

Your Name: \_\_\_\_\_

**PHY203**  
**Exam #4**  
**Chapters 5,9,10,14**  
**Mon., 12/9/13**

# Solutions

1. A planet has a mass of  $7.50 \times 10^{25}$  kg and a radius of  $5.00 \times 10^3$  km.  
 a. Find the magnitude of the acceleration due to gravity on the surface of the planet.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(7.50 \times 10^{25})}{(5.00 \times 10^6 \text{ m})^2} = 200 \text{ m/s}^2 \quad \mathbf{10}$$

- b. Find the escape speed from the surface of the planet.

$$\frac{1}{2}mv_e^2 - \frac{mGM}{R} = 0$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(7.50 \times 10^{25})}{(5.00 \times 10^6 \text{ m})}} = \mathbf{10}$$

$$4.47 \times 10^4 \text{ m/s}$$

- c. If a 550 kg rocket is propelled from the surface with an initial speed of 25.0 km/s, find the greatest height above the planet's surface that the rocket reaches.

$$\frac{1}{2}mv^2 - \frac{mGM}{R} = -\frac{mGM}{R+h}$$

$$v^2 = \frac{2GM}{R} \left[1 - \frac{1}{1+h/R}\right] = v_e^2 \left[1 - \frac{1}{1+h/R}\right] \quad \mathbf{15}$$

$$h = 0.460R = 2.30 \times 10^6 \text{ m}$$

2. A simple harmonic oscillator consisting of a block of mass 2.50 kg moving in the x-direction has amplitude 6.50 cm and period 3.00 s.

a. Find the frequency and angular frequency of the oscillator.

$$f = \frac{1}{T} = \frac{1}{3.00} = 0.333\text{Hz} \quad \mathbf{5}$$
$$\omega = 2\pi f = 2\pi(0.333) = 2.09\text{rad/s}$$

b. Write an equation for the position of the block vs. time.

$$x(t) = (0.0650)\cos(2.09t) \quad \mathbf{5}$$

c. Write an equation for the velocity of the block vs. time.

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t) = -(2.09)(0.0650)\sin(2.09t) \quad \mathbf{10}$$
$$= -0.136 \sin(2.09t) \text{m/s}$$

d. Find the maximum speed of the block.

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t) \quad \mathbf{5}$$
$$v_{\max} = \omega A = 0.136 \text{m/s}$$

e. Find the kinetic energy of the block at a time of 5.50 s.

$$v(5.50) = -0.136 \sin(2.09 \times 5.50) \quad \mathbf{10}$$
$$= 0.119 \text{m/s}$$
$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.50)(-0.119)^2 = 1.78 \times 10^{-2} \text{J}$$

3. A simple pendulum on the surface of Earth consists of a block of mass 2.50 kg suspended on a string of length 3.50 m. The amplitude of the pendulum is  $25.0^\circ$ .

a. Find the period of the pendulum.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{3.50}{9.81}} = 3.75s \quad \mathbf{10}$$

b. Find the frequency and angular frequency of the pendulum.

$$f = \frac{1}{T} = \frac{1}{3.75} = 0.266Hz \quad \mathbf{5}$$

$$\omega = 2\pi f = 2\pi(0.266) = 1.67rad/s$$

A pendulum on the surface of the Earth consists of a thin rod of mass 2.50 kg and length 3.50 m. It pivots about an axis through one end. The amplitude of the pendulum is  $25.0^\circ$ .

c. Find the period of the pendulum.

$$T = 2\pi\sqrt{\frac{I}{MgD}} = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}} \quad \mathbf{15}$$

$$= 2\pi\sqrt{\frac{2(3.50)}{3(9.81)}} = 3.07s$$

Alternate

1. A planet has a mass of  $6.50 \times 10^{25}$  kg and a radius of  $4.00 \times 10^3$  km.  
a. Find the magnitude of the acceleration due to gravity on the surface of the planet.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.50 \times 10^{25})}{(4.00 \times 10^6 \text{ m})^2} = 271 \text{ m/s}^2 \quad \mathbf{10}$$

- b. Find the escape velocity from the surface of the planet.

$$\frac{1}{2}mv_e^2 - \frac{mGM}{R} = 0$$
$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(6.50 \times 10^{25})}{(4.00 \times 10^6 \text{ m})}} = \mathbf{10}$$
$$4.65 \times 10^4 \text{ m/s}$$

- c. If a 650 kg rocket is propelled from the surface with an initial speed of 35.0 km/s, find the greatest height above the planet's surface that the rocket reaches.

$$\frac{1}{2}mv^2 - \frac{mGM}{R} = -\frac{mGM}{R+h}$$
$$v^2 = \frac{2GM}{R} \left[1 - \frac{1}{1+h/R}\right] = v_e^2 \left[1 - \frac{1}{1+h/R}\right] \quad \mathbf{15}$$
$$h = 1.31R = 5.24 \times 10^6 \text{ m}$$

2. A simple harmonic oscillator consisting of a block of mass 3.50 kg moving in the x-direction has amplitude 7.50 cm and period 4.00 s.
- a. Find the frequency and angular frequency of the oscillator.

$$f = \frac{1}{T} = \frac{1}{4.00} = 0.250 \text{ Hz} \quad \mathbf{5}$$

$$\omega = 2\pi f = 2\pi(0.250) = 1.57 \text{ rad/s}$$

- b. Write an equation for the position of the block vs. time.

$$x(t) = (0.0750)\cos(1.57t) \quad \mathbf{5}$$

- c. Write an equation for the velocity of the block vs. time.

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t) = -(1.57)(0.0750)\sin(1.57t) \quad \mathbf{10}$$

$$= -0.118 \sin(1.57t) \text{ m/s}$$

- d. Find the maximum speed of the block.

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t) \quad \mathbf{5}$$

$$v_{\max} = \omega A = 0.118 \text{ m/s}$$

- e. Find the kinetic energy of the block at a time of 4.50 s.

$$v(4.50) = -0.118 \sin(1.57 \times 4.50)$$

$$= 8.31 \times 10^{-2} \text{ m/s} \quad \mathbf{10}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(3.50)(8.31 \times 10^{-2})^2 = 1.21 \times 10^{-2} \text{ J}$$

3. A simple pendulum on the surface of Earth consists of a block of mass 3.50 kg suspended on a string of length 4.50 m. The amplitude of the pendulum is  $35.0^\circ$ .

a. Find the period of the pendulum.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{4.50}{9.81}} = 4.26s \quad \mathbf{10}$$

b. Find the frequency and angular frequency of the pendulum.

$$f = \frac{1}{T} = \frac{1}{4.26} = 0.235Hz \quad \mathbf{5}$$

$$\omega = 2\pi f = 2\pi(0.235) = 1.48rad/s$$

A pendulum on the surface of the Earth consists of a thin rod of mass 3.50 kg and length 4.50 m. It pivots about an axis through one end. The amplitude of the pendulum is  $35.0^\circ$ .

c. Find the period of the pendulum.

$$T = 2\pi\sqrt{\frac{I}{MgD}} = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}} \quad \mathbf{15}$$

$$= 2\pi\sqrt{\frac{2(4.50)}{3(9.81)}} = 3.48s$$