

Your Name: _____

PHY203
Exam #4
Chapters 5,9,10,14
Mon., 12/8/14

Solutions

1. Imagine the Earth doubled its mass and expanded to 6.00 times its original radius.

a. Find the acceleration of gravity of the surface of the new Earth.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(2)(5.97 \times 10^{24})}{6^2(6.37 \times 10^6 \text{ m})^2} = 0.545 \text{ m/s}^2 \quad \mathbf{10}$$

b. Find the escape speed of a 350 kg rocket from the surface of the new Earth.

$$\frac{1}{2}mv_e^2 - \frac{mGM}{R} = 0$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(2)(5.97 \times 10^{24})}{6(6.37 \times 10^6 \text{ m})}} = \mathbf{15}$$

$$6.46 \times 10^3 \text{ m/s}$$

c. If a 350 kg rocket is propelled from the surface of the new Earth with an initial speed of 2.50 km/s, find the greatest height above the planet's surface that the rocket reaches.

$$\frac{1}{2}mv^2 - \frac{mGM}{R} = -\frac{mGM}{R+h}$$

$$v^2 = \frac{2GM}{R} \left[1 - \frac{1}{1+h/R}\right] = v_e^2 \left[1 - \frac{1}{1+h/R}\right] \quad \mathbf{15}$$

$$(2.50 \times 10^3 \text{ m/s})^2 = (6.46 \times 10^3 \text{ m/s})^2 \left[1 - \frac{1}{1+h/6R_E}\right]$$

$$h = 6.73 \times 10^6 \text{ m}$$

2. A block of mass 1.25 kg is attached to a spring. The block is stretched and released. The equation of motion of the block is given by:

$$x = (0.350\text{ m}) \cos(2.50t)$$

a. Find the spring constant of the spring.

$$\omega = \sqrt{\frac{k}{m}} \quad 10$$

$$k = \omega^2 m = (2.50)^2 (1.25) = 7.81\text{ N / m}$$

b. Find the total energy of the system.

$$\omega = \sqrt{\frac{k}{m}} \quad 5$$

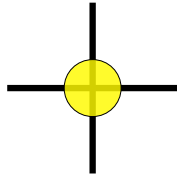
$$E = \frac{1}{2} k A^2 = \frac{1}{2} (7.81\text{ N / m}) (0.350\text{ m})^2 = 0.479\text{ J}$$

c. Find an expression for the velocity of the block as a function of time.

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t) = -(2.50)(0.350) \sin(2.50t) \quad 10$$
$$= -0.875 \sin(2.50t)\text{ m / s}$$

d. Find the maximum velocity of the block.

$$v_{\max} = \omega A = 0.875\text{ m / s} \quad 5$$



3. An object consists of a solid disk of mass 3.50 kg and radius 0.500 m with 4 spokes which each have a mass of 1.50 kg and length of 0.850 m.
- a. Find the moment of inertia of the object for an axis perpendicular to the plane of the object and through its center of mass.

$$I = \frac{1}{2}M_i R_i^2 + 4M_o R_o^2 = \frac{1}{2}(3.50)(0.500)^2 + \frac{4}{3}(1.50)(0.850)^2$$

$$= 1.88 \text{ kgm}^2$$

10

The object is suspended by a nail through the end of one of the spokes and set into small oscillations.

- b. Find the moment of inertia about the axis through the nail.

$$I = I_{CM} + Mh^2 = 1.88 + (9.50)(0.850)^2$$

$$= 8.74 \text{ kgm}^2$$

10

- c. Find the period of the motion of the oscillating object about the nail.

$$T = 2\pi \sqrt{\frac{I}{MgD}} = 2\pi \sqrt{\frac{8.74}{(9.50)g(0.850)}} =$$

$$= 2.09s$$

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Alt Version

1. Imagine the Earth tripled its mass and expanded to 4.00 times its original radius.

a. Find the acceleration of gravity of the surface of the new Earth.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(3)(5.97 \times 10^{24})}{4^2(6.37 \times 10^6 \text{ m})^2} = 1.84 \text{ m/s}^2 \quad \mathbf{10}$$

b. Find the escape speed of a 450 kg rocket from the surface of the new Earth.

$$\begin{aligned} \frac{1}{2}mv_e^2 - \frac{mGM}{R} &= 0 \\ v_e &= \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(3)(5.97 \times 10^{24})}{4(6.37 \times 10^6 \text{ m})}} = \mathbf{15} \\ &9.68 \times 10^3 \text{ m/s} \end{aligned}$$

c. If a 450 kg rocket is propelled from the surface of the new Earth with an initial speed of 2.00 km/s, find the greatest height above the planet's surface that the rocket reaches.

$$\begin{aligned} \frac{1}{2}mv^2 - \frac{mGM}{R} &= -\frac{mGM}{R+h} \\ v^2 &= \frac{2GM}{R} \left[1 - \frac{1}{1+h/R}\right] = v_e^2 \left[1 - \frac{1}{1+h/R}\right] \quad \mathbf{15} \\ (2.00 \times 10^3 \text{ m/s})^2 &= (9.68 \times 10^3 \text{ m/s})^2 \left[1 - \frac{1}{1+h/4R_E}\right] \\ h &= 1.14 \times 10^6 \text{ m} \end{aligned}$$

2. A block of mass 1.50 kg is attached to a spring. The block is stretched and released. The equation of motion of the block is given by:

$$x = (0.550\text{ m}) \cos(3.50t)$$

a. Find the spring constant of the spring.

$$\omega = \sqrt{\frac{k}{m}} \quad \mathbf{10}$$

$$k = \omega^2 m = (3.50)^2 (1.50) = 18.4\text{ N / m}$$

b. Find the total energy of the system.

$$\omega = \sqrt{\frac{k}{m}} \quad \mathbf{5}$$

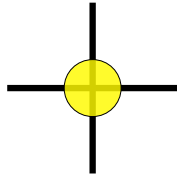
$$E = \frac{1}{2} k A^2 = \frac{1}{2} (18.4\text{ N / m}) (0.550\text{ m})^2 = 2.78\text{ J}$$

c. Find an expression for the velocity of the block as a function of time.

$$\begin{aligned} v &= \frac{dx}{dt} = -\omega A \sin(\omega t) = -(3.50)(0.550) \sin(3.50t) \\ &= -1.92 \sin(3.50t)\text{ m / s} \end{aligned} \quad \mathbf{10}$$

d. Find the maximum velocity of the block.

$$v_{\max} = \omega A = 1.92\text{ m / s} \quad \mathbf{5}$$



3. An object consists of a solid disk of mass 2.50 kg and radius 0.500 m with 4 spokes which each have a mass of 1.25 kg and overall length of 0.750 m.
- a. Find the moment of inertia of the object for an axis perpendicular to the plane of the object and through its center of mass.

$$I = \frac{1}{2}M_i R_i^2 + 4M_o R_o^2 = \frac{1}{2}(2.50)(0.500)^2 + \frac{4}{3}(1.25)(0.750)^2$$

$$= 1.25 \text{ kgm}^2$$

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The object is suspended by a nail through the end of one of the spokes and set into small oscillations.

- b. Find the moment of inertia about the axis through the nail.

$$I = I_{CM} + Mh^2 = 1.25 + (7.50)(0.750)^2$$

$$= 5.47 \text{ kgm}^2$$

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- c. Find the period of the motion of the oscillating object about the nail.

$$T = 2\pi \sqrt{\frac{I}{MgD}} = 2\pi \sqrt{\frac{5.47}{(7.50)g(0.750)}} =$$

$$= 1.98 \text{ s}$$

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