

Problem

Mars' moon Phobos has a mass of approximately 2×10^{-9} the mass of Earth and orbits around Mars at a distance of 1.5 times Earth radius from the center of the planet. Knowing that Mars has 0.1 times the mass of Earth, find:

The potential energy of Phobos.

$$U = \frac{G(2 \times 10^{-9} M_E)(0.1 M_E)}{1.5 R_E} = \frac{(6.67 \times 10^{-11})(2 \times 10^{-9})(5.98 \times 10^{24})^2}{(1.5)(6.37 \times 10^6)}$$
$$= 4.99 \times 10^{22} \text{ J}$$

The orbital speed of Phobos assuming that the orbit is a perfect circle.

$$v = \sqrt{\frac{GM_{\text{MARS}}}{R_{\text{ORBIT}}}} = \sqrt{\frac{G(0.1 M_E)}{1.5 R_E}} = \sqrt{\left(\frac{0.1}{3}\right) \frac{2GM_E}{R_E}}$$
$$= \sqrt{\frac{0.1}{3}} v_e^{\text{EARTH}} = \sqrt{\frac{0.1}{3}} 11.2 \text{ km/s} = 2.04 \text{ km/s}$$

The orbital period of Phobos.

$$T = \frac{2\pi(1.5 R_E)}{2.04 \text{ km/s}} = \frac{2\pi(1.5)(6370 \text{ km})}{2.04 \text{ km/s}} = 2.9 \times 10^4 \text{ s}$$

If Phobos has a radius of just 11 km (i.e. 0.0017 times Earth radius), find the escape speed from Phobos' surface.

$$v_e^{\text{PHOBOS}} = \sqrt{\frac{2G(2 \times 10^{-9} M_E)}{0.0017 R_E}} = \sqrt{\frac{2 \times 10^{-9}}{0.0017}} v_e^{\text{EARTH}}$$
$$= 0.012 \text{ km/s} = 12 \text{ m/s}$$