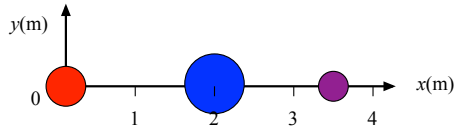


Your Name: _____

**PHY203
Exam #3
Chapters 6-8
Fri., 11/22/13**

Solutions



1. Balls are arranged along the x-axis as follows: 3.50 kg at $x=0$; 5.50 kg at $x=2.00\text{m}$; and 2.00 kg at $x=3.50\text{m}$.

a. Find the center of mass of the system and write it *in vector notation*.

$$x_{CM} = \frac{(3.50\text{kg})(0) + (5.50\text{kg})(2.00\text{m}) + (2.00\text{kg})(3.50\text{m})}{3.50\text{kg} + 5.50\text{kg} + 2.00\text{kg}}$$

$$= 1.64\text{m}$$

10

$$\vec{x}_{CM} = 1.64\text{m}\hat{i}$$

b. The 5.50 kg ball is moved to a position of $x=2.00\text{m}$ and $y=2.50\text{m}$. Find the center of mass of the system and write it *in vector notation*.

$$x_{CM} = \frac{(3.50\text{kg})(0) + (5.50\text{kg})(2.00\text{m}) + (2.00\text{kg})(3.50\text{m})}{3.50\text{kg} + 5.50\text{kg} + 2.00\text{kg}}$$

$$= 1.64\text{m}$$

10

$$y_{CM} = \frac{(3.50\text{kg})(0) + (5.50\text{kg})(2.50\text{m}) + (2.00\text{kg})(0)}{3.50\text{kg} + 5.50\text{kg} + 2.00\text{kg}}$$

$$= 1.25\text{m}$$

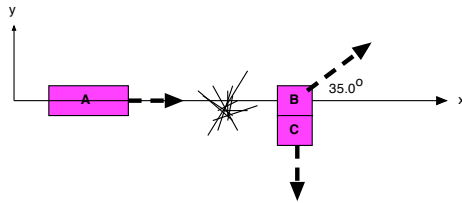
$$\vec{r}_{CM} = 1.64\text{m}\hat{i} + 1.25\text{m}\hat{j}$$

c. Find the moment of inertia about the z-axis (positive out of page) for the arrangement of balls described in part b.

$$I = (3.50\text{kg})(0) + (5.50\text{kg})[(2.00\text{m})^2 + (2.50\text{m})^2] + (2.00\text{kg})(3.50\text{m})^2$$

$$= 80.9\text{kgm}^2$$

10



2. A 7.50 kg block (A) is traveling in the positive x-direction with a speed of 4.00m/s. At some point it explodes and breaks into 2 pieces. After the explosion block C (5.00 kg) moves off in the negative y-direction and block B (2.50 kg) moves off at an angle of 35° with respect to the x-axis, as shown.

a. Find the momentum of block A before the explosion and write it *in vector notation*.

$$\mathbf{p} = (7.50)(4.00) = 30.0 \text{ kgm/s} \mathbf{i} \quad \mathbf{5}$$

b. Find the momentum of the 2-block combination after the explosion and write it *in vector notation*.

$$\text{same} = 30.0 \text{ kgm/s} \mathbf{i} \quad \mathbf{5}$$

c. Find the speeds of blocks B and C after the explosion.

$$x\text{-dir} : 30.0 = m_b v_b \cos(35.0)$$

$$v_b = 14.6 \text{ m/s}$$

$$y\text{-dir} : 0 = m_b v_b \sin(35.0) - m_c v_c$$

$$v_c = 4.20 \text{ m/s}$$

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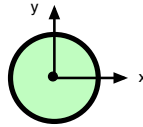
d. Find the change in kinetic energy of the system before vs. after the explosion.

$$K_i = \frac{1}{2}(7.50)(4.00)^2 = 60.0 \text{ J}$$

$$K_f = \frac{1}{2}(2.50)(14.6)^2 + \frac{1}{2}(5.00)(4.20)^2 = 311 \text{ J}$$

$$\Delta K = 251 \text{ J}$$

5



3. A thin spherical shell has a mass of 4.50 kg and a radius of 0.750 m. It is rotating about an axis through its center with angular speed of 5.50 rad/s.
- a. Assuming that the shell is rotating in a clockwise direction as viewed from above (looking down on the paper), in which direction (+x,-x,+y,-y,+z,-z) is the angular velocity vector pointing? Take the z-direction as positive out of the page.

-z 5

- b. Find the moment of inertia of the shell.

$$I = \frac{2}{3}(4.50\text{kg})(0.750\text{m})^2 = 1.69\text{kgm}^2 \quad 5$$

- c. Find the magnitude of the angular momentum of the shell.

$$L = (1.69\text{kgm}^2)(5.50\text{rad / s}) = 9.29\text{kgm}^2 / \text{s} \quad 5$$

- d. Find the kinetic energy of the shell.

$$K = \frac{1}{2}(1.69)(5.50)^2 = 25.6\text{J} \quad 5$$

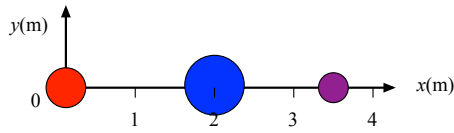
- e. If a torque of 6.00 Nm is applied at the edge of the sphere in the same direction as it is rotating, find the new angular speed of the sphere after the torque has been applied for 1.50 s.

$$\alpha = \frac{\tau}{I} = \frac{6.00}{1.69} = 3.55\text{rad / s}^2$$

$$\omega = 5.50\text{rad / s} + (3.55\text{rad / s}^2)(1.50\text{s}) \quad 15$$

$$= 10.8\text{rad / s}$$

Alternate



1. Balls are arranged along the x-axis as follows: 4.50 kg at $x=0$; 7.50 kg at $x=2.00\text{m}$; and 4.00 kg at $x=3.50\text{m}$.

a. Find the center of mass of the system and write it *in vector notation*.

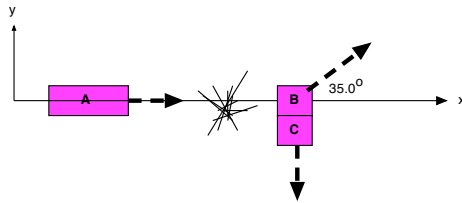
$$x_{CM} = \frac{(4.50\text{kg})(0) + (7.50\text{kg})(2.00\text{m}) + (4.00\text{kg})(3.50\text{m})}{4.50\text{kg} + 7.50\text{kg} + 4.00\text{kg}}$$
$$= 1.81\text{m} \quad \mathbf{10}$$
$$\vec{x}_{CM} = 1.81\text{m}\hat{i}$$

b. The 5.50 kg ball is moved to a position of $x=2.00\text{m}$ and $y=2.50\text{m}$. Find the center of mass of the system and write it *in vector notation*.

$$x_{CM} = \frac{(4.50\text{kg})(0) + (7.50\text{kg})(2.00\text{m}) + (4.00\text{kg})(3.50\text{m})}{4.50\text{kg} + 7.50\text{kg} + 4.00\text{kg}} \quad \mathbf{10}$$
$$= 1.81\text{m}$$
$$y_{CM} = \frac{(4.50\text{kg})(0) + (7.50\text{kg})(2.50\text{m}) + (4.00\text{kg})(0)}{4.50\text{kg} + 7.50\text{kg} + 4.00\text{kg}}$$
$$= 1.17\text{m}$$
$$\vec{r}_{CM} = 1.81\text{m}\hat{i} + 1.17\text{m}\hat{j}$$

c. Find the moment of inertia about the z-axis (positive out of page) for the arrangement of balls described in part b.

$$I = (4.50\text{kg})(0) + (7.50\text{kg})[(2.00\text{m})^2 + (2.50\text{m})^2] + (4.00\text{kg})(3.50\text{m})^2$$
$$= 126\text{kgm}^2 \quad \mathbf{10}$$



2. A 9.50 kg block (A) is traveling in the positive x-direction with a speed of 3.00m/s. At some point it explodes and breaks into 2 pieces. After the explosion block C (6.00 kg) moves off in the negative y-direction and block B (3.50 kg) moves off at an angle of 35° with respect to the x-axis, as shown.

a. Find the momentum of block A before the explosion and write it *in vector notation*.

$$\mathbf{p} = (9.50)(3.00) = 28.5 \text{ kgm/s} \mathbf{i} \quad \mathbf{5}$$

b. Find the momentum of the 2-block combination after the explosion and write it *in vector notation*.

$$\text{same} = 28.5 \text{ kgm/s} \mathbf{i} \quad \mathbf{5}$$

c. Find the speeds of blocks B and C after the explosion.

$$x\text{-dir} : 28.5 = m_b v_b \cos(35.0)$$

$$v_b = 9.94 \text{ m/s}$$

$$y\text{-dir} : 0 = m_b v_b \sin(35.0) - m_c v_c$$

$$v_c = 3.33 \text{ m/s}$$

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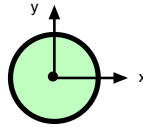
d. Find the change in kinetic energy of the system before vs. after the explosion.

$$K_i = \frac{1}{2}(9.50)(3.00)^2 = 42.8 \text{ J}$$

$$K_f = \frac{1}{2}(3.50)(9.94)^2 + \frac{1}{2}(6.00)(3.33)^2 = 206 \text{ J}$$

$$\Delta K = 163 \text{ J}$$

5



3. A thin spherical shell has a mass of 5.50 kg and a radius of 0.850 m. It is rotating about an axis through its center with angular speed of 6.50 rad/s.
- a. Assuming that the shell is rotating in a clockwise direction as viewed from above (looking down on the paper), in which direction (+x, -x, +y, -y, +z, -z) is the angular velocity vector pointing? Take the z-direction as positive out of the page.

-z 5

- b. Find the moment of inertia of the shell.

$$I = \frac{2}{3}(5.50\text{kg})(0.850\text{m})^2 = 2.65\text{kgm}^2 \quad 5$$

- c. Find the magnitude of the angular momentum of the shell.

$$L = (2.65\text{kgm}^2)(6.50\text{rad/s}) = 17.2\text{kgm}^2/\text{s} \quad 5$$

- d. Find the kinetic energy of the shell.

$$K = \frac{1}{2}(2.65)(6.50)^2 = 56.0\text{J} \quad 5$$

- e. If a torque of 8.00 Nm is applied at the edge of the sphere in the same direction as it is rotating, find the new angular speed of the sphere after the torque has been applied for 2.50 s.

$$\alpha = \frac{\tau}{I} = \frac{8.00}{2.65} = 3.02\text{rad/s}^2$$

$$\omega = 6.50\text{rad/s} + (3.02\text{rad/s}^2)(2.50\text{s}) \quad 15$$

$$= 14.0\text{rad/s}$$