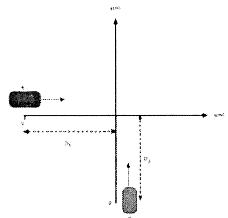
Your Name: ______________________

PHY203
Final Exam
Chapters 1-11,15
~~**????????**~~
Mon., Dec. 16, 2024

- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use g = 9.81 m/s^2

Solutions

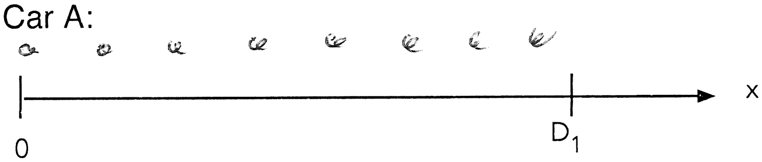
MakeupFinalS24



1. Two cars travel on perpendicular tracks. At a time of t=0, car A passes the x=0 point with a constant speed of 18.5 m/s. At a distance of D_1 = 195 m car A crosses the track that car B travels on. Car B passes y=0 at t = 0 at a distance of D_2 from the point at which the roads cross. Car B is initially traveling in the +y direction with ~~a constant~~ speed of 5.00 m/s and at t = 0 accelerates at 3.00 m/s^2.

a. Produce motion diagrams of the cars above the axes. (Assume the cars crash and stop at the intersection.) **10**

Car A:



Car B:

b. Fill out the tables of known quantities for the two cars. Take the final position as when they crash at the intersection.

Parameter (Car A)	Known Value	Parameter (Car B)	Known Value
x_0	0	y_0	0
x_f	195 m	yf	D_2
v_0	18.5 m/s	v_0	5.00 m/s
v_f	18.5 m/s	v_f	
a	0	a	3.00 m/s^2
t		t	

b. Using the coordinate system depicted above, write an equation of motion (x vs. t) for car A: **10**

x_A= $18.5t$

c. Using the coordinate system depicted above, write an equation of motion (y vs. t) for car B: **15**

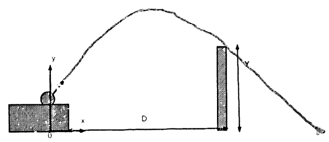
y_B= $5.00t + \frac{1}{2}(3.00)t^2 = 5.00t + 1.50t^2$

d. Find the distance, D_2, such that the centers of the cars collide. (Hint: the distances traveled are different so you can't solve this by setting $x_A = y_B$.) **15**

$195 = 18.5t$, $t = 10.5$ s

$D_2 = 5.00(10.5) + 1.50(10.5)^2$

$= 219$ m



2. A cannonball is shot from a cliff of height H = 25.0 m at a castle wall, as shown above. The castle wall is Y = 85.0 m high and a horizontal distance D = 225 m from the cannon. The initial vertical component of velocity of the ball is 55.0 m/s in magnitude. Take y = 0 at ground level. Assume the ball just grazes the top of the wall on the way down. (Ignore air resistance.)

a. On the figure above sketch above and the trajectory of the ball from start until it hits the ground. **5**

b. Fill out the tables of known values.(Take the "final" position when the ball grazes the wall.)

Parameter	Known Value
x_0	0
x_f	225 m
v_{x0}	
v_{xf}	
a_x	0
t	

Parameter	Known Value
y_0	25.0 m
y_f	85.0 m
v_{y0}	55.0 m/s
v_{yf}	
a_y	$-9.81 m/s^2$
t	

c. Find the initial velocity of the cannonball and write it in vector notation using the coordinate system above. **20**

$y: \quad 85 = 25 + 55t - \frac{1}{2}gt^2$

$4.905t^2 - 55t + 60 = 0$

$t = \frac{55 \pm \sqrt{55^2 - 4 \cdot 60 \cdot 4.905}}{9.81} = 9.99s$

$x: \quad 225 = v_0 t(9.99), \quad v_{0x} = 22.5 m/s$

$\vec{v_0} = (22.5\hat{i} + 55.0\hat{j})\ m/s$

d. Find the velocity and acceleration of the cannonball at its highest point in vector notation. **10**

$\vec{v} = 22.5\hat{i}\ m/s, \quad \vec{a} = -9.81\hat{j}\ m/s^2$

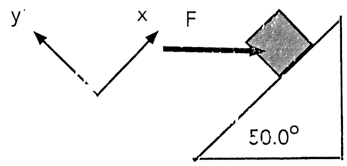
e. Find the range of the cannonball, assuming it missed the wall. **15**

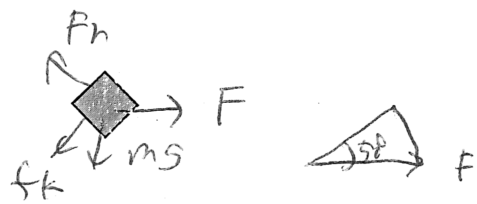
$y: \quad 0 = 25 + 55t - \frac{1}{2}gt^2$

$4.905t^2 - 55t - 25 = 0$

$t = \frac{55 \pm \sqrt{55^2 + 4 \cdot 25 \cdot 4.905}}{9.81} = 11.7s$

$x: \quad Range = 22.5(11.7) = 262 m$





3. A block on a <u>rough</u> ramp is pushed with a horizontal force, F.

a. Above and to the right draw a free body diagram of the block while it is sliding up the ramp. **5**

b. Write out Newton's 2nd Law for the block in both directions. **30**

x: $F\cos 50^\circ - mg\sin 50^\circ - \mu_k F_n = ma$

y: $F_n - mg\cos 50^\circ - F\sin 50^\circ = 0$

Assume m=1.50 kg, F=35.0 N, and μ_k= 0.100.

c. Find the magnitude of the normal force on the block. **5**

$$F_n = (1.50)g\cos 50^\circ + 35\sin 50^\circ$$

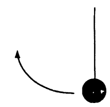
$$= 9.46 + 26.8 = 36.3\ N$$

d. Find the magnitude of the acceleration of the block. **10**

$$a = \frac{1}{1.50}\left[35\cos 50^\circ - 1.50g\sin 50^\circ - (.1)(36.3)\right]$$

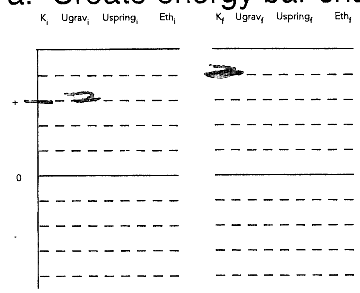
$$= \frac{1}{1.50}\left[22.5 - 11.3 - 3.63\right]$$

$$= 5.07\ m/s^2$$



4. A ball of mass 2.50 kg is attached to the end of a string of length L=1.50 m and is given a push so that it swings around in a vertical circle in a clockwise direction. The speed of the ball at the top of the circle is 9.50 m/s. We want to find the speed of the ball at the bottom of the circle and the tension in the string. Take initial position at top; $y = 0$ at bottom.

a. Create energy bar charts: **5**



b. Find the speed of the ball at the bottom of the circle. **20**

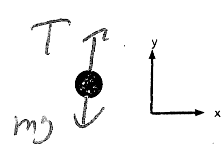
$$\frac{1}{2}\cancel{m}v_T^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$$

$$v_B^2 = v_T^2 + 2gh$$

$$= (9.50)^2 + 2g(3.00) = 149$$

$$v_B = 12.2 \text{ m/s}$$

c. On the figure below, draw a free body diagram of the ball when the ball is at the bottom.**5**



d. Write out Newton's 2nd Law in the y-direction for the ball at the bottom and find the magnitude of the tension in the string. **20**

$$T - mg = ma = \frac{mv^2}{r}$$

$$T = m\left(\frac{v^2}{r} + g\right)$$

$$= 2.50\left(\frac{12.2^2}{1.50} + g\right)$$

$$= 273 \text{ N}$$

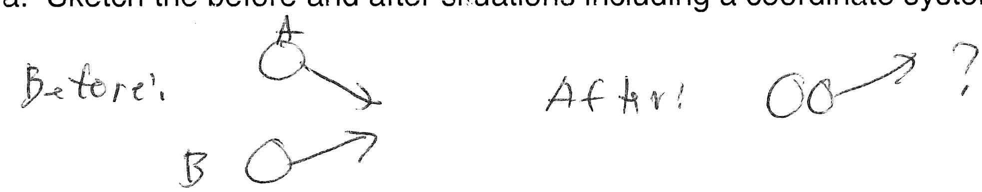
5. On a horizontal, frictionless table a puck (A) of mass 1.50 kg is traveling with a velocity given by:

$$v_A = (2.00m/s)\hat{i} - (5.50m/s)\hat{j}$$

it strikes and sticks to a puck (B) of mass 2.25 kg initially (before the collision) traveling with a velocity given by:

$$v_B = (1.50m/s)\hat{i} + (2.50m/s)\hat{j}$$

a. Sketch the before and after situations including a coordinate system. **10**



b. List the known quantities before and after the collision:

Parameter	Known Value
M_A	1.50 kg
V_A	$(2\hat{i} - 5.5\hat{j})$ m/s
M_B	2.25 kg
V_B	$(1.5\hat{i} + 2.5\hat{j})$ m/s

Parameter	Known Value
M_{AB}	3.75 kg

c. Find the momenta of pucks A and B separately before the collision in vector notation. **10**

$$\vec{p_A} = (1.50)(2\hat{i} - 5.5\hat{j}) = (3.00\hat{i} - 8.25\hat{j}) \text{ kg m/s}$$

$$\vec{p_B} = (2.25)(1.5\hat{i} + 2.5\hat{j}) = (3.38\hat{i} + 5.62\hat{j}) \text{ kg m/s}$$

d. Find the velocity of the 2-puck combination after the collision in vector notation. **15**

$$\vec{p_A} + \vec{p_B} = 6.38\hat{i} - 2.62\hat{j} = (3.75)\vec{v_f}$$

$$\vec{v_f} = \frac{1}{3.75}(6.38\hat{i} - 2.62\hat{j}) = (1.70\hat{i} - 0.700\hat{j}) \text{ kg m/s}$$

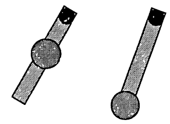
e. Determine quantitatively if the collision was elastic or inelastic. **15**

$$K_i = \frac{1}{2}(1.5)[2^2 + 5.5^2] + \frac{1}{2}(2.25)[1.5^2 + 2.5^2]$$

$$= 25.7 + 9.56 = 35.3 \text{ J}$$

$$K_f = \frac{1}{2}(3.75)[1.70^2 + 0.700^2] = 6.34 \text{ J}$$

$$K_i \neq K_f \Rightarrow \text{inelastic collision}$$



6. A rod of mass 1.50 kg and length 2.00 m has a small ball of mass 0.500 kg (treat it as a point mass) attached to the midpoint of the rod. The rod/ball is initially swinging with an angular speed of 7.50 rad/s about the end of the rod in a clockwise direction.

a. List the known quantities below:

Parameter	Known Value
L_{rod}	2.00 m
M_{rod}	1.50 kg
M_{ball}	0.500 kg
ω_o	7.50 rad/s

b. Find the moment of inertia of the rod/ball combination and the angular momentum (in <u>vector notation</u>) of the rod/ball with respect to the pivot point. Take the +z direction as out of the paper. **25**

$$\vec{L} = I\vec{\omega} = -I(7.50)\hat{k}$$

$$I = \frac{1}{3}(1.50)(2.00)^2 + (0.500)(1.00)^2$$

$$= 2.00 + .500 = 2.50 \text{ kg m}^2$$

$$\vec{L} = -(2.50)(7.50)\hat{k} = -18.8 \text{ kg} \frac{\text{m}}{\text{s}} \hat{k}$$

Now the ball is released and slides to the end of the rod as shown above.

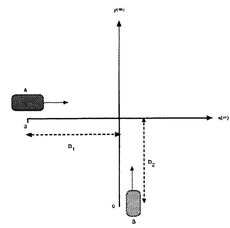
c. Find the new angular speed of the rod/ball system. **25**

$$18.8 = I_f \omega_f$$

$$I_f = 2.00 + (0.500)(2.00)^2$$

$$= 2.00 + 2.00 = 4.00$$

$$\omega_f = \frac{18.8}{4.00} = 4.69 \text{ rad/s}$$



1. Two cars travel on perpendicular tracks. At a time of t=0, car A passes the x=0 point with a constant speed of 20.5 m/s. At a distance of D_1 = 175 m car A crosses the track that car B travels on. Car B passes y=0 at t = 0 at a distance of D_2 from the point at which the roads cross. Car B is initially traveling in the +y direction with a constant speed of 5.00 m/s and at t = 0 accelerates at 3.50 m/s^2.

a. Produce motion diagrams of the cars above the axes. (Assume the cars crash and stop at the intersection.) **10**

Car A:

0 D_1 x

Car B:

0 D_2 y

b. Fill out the tables of known quantities for the two cars. Take the final position as when they crash at the intersection.

Parameter (Car A)	Known Value	Parameter (Car B)	Known Value
x_0	0	y_0	0
x_f	175 m	yf	D_2
v_0	20.5 m/s	v_0	5.00 m/s
v_f	20.5 m/s	v_f	
a	0	a	3.50 m/s^2
t		t	

b. Using the coordinate system depicted above, write an equation of motion (x vs. t) for car A: **10**

x_A= $20.5t$

c. Using the coordinate system depicted above, write an equation of motion (y vs. t) for car B: **15**

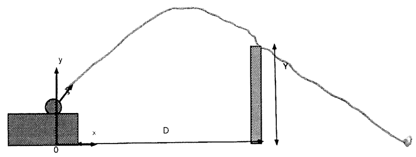
y_B= $5.00t + \frac{1}{2}(3.50)t^2 = 5.00t + 1.75t^2$

d. Find the distance, D_2, such that the centers of the cars collide. (Hint: the distances traveled are different so you can't solve this by setting $x_A = y_B$.) **15**

x: $175 = 20.5t, \quad t = 8.54\text{ s}$

y: $D_2 = 5.00(8.54) + 1.75(8.54)^2$

$= 170\text{ m}$



2. A cannonball is shot from a cliff of height H = 35.0 m at a castle wall, as shown above. The castle wall is Y = 80.0 m high and a horizontal distance D = 205 m from the cannon. The initial vertical component of velocity of the ball is 50.0 m/s in magnitude. Take y = 0 at ground level. Assume the ball just grazes the top of the wall on the way down. (Ignore air resistance.)

a. On the figure above sketch above and the trajectory of the ball from start until it hits the ground. **5**

b. Fill out the tables of known values.(Take the "final" position when the ball grazes the wall.)

Parameter	Known Value
x_0	0
x_f	205 m
v_{x0}	
v_{xf}	
a_x	0
t	

Parameter	Known Value
y_0	35.0 m
y_f	80.0 m
v_{y0}	50.0 m/s
v_{yf}	
a_y	$-9.81\ m/s^2$
t	

c. Find the initial velocity of the cannonball and write it in <u>vector notation</u> using the coordinate system above. **20**

y: $80 = 35 + 50t - \frac{1}{2}gt^2$

$4.905t^2 - 50t + 45 = 0$

$t = \dfrac{50 \pm \sqrt{50^2 - 4 \cdot 45 \cdot 4.905}}{9.81} = 9.20\ s$

x: $205 = v_{0x}(9.20)$ $v_{0x} = 22.3\ m/s$

$\vec{v}_0 = (22.3\hat{i} + 50.0\hat{j})\ m/s$

d. Find the velocity and acceleration of the cannonball at its highest point in vector notation. **10**

$\vec{a} = -9.81\hat{j}\ m/s^2$ $\vec{v} = 22.3\hat{i}\ m/s$

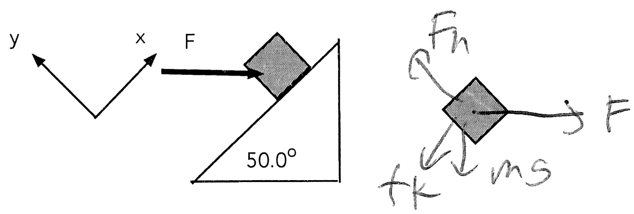
e. Find the range of the cannonball, assuming it missed the wall. **15**

y: $0 = 35.0 + 50t - \frac{1}{2}gt^2$

$4.905t^2 - 50t - 35 = 0$

$t = \dfrac{50 \pm \sqrt{50^2 + 4 \cdot 35 \cdot 4.905}}{9.81} = 10.85\ s$

$R = x = 22.3(10.85) = 242\ m$



3. A block on a rough ramp is pushed with a horizontal force, F.

a. Above and to the right draw a free body diagram of the block while it is sliding up the ramp. **5**

b. Write out Newton's 2nd Law for the block in both directions. **30**

x: $F\cos 50° - mg\sin 50° - \mu_k F_n = ma$

y: $F_n - mg\cos 50° - F\sin 50° = 0$

Assume m=2.50 kg, F=45.0 N, and μ_k= 0.100.

c. Find the magnitude of the normal force on the block. **5**

$F_n = mg\cos 50° + F\sin 50°$

$= 2.5g\cos 50° + 45\sin 50° =$

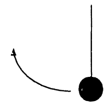
$= 15.76 + 34.47 = 50.2\ N$

d. Find the magnitude of the acceleration of the block. **10**

$a = \frac{1}{2.5}\left[45\cos 50° - 2.5g\sin 50° - .1(50.2)\right]$

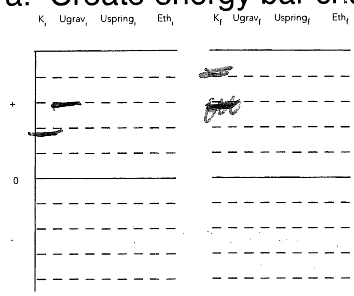
$= \frac{1}{2.5}\left[28.9 - 18.8 - 5.02\right]$

$= 2.04\ m/s^2$



4. A ball of mass 3.50 kg is attached to the end of a string of length L=2.00 m and is given a push so that it swings around in a vertical circle in a clockwise direction. The speed of the ball at the top of the circle is 8.50 m/s. We want to find the speed of the ball at the bottom of the circle and the tension in the string. Take the initial position at the top with y=0 at the bottom.

a. Create energy bar charts: **5**



b. Find the speed of the ball at the bottom of the circle. **20**

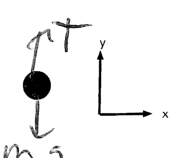
$$\frac{1}{2}\cancel{m}v_T^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$$

$$v_B^2 = v_T^2 + 2gh$$

$$= 8.5^2 + 2g(4)$$

$$v_B = 12.3 \text{ m/s}$$

c. On the figure below, draw a free body diagram of the ball when the ball is at the bottom.**5**



d. Write out Newton's 2nd Law in the y-direction for the ball at the bottom and find the magnitude of the tension in the string. **20**

$$T - mg = ma = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{R}\right) = 3.50\left(g + \frac{12.3^2}{2}\right)$$

$$= 298 \text{ N}$$

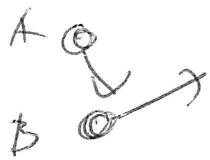
5. On a horizontal, frictionless table a puck (A) of mass 2.50 kg is traveling with a velocity given by:

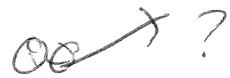
$$v_A = (2.00 m/s)\hat{i} - (5.50 m/s)\hat{j}$$

It strikes and sticks to a puck (B) of mass 3.75 kg initially (before the collision) traveling with a velocity given by:

$$v_B = (1.50 m/s)\hat{i} + (2.50 m/s)\hat{j}$$

a. Sketch the before and after situations including a coordinate system. **10**





b. List the known quantities before collision:

Parameter	Known Value
M_A	2.50 kg
v_A	$(2\hat{i} - 5.5\hat{j})$ m/s
M_B	3.75 kg
V_B	$(1.5\hat{i} + 2.5\hat{j})$ m/s

and after the

Parameter	Known Value
M_{AB}	6.25 kg

c. Find the momenta of pucks A and B separately before the collision in vector notation. **10**

$$\vec{pA} = 2.50(2\hat{i} - 5.5\hat{j}) = (5.00\hat{i} - 13.8\hat{j})\ kg\ m/s$$

9.38

$$\vec{pB} = 3.75(1.5\hat{i} + 2.5\hat{j}) = (5.62\hat{i} + \hat{j})\ kg\ m/s$$

d. Find the velocity of the 2-puck combination after the collision in vector notation. **15**

$$6.25\,\vec{V_{AB}} = (5\hat{i} - 13.8\hat{j}) + (5.62\hat{i} + 9.38\hat{j})$$

$$= (10.62\hat{i} - 4.42\hat{j})$$

$$\vec{V_{AB}} = (1.70\hat{i} - 0.708\hat{j})\ m/s$$

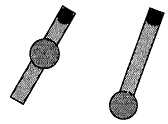
e. Determine <u>quantitatively</u> if the collision was elastic or inelastic. **15**

$$K_i = \frac{1}{2}(2.5)(2^2 + 5.5^2) + \frac{1}{2}(3.75)(1.5^2 + 2.5^2)$$

$$= 42.8 + 15.9 = 58.75$$

$$K_f = \frac{1}{2}(6.25)(1.7^2 + .708^2) = 10.65$$

$58.7 \neq 10.6$ inelastic



6. A rod of mass 2.50 kg and length 3.00 m has a small ball of mass 0.600 kg (treat it as a point mass) attached to the midpoint of the rod. The rod/ball is initially swinging with an angular speed of 8.50 rad/s about the end of the rod in a clockwise direction.

a. List the known quantities below:

Parameter	Known Value
L_{rod}	3.00 m
M_{rod}	2.50 kg
M_{ball}	0.600 kg
ω_o	-8.50 rad/s $\hat{k}$

b. Find the moment of inertia of the rod/ball combination and the angular momentum (in vector notation) of the rod/ball with respect to the pivot point. Take the +z direction as out of the paper. **25**

$$I = \frac{1}{3} M L^2 + m r^2 = \frac{1}{3}(2.50)(3)^2 + (.6)(1.5)^2$$

$$= 8.85 \text{ kg m}^2$$

$$\vec{L} = -\hat{k}(8.85)(8.50)$$

$$= -75.2\,\hat{k} \text{ kg m}^2/\text{s}$$

Now the ball is released and slides to the end of the rod as shown above.

c. Find the new angular speed of the rod/ball system. **25**

$$I_f = \frac{1}{3} M L^2 + m r_f^2 = \frac{1}{3}(2.50)(3)^2 + (.6)(3)^2$$

$$= 12.9 \text{ kg m}^2$$

$$I_f \omega_f = I_i \omega_i = L_i$$

$$\omega_f = \frac{75.2}{12.9} = 5.83 \text{ rad/s}$$