

Your Name: _____

PHY203
Exam #3
Chapters 9-11,15
Mon., 5/1/23

Solutions

Exam3S23

1. A wheel released from rest is rotating with constant angular acceleration of 2.80 rad/s^2 . We want to find some properties of the wheel after it has been rotating for 6.50 s.

a. List the known quantities below:

Parameter	Known Value
θ_0	0
θ_f	
ω_0	0
ω_f	
α	2.80 rad/s^2
t	6.50 s

b. Find the angular velocity after the wheel has been rotating for 6.50 s. **10**

$$\begin{aligned}\omega &= 0 + 2.80(6.50) \\ &= 18.2 \text{ rad/s}\end{aligned}$$

c. Find the number of revolutions the wheel has turned through after the wheel has been rotating for 6.50 s. **10**

$$\begin{aligned}\theta &= 0 + 0 + \frac{1}{2}(2.80)(6.50)^2 \\ &= \frac{59.15 \text{ rad}}{2\pi} = 9.41 \text{ rev}\end{aligned}$$

d. Find the tangential speed of a point 0.800 m from the axis of rotation after the wheel has been rotating for 6.50 s. **10**

$$\begin{aligned}v_{\text{tan}} &= \omega r = (18.2)(0.800) \\ &= 14.6 \text{ m/s}\end{aligned}$$

2. A solid sphere with mass 5.00 kg and radius 1.50 m is rolling without slipping on a horizontal surface. The linear speed of the sphere is 6.50 m/s.

a. List the known quantities below:

Parameter	Known Value
F	
r	1.50 m
m	5.00 kg
v	6.50 m/s

b. Find the moment of inertia of the sphere about its central axis. 5

$$I = \frac{2}{5} (5.00) (1.50)^2 = 4.50 \text{ kg m}^2$$

c. Find the angular speed of the sphere about an axis through the center of mass. 5

$$\omega = \frac{v_{cm}}{r} = \frac{6.50}{1.50} = 4.33 \text{ rad/s}$$

d. Find the kinetic energy of the sphere. 10 / 15

$$\begin{aligned} E &= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \\ &= \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{2}{5} m r^2 \right) \frac{v^2}{r^2} \\ &= \frac{1}{2} m v^2 \left(1 + \frac{2}{5} \right) = \frac{1}{2} (5.00) (6.50)^2 (1.4) = 108 \text{ J} \end{aligned}$$

e. An additional kinetic energy of 25.0 J is added to the sphere. Find its new linear speed. 15 / 10

$$\begin{aligned} 108 + 25 &= 133 \text{ J} \\ 133 &= \frac{1}{2} m v'^2 (1.4) \\ &= \frac{1}{2} (5.00) v'^2 (1.4) \\ v'^2 &= \frac{133}{(2.50)(1.4)}, \quad v' = 7.03 \text{ m/s} \end{aligned}$$

3. A block of mass 2.50 kg is attached to a spring, stretched by 0.450 m and released at $t=0$. At a time of 0.330 s, the block is at an x -position of -0.150 m.

a. Identify and list the known quantities (in SI units) (these could include $m, A, x, t, v, a, E, T, f, \omega, \dots$)

$$m = 2.50 \text{ kg}, \quad A = 0.450 \text{ m}, \quad t = 0.330 \text{ s}, \quad x = -0.150 \text{ m}$$

b. Find the angular frequency of the block-spring combination. 15

$$\begin{aligned}x &= A \cos \omega t \\-0.150 &= 0.450 \cos \omega (0.330) \\ \cos(0.33\omega) &= -0.333 \\ 0.33\omega &= 1.91 \\ \omega &= 5.79 \text{ rad/s}\end{aligned}$$

c. Find the maximum velocity of the block. 10

$$\begin{aligned}v &= -\omega A \sin \omega t \\ v_{\text{max}} &= \omega A = (5.79)(0.450) \\ &= 2.60 \text{ m/s}\end{aligned}$$

d. Find the energy of the block-spring combination. 10

$$\begin{aligned}k &= m\omega^2 = (2.50)(5.79)^2 = 83.8 \text{ N/m} \\ E &= \frac{1}{2} k A^2 = \frac{1}{2} (83.8)(0.450)^2 \\ &= 8.49 \text{ J}\end{aligned}$$

Exam3S23alt

1. A wheel released from rest is rotating with constant angular acceleration of 3.80 rad/s^2 . We want to find some properties of the wheel after it has been rotating for 6.00 s .

a. List the known quantities below:

Parameter	Known Value
θ_0	0
θ_f	
ω_0	0
ω_f	
α	3.80 rad/s^2
t	6.00 s

b. Find the angular velocity after the wheel has been rotating for 6.00 s . **10**

$$\begin{aligned}\omega &= 0 + 3.80(6.00) \\ &= 22.8 \text{ rad/s}\end{aligned}$$

c. Find the number of revolutions the wheel has turned through after the wheel has been rotating for 6.00 s . **10**

$$\begin{aligned}\theta &= 0 + 0 + \frac{1}{2}(3.80)(6.00)^2 \\ &= \frac{68.4 \text{ rad}}{2\pi} = 10.9 \text{ rev}\end{aligned}$$

d. Find the tangential speed of a point 0.800 m from the axis of rotation after the wheel has been rotating for 6.00 s . **10**

$$\begin{aligned}v_{\text{tan}} &= \omega r = (22.8)(0.800) \\ &= 18.2 \text{ m/s}\end{aligned}$$

2. A solid sphere with mass 6.00 kg and radius 1.25 m is rolling without slipping on a horizontal surface. The linear speed of the sphere is 8.50 m/s.

a. List the known quantities below:

Parameter	Known Value
F	
r	1.25 m
m	6.00 kg
v	8.50 m/s

b. Find the moment of inertia of the sphere about its central axis. 5

$$I = \frac{2}{5} (6.00)(1.25)^2 = 3.75 \text{ kg m}^2$$

c. Find the angular speed of the sphere about an axis through the center of mass. 5

$$\omega = \frac{v_{cm}}{r} = \frac{8.50}{1.25} = 6.80 \text{ rad/s}$$

d. Find the kinetic energy of the sphere. 10/15

$$\begin{aligned} E &= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{2}{5} m r^2 \right) \frac{v^2}{r^2} \\ &= \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \left(\frac{2}{5} \right) = \frac{1}{2} m v^2 (1.4) \\ &= \frac{1}{2} (6.00) (8.50)^2 (1.4) = 303 \text{ J} \end{aligned}$$

e. An additional kinetic energy of 25.0 J is added to the sphere. Find its new linear speed. 15/10

$$\begin{aligned} 303 + 25 &= 328 \text{ J} \\ 328 \text{ J} &= \frac{1}{2} m v'^2 (1.4) \\ &= \frac{1}{2} (6.00) v'^2 (1.4) \\ v'^2 &= \frac{328}{(3.00)(1.4)} \quad v' = 8.84 \text{ m/s} \end{aligned}$$

3. A block of mass 3.50 kg is attached to a spring, stretched by 0.500 m and released at $t=0$. At a time of 0.430 s, the block is at an x -position of -0.150 m.

a. Identify and list the known quantities (in SI units) (these could include $m, A, x, t, v, a, E, T, f, \omega, \dots$)

$$m = 3.50 \text{ kg}, A = 0.500 \text{ m}, t = 0.430 \text{ s}, x = -0.150 \text{ m}$$

b. Find the angular frequency of the block-spring combination. 15

$$x = A \cos \omega t$$

$$-0.150 = (0.500) \cos \omega (0.430)$$

$$\cos (0.430 \omega) = -0.300$$

$$0.430 \omega = 1.875$$

$$\omega = 4.36 \text{ rad/s}$$

c. Find the maximum velocity of the block. 10

$$v = -\omega A \sin \omega t$$

$$v_{\text{max}} = \omega A$$

$$= (4.36)(0.500)$$

$$= 2.18 \text{ m/s}$$

d. Find the energy of the block-spring combination. 10

$$E = \frac{1}{2} k A^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \frac{1}{2} (66.5) (0.500)^2$$

$$= 8.32 \text{ J}$$

$$k = m \omega^2 = (3.50) (4.36)^2$$
$$= 66.5 \text{ N/m}$$