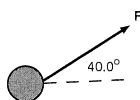


Your Name: _____

PHY203
Exam #3
Chapters 9-11,15
Mon., 12/12/22

Solutions

Exam3F22



1. A wheel consists of a solid disk of radius 0.850 m and mass 3.50 kg. The wheel is pinned through its axis and allowed to rotate freely. The wheel is initially at rest. A force of 15.5 N is applied at an angle of 40.0° as shown. We want to know some properties of the wheel after it has been spinning for 15.0 s.

List the known quantities below:

Parameter	Known Value
F	15.5 N
r	0.850 m
m	3.50 kg
θ	40.0°
t	15.0 s

a. Find the moment of inertia of the wheel about its axis. 5

$$I = \frac{1}{2} MR^2 = \frac{1}{2} (3.50)(0.850)^2 = 1.26 \text{ kg m}^2$$

b. Find the magnitude of the torque on the wheel. 5

$$\tau = rF \sin \theta = (0.850)(15.5) \sin(40^\circ) = 8.47 \text{ Nm}$$

c. Find the magnitude of the angular acceleration of the wheel. 5

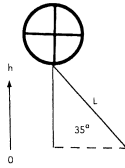
$$\alpha = \frac{\tau}{I} = \frac{8.47}{1.26} = 6.72 \text{ rad/s}^2$$

d. Find the angular speed after the force has been applied for 15.0 s. 5

$$\omega = \omega_0 + \alpha t = 0 + 6.72(15.0) = 101 \text{ rad/s}$$

e. Find how many revolutions the wheel has made after 15.0 s. 10

$$\begin{aligned} \theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ &= 0 + 0 + \frac{1}{2} (6.72)(15.0)^2 \\ &= \frac{756 \text{ rad}}{2\pi} = 120 \text{ rev} \end{aligned}$$

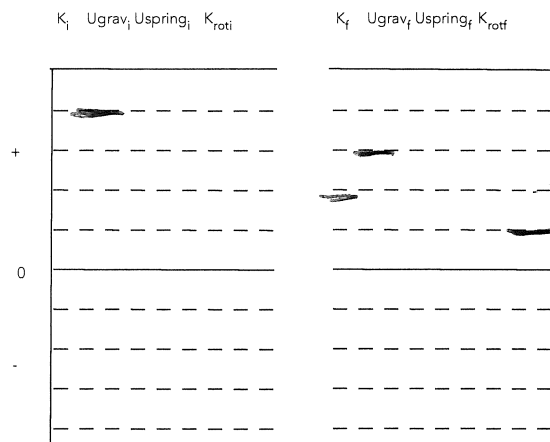


2. A wheel starts from rest at the top of a ramp which makes an angle of 35.0° with respect to the horizontal and has a height of $h = 5.00 \text{ m}$. The wheel consists of a thin rim with mass 3.50 kg , radius 1.50 m , and 4 spokes of length 1.50 m and mass 0.450 kg apiece. The wheel is released and starts to roll down the ramp.

List the known quantities below:

Parameter	Known Value
h	5.00 m
r	1.50 m
M	3.50 kg
L	1.50 m
m	0.450 kg

a. Create energy bar charts for the wheel at the top of the ramp and after it has rolled 3.00 m along the ramp: **5**



b. Find the moment of inertia of the wheel. **10**

$$\begin{aligned}
 I &= MR^2 + 4\left(\frac{1}{3}mL^2\right) \\
 &= R^2\left[m + \frac{4}{3}m\right] = (1.50)^2\left[3.50 + \frac{4}{3}(0.450)\right] \\
 &= 9.22 \text{ kg m}^2 \qquad 7.88 + 1.35
 \end{aligned}$$

d. Use Conservation of Energy to find the kinetic energy of the wheel once it has rolled 3.00 m along the ramp. (Note: 3.00 m is the distance along the ramp, not the height change.) 15

$$M_{\text{tot}} = 3.50 \text{ kg} + 1(1.45 \text{ kg}) = 5.3 \text{ kg}$$

$$h_{\text{change}} = 3.00 \sin(35^\circ) = 1.72 \text{ m}$$

$$M_{\text{tot}} g (5.00) = M_{\text{tot}} (5 - 1.72)g + KE$$

$$KE = (5.30)g(1.72)$$

$$= 89.5 \text{ J}$$

e. Find the linear speed of the center of mass of the wheel at this point. 15

$$89.5 = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} v^2 \left[M_{\text{tot}} + \frac{I}{v^2} \right]$$

$$= \frac{1}{2} v^2 \left[5.30 + \frac{9.22}{1.50^2} \right]$$

$$= \frac{1}{2} v^2 (9.40)$$

$$v = 4.36 \text{ m/s}$$

3. The position of a particle is given by $x = 4.20 \cos 3.00t$, where x is in meters and t is in seconds.

a. Identify and list the known quantities (in SI units) (these could include $m, A, x, t, v, a, E, T, f, \omega, \dots$) **5**

$$A = 4.20 \text{ m}, \quad \omega = 3.00 \text{ rad/s}$$

b. Find the period of the motion. **5**

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3} = 2.10 \text{ s}$$

c. Find the maximum speed of the particle. **5**

$$v = -\omega A \sin \omega t$$

$$v_{\max} = \omega A = (3.00)(4.20) \\ = 12.6 \text{ m/s}$$

d. Find the speed of the particle when it is at a position of 1.30 m. **10**

$$x = 1.30 = 4.20 \cos 3.00t$$

$$\cos(3.00t) = \frac{1.30}{4.20} = 0.310$$

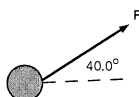
$$3.00t = \cos^{-1}(0.310) = 1.26$$

$$v = -\omega A \sin \omega t$$

$$= -(3.00)(4.20) \sin(1.26)$$

$$= -12.0 \text{ m/s}$$

Exam3F22alt



1. A wheel consists of a solid disk of radius 0.950 m and mass 4.50 kg. The wheel is pinned through its axis and allowed to rotate freely. The wheel is initially at rest. A force of 17.5 N is applied at an angle of 40.0° as shown. We want to know some properties of the wheel after it has been spinning for 15.0 s.

List the known quantities below:

Parameter	Known Value
F	17.5 N
r	0.950 m
m	4.50 kg
θ	40.0°
t	15.0 s

a. Find the moment of inertia of the wheel about its axis. 5

$$I = \frac{1}{2} MR^2 = \frac{1}{2} (4.50) (0.950)^2 = 2.03 \text{ kg m}^2$$

b. Find the magnitude of the torque on the wheel. 5

$$\tau = r F \sin \theta = (0.950) (17.5) \sin (40.0^\circ) = 10.7 \text{ N m}$$

c. Find the magnitude of the angular acceleration of the wheel. 5

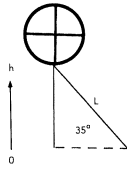
$$\alpha = \frac{\tau}{I} = \frac{10.7}{2.03} = 5.26 \text{ rad/s}^2$$

d. Find the angular speed after the force has been applied for 15.0 s. 5

$$\omega = \omega_0 + \alpha t = 0 + (5.26) (15.0) = 79.0 \text{ rad/s}$$

e. Find how many revolutions the wheel has made after 15.0 s. 10

$$\begin{aligned} \theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ &= 0 + 0 + \frac{1}{2} (5.26) (15.0)^2 = 592 \text{ rad} \\ \frac{592}{2\pi} &= 94.2 \text{ rev} \end{aligned}$$

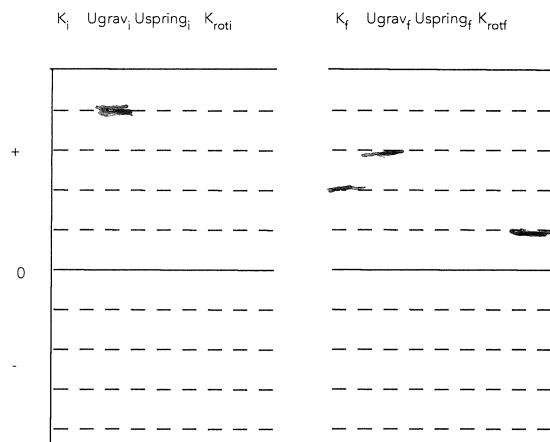


2. A wheel starts from rest at the top of a ramp which makes an angle of 35.0° with respect to the horizontal and has a height of $h = 6.00$ m. The wheel consists of a thin rim with mass 2.50 kg, radius 1.75 m, and 4 spokes of length 1.75 m and mass 0.450 kg apiece. The wheel is released and starts to roll down the ramp.

List the known quantities below:

Parameter	Known Value
h	6.00 m
r	1.75 m
M	2.50 kg
L	1.75 m
m	0.450 kg

a. Create energy bar charts for the wheel at the top of the ramp and after it has rolled 3.00 m along the ramp: **5**



b. Find the moment of inertia of the wheel. **10**

$$\begin{aligned}
 I &= Mr^2 + 4 \left(\frac{1}{3} mL^2 \right) \\
 &= r^2 \left[2.50 + \frac{4}{3} (0.450) \right] \\
 &= (1.75)^2 (3.1) = 9.49 \text{ kg}\cdot\text{m}^2
 \end{aligned}$$

$$7.66 + 1.84$$

d. Use Conservation of Energy to find the kinetic energy of the wheel once it has rolled 3.00 m along the ramp. (Note: 3.00 m is the distance along the ramp, not the height change.) **15**

$$M_{\text{tot}} = 2.50 + 4(0.750) = 4.30 \text{ kg}$$

$$h' = 6.00 \text{ m} - 3.00 \sin(35^\circ) = 1.28 \text{ m}$$

$$M_{\text{tot}} g (6.00) = M_{\text{tot}} g (1.28) + KE$$

$$KE = M_{\text{tot}} g (1.72)$$

$$= (4.30) g (1.72)$$

$$= 72.6 \text{ J}$$

e. Find the linear speed of the center of mass of the wheel at this point. **15**

$$72.6 = \frac{1}{2} M_{\text{tot}} v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} v^2 \left[M_{\text{tot}} + \frac{I}{r^2} \right]$$

$$= \frac{1}{2} v^2 \left[4.30 + \frac{9.77}{1.75^2} \right]$$

$$= \frac{1}{2} v^2 (7.80)$$

$$v = 4.43 \text{ m/s}$$

3. The position of a particle is given by $x = 5.20 \cos 3.50t$, where x is in meters and t is in seconds.

a. Identify and list the known quantities (in SI units) (these could include $m, A, x, t, v, a, E, T, f, \omega, \dots$) **5**

$$A = 5.20 \text{ m}, \quad \omega = 3.50 \text{ rad/s}$$

b. Find the period of the motion. **5**

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3.50} = 1.80 \text{ s}$$

c. Find the maximum speed of the particle. **5**

$$v = -\omega A \sin \omega t$$

$$v_{\max} = \omega A = (3.50)(5.20) \\ = 18.2 \text{ m/s}$$

d. Find the speed of the particle when it is at a position of 1.40 m. **10**

$$1.40 = 5.20 \cos 3.50t$$

$$\cos 3.50t = \frac{1.40}{5.20} = 0.269$$

$$3.50t = 1.30$$

$$v = -(3.50)(5.20) \sin(1.30)$$

$$= -17.5 \text{ m/s}$$