ball system after the collision. **20**

$$5.00 = (I_{rod} + I_{ball})\,\omega$$
$$= (0.500 + .5(.8)^2)\,\omega$$
$$= (0.500 + 0.32)\,\omega = .82\,\omega$$
$$\omega = 6.10 \text{ rad/s}$$

e. Find how many revolutions the rod/ball system makes in 45.0 s after the collision. **10**

$$\theta = 0 + \omega_0 t + 0 = (6.10)(45)$$
$$= \frac{274 \text{ rad}}{2\pi} = 43.7 \text{ rev}$$

PHY203FinalF25alt



1. Two stones are initially at rest on cliffs of heights H_A=85.0m (stone A) and H_B=125 (stone B). At t=0, stone A is dropped straight down. At t=0, stone B is thrown down with an initial speed of 17.0 m/s. Take the y-axis as positive up and 0 at ground level.

a. Fill out the tables of known quantities for the two stones:

Stone A: Stone B:

Parameter	Known Value	Parameter	Known Value
x_0	85.0 m	x_0	125 m
x_f		x_f	
v_0	0	v_0	−17.0 m/s
v_f		v_f	
a	$-9.81\ m/s^2$	a	$-9.81\ m/s^2$
t		t	

b. Write the equation for the position of stone A as a function of time: **15**

$y_A = 85.0 - \frac{1}{2}gt^2 = 85.0 - 4.90t^2$

c. Write the equation for the position of stone B as a function of time: **15**

$y_B = 125 - 17.0t - \frac{1}{2}gt^2 = 125 - 17t - 4.90t^2$

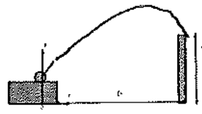
d. Find the time and the position when the stones are side-by-side. **20**

$$85 - \cancel{\frac{1}{2}gt^2} = 125 - 17t - \cancel{\frac{1}{2}gt^2}$$

$$17t = 40$$

$$t = \frac{40}{17} = 2.35\ s$$

$$y_A = 85 - \frac{1}{2}g(2.35)^2 = 57.9\ m$$



2. A cannonball is shot from a cliff of height H = 135 m at a castle wall. The castle wall is Y = 175 m high and a horizontal distance D = 225 m from the cannon. Take y = 0 at ground level. Assume the ball just misses the top of the wall on the way down. Assume the takeoff angle is 45.0° with respect to the horizontal direction.

a. Make a sketch of the trajectory of the ball from start until it grazes the wall. **5**

b. Fill out the tables of known values. (Take the "final" position as when the ball grazes the top of the wall.)

Parameter	Known Value
x_0	0
x_f	225 m
v_{x0}	$V_0 \sin 45°$
v_{xf}	''
a_x	0
t	

Parameter	Known Value
y_0	135 m
y_f	175 m
v_{y0}	$V_0 \sin 45°$
v_{yf}	
a_y	-9.81 m/s^2
t	

$v_{x0} = v_{y0}$
$= V_0 \sin 45°$

c. Find the initial velocity of the cannonball in vector notation using the coordinate system above. **15**

x: $225 = 0 + v_0 \sin 45° t$

y: $175 = 135 + v_0 \sin 45° t - \frac{1}{2} g t^2$

$175 = 135 + 225 - \frac{1}{2} g t^2$

$\frac{1}{2} g t^2 = 185, \quad t = 6.14 s$

$v_0 \sin 45° = \frac{225}{6.14} = 36.6 \quad \vec{v} = (36.6 \hat{i} + 36.6 \hat{j}) \text{ m/s}$

d. Find the acceleration, velocity, and position of the cannonball in vector notation at its highest point. **15**

$\vec{a} = -9.81 \text{ m/s}^2 \hat{j}, \quad \vec{v} = 36.6 \text{ m/s } \hat{i}$

y: $0 = 36.6 - gt, \quad t = 3.73$

x: $x = 0 + 36.6(3.73) = 137$

y: $0 = 36.6^2 - 2g\Delta y, \quad \Delta y = 68.3$

$+135$

203

$\vec{r} = (137\hat{i} + 203\hat{j}) \text{ m}$

e. Find the 2 times the ball is at the height of the castle wall. **15**

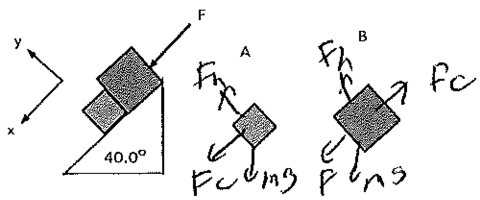
y: $175 = 135 + 36.6t - \frac{1}{2} g t^2$

$4.905t^2 - 36.6t + 40 = 0$

23.6

$t = \frac{36.6 \pm \sqrt{36.6^2 - 4 \cdot 40 \cdot 4.905}}{9.81}$

$= 1.34 s$ or $6.13 s$



3. Blocks A and B are being held at rest on a smooth ramp and then released and they slide down the ramp in contact with each other and with a force pushing on block B parallel to the ramp. The ramp makes an angle of 40.0° with respect to the horizontal.

a. Draw free body diagrams on the blocks shown above and on the right. **5**

b. Write out Newton's 2nd Law for both blocks in the x- and y-directions. **30**

A: x: $F_c + m_A g \sin 40^\circ = m_A a$

y: $F_n - m_A g \cos 40^\circ = 0$

B: x: $F - F_c + m_B g \sin 40^\circ = m_B a$

y: $F_n - m_B g \cos 40^\circ = 0$

c. Assuming that M_A = 4.50 kg, M_B = 9.50 kg, and F = 60.0 N, find the magnitude of the acceleration of the blocks and the magnitude of the contact force between the blocks. **15**

combine: $F + (m_A + m_B) g \sin 40^\circ = (m_A + m_B) a$

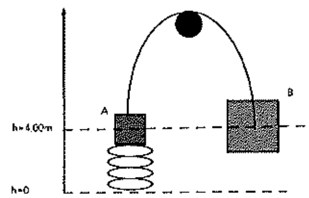
$$a = \frac{F}{(m_A + m_B)} + g \sin 40^\circ$$

$$= \frac{60}{14} + g \sin 40^\circ = 10.6 \text{ m/s}^2$$

A: x $F_c = m_A (a - g \sin 40^\circ)$

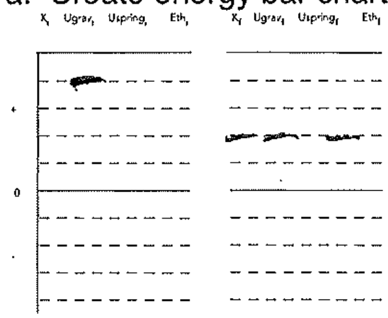
$= 4.5(10.6 - g \sin 40^\circ)$

$= 19.3$ N



4. Two blocks with masses m_A = 4.00 kg and m_B = 7.50 kg are connected by a light string and suspended over a massless pulley. Assume the initial height of both blocks is h= 4.00 m. Block A is attached to a spring with spring constant k = 15.0 N/m. At the start of the problem the spring is unstretched. The blocks are released. We want to find the speed of the blocks after they have traveled 3.00 m.

a. Create energy bar charts: **10**



b. Find the initial energy of the blocks before they start moving. **10**

$U = (4.00 + 7.50) g (4.00) = 451 J$

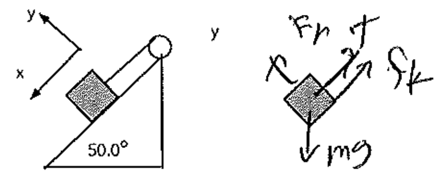
c. Find the speed of the blocks after they have traveled 3.00 m. **30**

$451 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}k(3)^2 + g(4\cdot 7 + 7.5\cdot 1)$

$= 5.75 v^2 + 67.5 + 348$

$5.75 v^2 = 35.2$

$v = 2.48$ m/s



5. A block on a rough ramp is attached to a light string wrapped around a pulley. Given m for the block and M and R for the pulley (a solid disk). Assume the block starts from rest at a height of 4.00 m. We want to find the acceleration of the block on the ramp.

a. Draw a free body diagram for the block above and to the right. **5**

b. Write out Newton's 2nd Law the block in all directions and the torque equation for the pulley. **30**

x: $mg\sin 50^\circ - T - \mu_k F_n = ma$

y: $F_n - mg\cos 50^\circ = 0$

τ: $TR = I\alpha = \frac{1}{2} M R^{\cancel{2}} \frac{a}{\cancel{R}}$

Given m=6.50 kg, M_{Pulley}=3.00 kg, R=0.500 m, and μ_k= 0.250.

c. Find the magnitude of the acceleration of the block. **15**

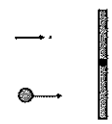
$mg\sin 50^\circ - \cancel{T} - \mu_k mg\cos 50^\circ = ma$

$\cancel{T} = \frac{1}{2} M a$

$mg(\sin 50^\circ - \mu_k \cos 50^\circ) = \left(m + \frac{M}{2}\right) a$

$$a = \frac{6.5g(\sin 50^\circ - .250\cos 50^\circ)}{6.50 + \frac{3.00}{2}}$$

$= 4.82\ m/s^2$



6. A thin rod of mass 1.80 kg and length 2.00 m is initially at rest on a horizontal frictionless surface and is pinned through its center by a frictionless shaft. (A top view is shown above.) A small ball of mass 0.600 kg is initially traveling with a speed of 13.5 m/s. It hits the rod and sticks to it at a distance of 0.800 m from the rod's center.

a. Find the moment of inertia of the rod about its pivot point point. **5**

$$I = \frac{1}{12} M L^2 = \frac{1}{12}(1.80)(2)^2 = 0.600 \text{ kg m}^2$$

b. Find the linear momentum of the ball before the collision. **5**

$$\vec{p} = (.600)(13.5) = 8.10 \text{ kg m/s } \hat{i}$$

c. Find the angular momentum of the ball about the rod's pivot point before the collision. Take +z as the direction out of the paper. **10**

$$L = r p \sin\theta = (.800)(8.10)(1) = 6.48$$

rt hand rule $\quad \vec{L} = 6.48 \text{ kg m}^2/\text{s } \hat{k}$ $\quad \vec{p}$ $\vec{r}$

d. Find the angular speed of the rod/ball system after the collision. **20**

$$6.48 = (I_{rod} + I_{ball})\omega$$
$$= (0.600 + .600(.8)^2)\omega$$
$$= (0.600 + .384)\omega = .984\omega$$
$$\omega = 6.59 \text{ rad/s}$$

e. Find how many revolutions the rod/ball system makes in 55.0 s after the collision. **10**

$$\theta = 0 + \omega t + 0 = 6.59(55)$$
$$= \frac{362 \text{ rad}}{2\pi} = 57.6 \text{ rev}$$