

Your Name: _____

PHY203
Final Exam
5/3/17

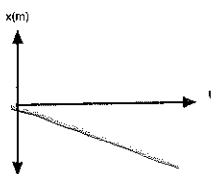
- Show work
- Use correct SI units
- Use scientific notation
- All answers with 3 significant figures
- use $g = 9.81 \text{ m/s}^2$

Solutions

FinalS17Part1

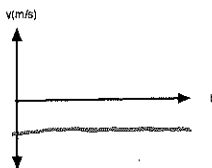
1. On the graphs below, fill in the appropriate sketches assuming 1D motion.

a. Sketch a plot consistent with constant negative velocity.



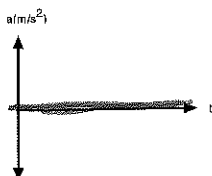
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b. Sketch a plot consistent with constant negative velocity.



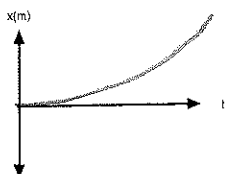
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c. Sketch a plot consistent with constant negative velocity.



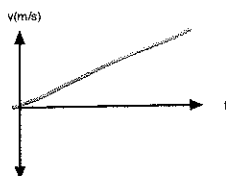
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d. Sketch a plot consistent with constant, positive acceleration.



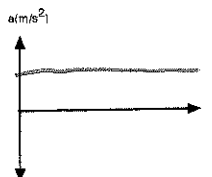
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e. Sketch a plot consistent with constant, positive acceleration.



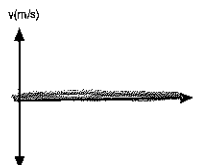
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f. Sketch a plot consistent with constant, positive acceleration.



3

g. Sketch a plot consistent with constant, positive position.



2

2. Two stones, A and B, are thrown straight up (call that the +x direction) from the ground ($x=0$). Stone A is thrown with an initial speed of 22.5 m/s at $t=0$. Stone B is thrown with an initial speed of 17.5 m/s at $t=1.50$ s.

a. Write the equation for the position of stone A as a function of time.

$$x_A = 22.5t - \frac{1}{2}(9.81)t^2 \quad 10$$

b. Write the equation for the position of stone B as a function of time.

$$x_B = 17.5(t - 1.50) - \frac{1}{2}(9.81)(t - 1.50)^2 \quad 15$$

$$(t^2 - 3.00t + 2.25)$$

c. Find the time(s) at which the stones are side-by-side.

$$22.5t - \cancel{4.905t^2} = 17.5t - 26.2 - \cancel{4.905t^2}$$

$$+ 14.7t - 11.0 \quad 10$$

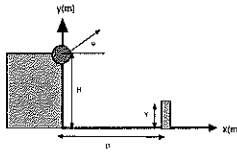
$$9.70t = 37.2$$

$$t = 3.84 \text{ s}$$

d. Find the velocity (magnitude and sign) of stone A at this time.

$$v = v_0 + at = 22.5 - 9.81(3.84) \quad 5$$

$$= -15.1 \text{ m/s}$$



3. A cannonball is shot from a cliff of height $H = 225 \text{ m}$ at a castle wall. The initial velocity of the ball is v_0 at an angle of θ with respect to the horizontal direction. The castle wall is $Y = 65.0 \text{ m}$ high and a horizontal distance of $D = 175 \text{ m}$ from the cannon. Assume the cannonball just grazes the top of the wall after a time of 7.50 s .

a. Find x-component of the initial velocity of the cannonball.

$$175 = v_{0x} (7.50) \quad 5$$

$$v_{0x} = 175 / 7.50 = 23.3 \text{ m/s}$$

b. Find y-component of the initial velocity of the cannonball.

$$65.0 = 225 + v_{70} (7.50) + \frac{1}{2} (-9.81) (7.50)^2$$

$$v_{70} = 15.5 \text{ m/s} \quad 10$$

c. Find the initial speed, v_0 , and angle, θ , of the cannonball.

$$v_0 = \sqrt{23.3^2 + 15.5^2} = 28.0 \text{ m/s} \quad 5$$

$$\theta = \tan^{-1} \left(\frac{15.5}{23.3} \right) = 33.6^\circ$$

d. Find the velocity in vector notation of the cannonball when it hits the ground.

$$y = v_{70} t + \frac{1}{2} a t^2$$

$$0 = 225 + (15.5) t + \frac{1}{2} (-9.81) t^2$$

$$4.905 t^2 - 15.5 t - 225 = 0 \quad 20$$

$$t = \frac{15.5 \pm \sqrt{15.5^2 + 4(225)(4.905)}}{9.81} = 8.53 \text{ s}$$

$$v_y = v_{70} + a t = 15.5 + (-9.81) (8.53) = -68.2 \text{ m/s}$$

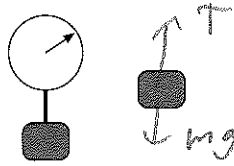
$$v_x = v_{x0} = 23.3$$

$$\vec{v} = (23.3 \hat{i} - 68.2 \hat{j}) \text{ m/s}$$

or $v_y^2 = 15.5^2 + 72(-9.81)(-225)$

$$v_y = \pm 68.2 \text{ m/s}$$

FinalPart2F17



4. A block of mass 6.50 kg is attached to a rod which is part of a spring scale which is suspended from the roof of an elevator car. From $t=0$ to $t=5.00$ s the elevator is stationary. From $t=5.00$ to $t=10.0$ s the elevator accelerates upwards with an acceleration of 3.50 m/s^2 in magnitude, from $t=10.0$ to $t=15.0$ s the elevator travels at constant speed, and from $t=15.0$ s to $t=25.0$ s the elevator accelerates until it is at rest. Assume constant acceleration in all cases.

a. Above and to the right, draw a free body diagram for the block during the 5-10 s time interval. 5

b. Find the scale reading during the 5-10 s interval. 5

$$T - mg = ma$$

$$T = (6.50)(3.50 + g) = 86.5 \text{ N}$$

c. Find the scale reading during the 10-15 s interval. 5

$$T = mg = (6.50)g = 63.8 \text{ N}$$

d. Find the magnitude of the acceleration of the block during the 15-25 s interval. 10

$$v_{10} = (5.00)(3.50) = 17.5 \text{ m/s}$$

$$0 = 17.5 - a_{15}(10)$$

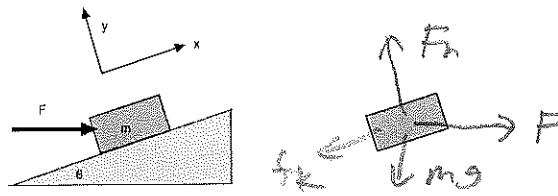
$$a_{15} = 1.75 \text{ m/s}^2$$

e. Find the scale reading during the 15-25 s interval. 5

$$T = m(g + a)$$

$$= (6.50)(g - 1.75)$$

$$= 52.4 \text{ N}$$



5. A horizontal force, F , pushes a block of mass, m , up a rough ramp which makes an angle of θ with respect to the horizontal. The coefficient of kinetic friction between block and ramp is μ_k .

a. Above and to the right draw a free body diagram of the block while it is sliding up the ramp. 5

b. Write out Newton's 2nd Law for the block in both directions.

$$x: F \cos \theta - mg \sin \theta - f_k = ma$$

$$y: F_n - mg \cos \theta - F \sin \theta = 0$$



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Assume $F=105 \text{ N}$, $m=4.50 \text{ kg}$, $\theta=40.0^\circ$, and $\mu_k=0.350$.

c. Find the magnitude of the normal force on the block.

$$F_n = mg \cos \theta + F \sin \theta$$

$$= (4.50)g \cos(40^\circ) + 105(\sin 40^\circ)$$

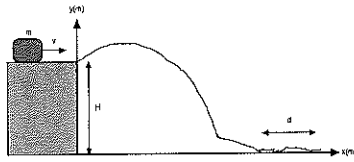
$$= 101 \text{ N}$$

d. Find the magnitude of the acceleration of the block.

$$0 = \frac{1}{m} [F \cos \theta - mg \sin \theta - \mu_k F_n]$$

$$= \frac{1}{4.50} [105 \cos 40^\circ - (4.50)g \sin 40^\circ - (0.350)(101)]$$

$$= 3.71 \text{ m/s}^2$$



6. A block of mass $m=7.50$ kg is traveling at a speed of $v=15.5$ m/s on a frictionless surface at a height $H=35.0$ m. It then slides down a frictionless curvy track. Then it slides along a horizontal rough surface a distance $d=250$ m before coming to rest. Take $y=0$ at the bottom of the track.

a. Find the total energy of the block before it reaches the frictionless track.

$$E = \frac{1}{2} m v^2 + m g h \quad 10$$

$$= (7.50) \left[\frac{15.5^2}{2} + g(35) \right] = 3.48 \times 10^3 \text{ J}$$

b. Find the speed of the block at the bottom of the ramp.

$$3476 = \frac{1}{2} m v^2 \quad 10$$

$$v = \left(\frac{2 \cdot 3476}{7.50} \right)^{1/2} = 30.4 \text{ m/s}$$

c. Draw a free body diagram of the block as it is sliding on the rough surface.



d. Using Conservation of Energy, find the coefficient of kinetic friction between block and rough surface.

$$3476 = f_k \Delta s \quad 10$$

$$= \mu_k F_n \Delta s = \mu_k m g \Delta s$$

$$\mu_k = \frac{3476}{(7.50)g(250)} = 0.189$$

FinalPart3 S17

7. On a horizontal, frictionless table a puck (A) of mass 1.50 kg is traveling with a velocity given by:

$$v_A = (2.00 \text{ m/s})\hat{i} - (5.50 \text{ m/s})\hat{j}$$

It strikes and sticks to a puck (B) of mass 2.25 kg initially (before the collision) traveling with a velocity given by:

$$v_B = (1.50 \text{ m/s})\hat{i} + (2.50 \text{ m/s})\hat{j}$$

All answers below should be in vector notation.

a. Find the momenta of pucks A and B before the collision.

$$\begin{aligned} \vec{p}_A &= m_A v_A = (1.50) (2.00\hat{i} - 5.50\hat{j}) \\ &= (3.00\hat{i} - 8.25\hat{j}) \text{ kg}\frac{\text{m}}{\text{s}} \end{aligned} \quad 10$$

$$\begin{aligned} \vec{p}_B &= m_B v_B = (2.25) (1.50\hat{i} + 2.50\hat{j}) \\ &= (3.38\hat{i} + 5.62\hat{j}) \text{ kg}\frac{\text{m}}{\text{s}} \end{aligned}$$

b. Find the momentum of the 2-puck combination after the collision.

$$\begin{aligned} \vec{p}_f &= \vec{p}_i = \vec{p}_A + \vec{p}_B \\ &= (6.38\hat{i} - 2.62\hat{j}) \text{ kg}\frac{\text{m}}{\text{s}} \end{aligned} \quad 10$$

c. Find the velocity of the 2-puck combination after the collision.

$$\begin{aligned} \vec{v}_f &= \frac{\vec{p}_f}{m_A + m_B} = \frac{\vec{p}_f}{3.75} \\ &= \frac{(6.38\hat{i} - 2.62\hat{j})}{3.75} \\ &= (1.70\hat{i} - 0.700\hat{j}) \frac{\text{m}}{\text{s}} \end{aligned} \quad 5$$

8. A wheel is at rest at the top of a ramp which makes an angle of 55.0° with respect to the horizontal. The height of the ramp is 2.50 m. The wheel consists of a thin rim with mass 1.25 kg at a radius of 1.50 m, and 6 spokes of length 1.50 m and mass 0.250 kg apiece.

a. Find the moment of inertia of the wheel about its axis.

$$\begin{aligned}
 I &= MR^2 + 6 \left(\frac{1}{3} mL^2 \right) \\
 &= (1.25)(1.50)^2 + 2(1.25)(1.50)^2 \\
 &= 3.94 \text{ kg m}^2
 \end{aligned}$$

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b. Assume the ramp is frictionless and the wheel slides to the bottom of the ramp. Use Conservation of Energy to find the speed of the wheel at the bottom of the ramp.

$$\begin{aligned}
 \frac{1}{2} m v^2 &= m g h \\
 v &= \sqrt{2 g h} = \sqrt{2 g (2.50)} \\
 &= 7.00 \text{ m/s}
 \end{aligned}$$

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c. Now assume the ramp is rough and the wheel rolls without slipping to the bottom of the ramp. Find the linear speed of the wheel at the bottom of the ramp.

$$\begin{aligned}
 m g h &= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \\
 &= \frac{1}{2} m v^2 + \frac{1}{2} I \left(\frac{v}{R} \right)^2 \\
 &= \frac{1}{2} v^2 \left[m + \frac{I}{R^2} \right] \\
 2.75 g (2.5) &= \frac{1}{2} v^2 \left[2.75 + \frac{3.94}{1.5^2} \right] \\
 v &= 5.47 \text{ m/s}
 \end{aligned}$$

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9. A block of 2.50 kg is attached to a spring with spring constant 650 N/m. At $t=0$, the spring is stretched by 25.0 cm and released at $t = 0$.

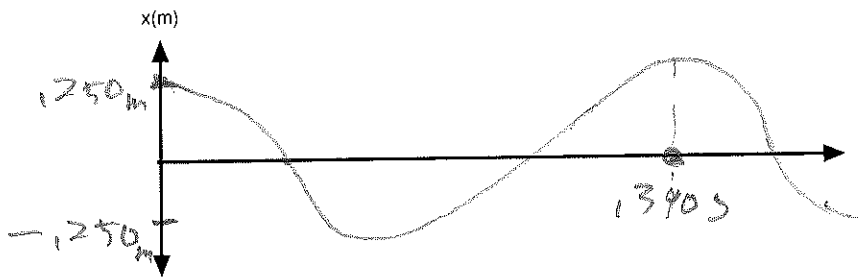
a. Write an equation for the position of the block as a function of time.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{650}{2.50}} = 16.1 \frac{\text{rad}}{\text{s}}$$

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$$x = (0.250 \text{ m}) \cos(16.1 t)$$

b. Plot the position vs. time of the block, labeling the axes appropriately.



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$$T = \frac{2\pi}{\omega} = \frac{2\pi}{16.1} = 0.390 \text{ s}$$

c. Find the velocity of the block at $t = 2.50$ s.

$$\begin{aligned} v &= \frac{dx}{dt} = -(16.1)(0.250) \sin(16.1 t) \\ &= -(16.1)(0.250) \sin[(16.1)(2.50)] \\ &= -2.124 \text{ m/s} \end{aligned}$$

10

d. Find the maximum acceleration of the block.

$$\begin{aligned} a &= \frac{dv}{dt} = -(16.1)^2 (0.250) \cos(16.1 t) \\ a_{\text{max}} &= (16.1)^2 (0.250) = 64.8 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

10