Your Name: ________________________

PHY203
Exam #3
Chapters 8-10
Mon., 5/1/17

Solutions

1. A runaway train (A) with mass 50,000 kg and traveling at a speed of 35.0 m/s in the +x-direction runs into a train car (B) with mass 30,000 kg that before the collision was moving in the opposite direction with a speed of 25.0 m/s. Assume the trains stick together after the collision.

a. Write the momenta of each train before the collision in vector notation.

$$\vec{p_A} = (50{,}000)(35.0)\,\hat{i} = 1.75 \times 10^6 \ \mathrm{kg\frac{m}{s}}\,\hat{i}$$

$$\vec{p_B} = (30{,}000)(-25.0)\,\hat{i} = -7.50 \times 10^5 \ \mathrm{kg\frac{m}{s}}\,\hat{i}$$

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b.. Find the momentum of the two-train combination after the collision in vector notation.

$$\vec{p}_{tot\,f} = \vec{p}_{tot\,i} = \vec{p_A} + \vec{p_B}$$

$$= (1.75 - 0.750) \times 10^6 \ \mathrm{kg\,m/s}\,\hat{i}$$

$$= 1.00 \times 10^6 \ \mathrm{kg\,m/s}\,\hat{i}$$

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c. Find the speed of the two-train combination after the collision.

$$V = \frac{p_{tot}}{m_{tot}} = \frac{1.00 \times 10^6}{80{,}000} = 12.5 \ \frac{m}{s}$$

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d. Find quantitatively what kind of collision this was, elastic or inelastic.

$$K_i = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$= \frac{1}{2}(50{,}000)(35.0)^2 + \frac{1}{2}(30{,}000)(25.0)^2$$

$$= 4.00 \times 10^7 \ J$$

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$$K_f = \frac{1}{2} m_{tot} v_{tot}^2 = \frac{1}{2}(80{,}000)(12.5)^2$$

$$= 6.25 \times 10^6 \ J$$

$K_i \neq K_f \Rightarrow$ Inelastic collision

2. A hollow sphere of mass 5.50 kg and radius 1.25 m is spinning about a axis through the center with an angular speed of 7.50 rad/s. Assume the sphere is spinning in a clockwise direction as viewed from above.

a. Find the moment of inertia of the sphere.

$$I = \frac{2}{3} M R^2 = \frac{2}{3} (5.50)(1.25)^2 = 5.73 \text{ kg m}^2$$

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b. Find the kinetic energy of the sphere.

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} (5.73)(7.50)^2$$

$$= 161 \text{ J}$$

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c. A frictional force of 8.50 N is applied tangent to the rim of the sphere in a direction such that the sphere slows down. Find the magnitude of the torque and the angular acceleration of the sphere.

$$\tau = |\vec{r} \times \vec{F}| = (1.25)(8.50) = 10.6$$

$$\alpha = \frac{\tau}{I} = \frac{10.6}{5.73} = 1.85 \ \frac{\text{rad}}{\text{s}^2}$$

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d. Find the time it takes for the angular speed of the sphere to change to 2.25 rad/s.

$$\omega = \omega_0 + \alpha t$$

$$2.25 = 7.50 + (-1.85) t$$

$$t = 2.84 \text{ s}$$

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e. Find the number of revolutions the sphere has made in this time.

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$= 0 + (7.50)(2.84) + \frac{1}{2} (-1.85)(2.84)^2$$

$$= 13.8 \text{ rad}$$

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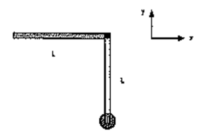
$$\frac{13.8}{2\pi} = 2.20 \text{ rev}$$



3. A thin rod of mass 2.50 kg and length L=0.750 m is pinned so that it pivots about its end and is initially at rest in a horizontal direction as shown above. it is released and swings down, eventually hitting and sticking to the 1.50 kg ball at the bottom of the swing.

a. Find the moment of inertia of the rod about its pivot point.

$I = \frac{1}{3} ML^2 = \frac{1}{3}(2.50)(.750)^2 = 0.468 \text{ kgm}^2$ 5

b. Find the change in potential energy of the rod from its initial position until just before it strikes the ball. Take the initial height of the rod as y=0. (Hint: think center of mass.)

$mg\Delta h_{cm} = (2.50)g\left(\frac{.750}{2}\right) = 9.20 \text{ J}$ 5

c. Find the angular speed of the rod at the bottom of its swing, just before it hits the ball.

$mg\Delta h = 9.20 = \frac{1}{2} I\omega^2 = \frac{1}{2}(0.468)\omega^2$ 5

$\omega = 6.27 \frac{\text{rad}}{\text{s}}$

d. Find the angular momentum of rod just before it hits the ball and write it in vector notation. Assume that the z-axis is positive "out of the paper".

$\vec{L} = I\vec{\omega} = (0.468)(6.27)(+\hat{k})$ 5

$= +2.93\hat{k}$

e. Find the angular momentum of rod/ball combination just the collision and write it in vector notation.

same $\vec{L_f} = \vec{L_i} = +2.93\hat{k}$ 5

f. Find the moment of inertia of the rod with the ball stuck to it.

$L_{tot} = L_{rod} + L_{ball} = 0.468 + mr^2$ 5

$= 0.468 + (1.5)(.750)^2 = 1.31 \text{ kgm}^2$

g. Find the angular speed of the rod/ball combination just as it starts to swing up.

$\omega_f = \frac{L_f}{I_f} = \frac{2.93}{1.31} = 2.23 \frac{\text{rad}}{\text{s}}$ 5