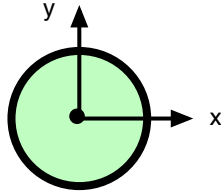


Your Name: _____

PHY203
Exam #3
Chapters 9-11,14
Thurs., 4/30/15

Solutions



1. A thin spherical shell has a mass of 4.50 kg and a radius of 0.750 m. It is rotating about an axis through its center with a constant angular speed of 5.50 rad/s.

a. Assuming that the sphere is rotating in a clockwise direction as viewed from above (looking down on the paper), in which direction is the angular velocity vector pointing? Take the z-direction as positive out of the page.

-z

5

b. Find the moment of inertia of the sphere about the axis through its center.

$$I = \frac{2}{3}MR^2 = \frac{2}{3}(4.50)(0.750)^2 = 1.69 \text{ kgm}^2$$

5

c. Find how long it takes the sphere to rotate through 900°.

$$\frac{900^\circ}{360^\circ}(2\pi) = 15.7 \text{ rad}$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

10

$$15.7 = 0 + (5.50)t + 0$$

$$t = 2.86 \text{ s}$$

d. If an energy of 15.0 J was added to the sphere, find the new angular speed of the sphere.

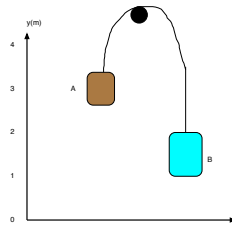
$$K_f = K_i + 15.0 \text{ J} = \frac{1}{2}I\omega_i^2 + 15.0$$

$$= \frac{1}{2}(1.69)(5.50)^2 + 15.0 = 40.6$$

10

$$40.6 = \frac{1}{2}I\omega_f^2$$

$$\omega_f = 6.93 \text{ rad/s}$$



2. At $t=0$ the blocks shown above are at rest. Block A has a mass of 2.50 kg, block B has a mass of 3.50 kg, the pulley is a solid disk with a mass of 8.50 kg and a radius of 0.200 m. The blocks are released. Find the magnitude of the linear acceleration of the blocks.

$$\text{block A: } T_A - m_A g = m_A a$$

$$\text{block B: } -T_B + m_B g = m_B a$$

$$\text{pulley: } (T_B R - T_A R) = I \alpha = I \frac{a}{R} = \frac{1}{2} M R^2 \frac{a}{R}$$

$$\text{Combine: } (m_B - m_A)g = a(m_A + m_B + \frac{1}{2} M)$$

$$(1.00)g = a(6.00 + \frac{1}{2} 8.50)$$

$$a = 0.957 \text{ m/s}^2$$

30

3. A planet has a radius of 4.00×10^3 km and a mass of 5.00×10^{24} kg.
 a. A simple pendulum with mass of 2.50 kg and length of 3.50 m is set up on the planet's surface. Find the period of the pendulum.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(5.00 \times 10^{24})}{(4.00 \times 10^6 \text{ m})^2} = 20.8 \text{ m/s}^2 \quad \mathbf{15}$$

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{3.50 \text{ m}}{20.8 \text{ m/s}^2}} = 2.57 \text{ s}$$

- b. Find the escape speed from the surface of the planet.

$$\frac{1}{2}mv_e^2 - \frac{mGM}{R} = 0 \quad \mathbf{10}$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.00 \times 10^{24})}{(4.00 \times 10^6 \text{ m})}} = 1.29 \times 10^4 \text{ m/s}$$

- c. Find the speed with which a rocket of mass 650 kg would have to be launched from the surface of the planet such that its speed at a very large distance would be 2.50 times the escape speed.

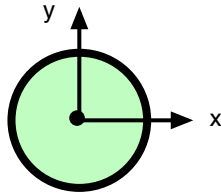
$$\frac{1}{2}mv^2 - \frac{mGM}{R} = \frac{1}{2}m(2.5v_e)^2 \quad \mathbf{15}$$

$$v^2 - v_e^2 = (2.5v_e)^2$$

$$v^2 = (v_e)^2(1 + 2.5^2)$$

$$v = 3.47 \times 10^4 \text{ m/s}$$

Alternate Solutions



1. A thin spherical shell has a mass of 4.50 kg and a radius of 0.850 m. It is rotating about an axis through its center with a constant angular speed of 6.50 rad/s.

a. Assuming that the sphere is rotating in a clockwise direction as viewed from above (looking down on the paper), in which direction is the angular velocity vector pointing? Take the z-direction as positive out of the page.

-z 5

b. Find the moment of inertia of the sphere about the axis through its center.

$$I = \frac{2}{3}MR^2 = \frac{2}{3}(4.50)(0.850)^2 = 2.17 \text{ kgm}^2 \quad 5$$

c. Find how long it takes the sphere to rotate through 900° .

$$\frac{900^\circ}{360^\circ}(2\pi) = 15.7 \text{ rad}$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 \quad 10$$

$$15.7 = 0 + (6.50)t + 0$$

$$t = 2.42 \text{ s}$$

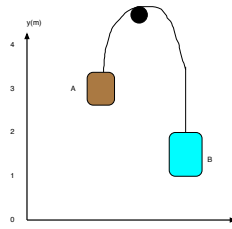
d. If an energy of 17.0 J was added to the sphere, find the new angular speed of the sphere.

$$K_f = K_i + 17.0 \text{ J} = \frac{1}{2}I\omega_i^2 + 17.0$$

$$= \frac{1}{2}(2.17)(6.50)^2 + 17.0 = 62.8 \quad 10$$

$$62.8 = \frac{1}{2}I\omega_f^2$$

$$\omega_f = 7.61 \text{ rad/s}$$



2. At $t=0$ the blocks shown above are at rest. Block A has a mass of 2.50 kg, block B has a mass of 4.50 kg, the pulley is a solid disk with a mass of 6.50 kg and a radius of 0.200 m. The blocks are released. Find the magnitude of the linear acceleration of the blocks.

$$\text{block A: } T_A - m_A g = m_A a$$

$$\text{block B: } -T_B + m_B g = m_B a$$

$$\text{pulley: } (T_B R - T_A R) = I \alpha = I \frac{a}{R} = \frac{1}{2} M R^2 \frac{a}{R}$$

$$\text{Combine: } (m_B - m_A)g = a(m_A + m_B + \frac{1}{2} M)$$

$$(2.00)g = a(7.00 + \frac{1}{2} 6.50)$$

$$a = 1.91 \text{ m/s}^2$$

30

3. A planet has a radius of 4.00×10^3 km and a mass of 6.00×10^{24} kg.
 a. A simple pendulum with mass of 2.50 kg and length of 4.50 m is set up on the planet's surface. Find the period of the pendulum.

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.00 \times 10^{24})}{(4.00 \times 10^6 \text{ m})^2} = 25.0 \text{ m/s}^2 \quad \mathbf{15}$$

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{4.50 \text{ m}}{25.0 \text{ m/s}^2}} = 2.67 \text{ s}$$

- b. Find the escape speed from the surface of the planet.

$$\frac{1}{2}mv_e^2 - \frac{mGM}{R} = 0 \quad \mathbf{10}$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(6.00 \times 10^{24})}{(4.00 \times 10^6 \text{ m})}} = 1.41 \times 10^4 \text{ m/s}$$

- c. Find the speed with which a rocket of mass 650 kg would have to be launched from the surface of the planet such that its speed at a very large distance would be 3.50 times the escape speed.

$$\frac{1}{2}mv^2 - \frac{mGM}{R} = \frac{1}{2}m(3.5v_e)^2 \quad \mathbf{15}$$

$$v^2 - v_e^2 = (3.5v_e)^2$$

$$v^2 = (v_e)^2(1 + 3.5^2)$$

$$v = 5.13 \times 10^4 \text{ m/s}$$