Your Name: ______________________

PHY203
Exam #1
Chapters 1-3
Fri., 2/16/18

Solutions

1. A ball is thrown straight up from a cliff of height 250 m, with an initial speed of 35.0 m/s at t=0. Take "up" as the positive x-direction and x=0 at ground level.

a. Find the velocity and acceleration at the highest point (magnitudes and signs).

$V = 0$

$a = -9.81\ m/s^2$

b. Find the position of the ball at its highest point.

$V^2 = V_0^2 + 2a\Delta x$

$0 = (35.0)^2 + 2(-9.81)\Delta x$

$\Delta x = 62.4\ m$

$62.4 + 250 = 312\ m$

c. Find the time at which the ball is at a position one third of the way to the highest point (from ground level).

$x = \frac{312}{3} = 104\ m$

$x = x_0 + v_0 t + \frac{1}{2} a t^2$

$104 = 250 + 35.0t + \frac{1}{2}(-9.81)t^2$

$t = \frac{35.0 \pm \sqrt{35.0^2 \cdot 2 \cdot 9.81 \cdot 146}}{9.81}$

$t = 10.1\ s$

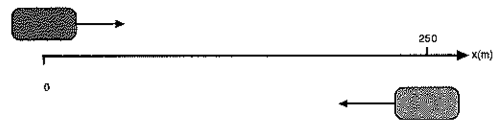
d. Find the velocity and acceleration (magnitudes and signs) of the ball when it is one third of the way to the highest point.

$a = -9.81\ m/s^2$ always

$V = v_0 + at$

$= 35.0 + (-9.81)(10.1)$

$= -64.0\ m/s$



2. Two trains approach each other on parallel tracks. At a time of t=0 the trains are separated by 250 m. At t=0 the train on the left is at rest and starts to travel with a constant acceleration of 2.50 m/s^2 to the right. At t=0 the train on the right is at rest and begins to accelerate at a magnitude of 3.50 m/s^2 to the left. The train accelerates for 4.00 s then continues at a constant speed.

a. Using the coordinate system depicted above, write an equation of motion (x vs. t) for the train on the left:

$x_l=$ $\frac{1}{2}(2.50)t^2 = 1.25t^2$

b. Using the coordinate system depicted above, write equations of motion (x vs. t) for the train on the right for t<4.00 s <u>and</u> t≥4.00s:

t<4.00 s $x_r=$ $250 - \frac{1}{2}(3.50)t^2 = 250 - 1.75t^2$

$x(4) = 250 - 1.75(4.00)^2 = 222\text{ m}$

$v(4) = (-3.50)(4) = -14.0\text{ m/s}$

t≥4.00 s $x_r=$

$222 - 14(t-4)$

c. Find the time at which the centers of the trains are side-by-side.

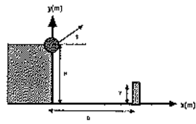
$1.25t^2 = 222 - 14.0(t-4)$

$= 222 - 14.0t + 56.0$

$1.25t^2 + 14.0t - 278 = 0$

$t = \dfrac{-14.0 \pm \sqrt{14.0^2 + 4 \cdot 1.25 \cdot 278}}{2.50}$

$= 10.3\text{ s}$



3. A cannonball is shot from a cliff of height H = 225 m at a castle wall. The initial velocity of the ball is v_o at an angle of θ with respect to the horizontal direction. The castle wall is 65.0 m high and a horizontal distance of D = 850 m from the base of the wall. Assume the initial vertical speed is 65.5 m/s and that the cannonball just grazes the top of the wall.

a. Find the time it takes the ball to reach the wall.

$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2$$
$$65 = 225 + 65.5t + \tfrac{1}{2}(-9.81)t^2$$
$$4.905 - 65.5t - 160 = 0$$
$$t = \frac{65.5 \pm \sqrt{65.5^2 + 4 \cdot 4.905 \cdot 160}}{9.81}$$
$$= 15.4\ s$$

b. Find the initial velocity of the cannonball and <u>write it in vector notation</u> using the coordinate system above.

x: $850 = v_{0x}(15.4)$

$v_{0x} = 55.1\ m/s$

$$\vec{v}_0 = (55.1\hat{i} + 65.5\hat{j})\ m/s$$

c. Find the y position of the cannonball at its highest point.

$$v_y^2 = v_{y0}^2 + 2a_y \Delta y$$
$$0 = 65.5^2 + 2(-9.81)\Delta y$$
$$\Delta y = 219\ m$$
$$+\ 225$$
$$y = 444\ m$$

d. Find the velocity and acceleration of the cannonball in vector notation at its highest point.

$\vec{a} = -9.81\ m/s^2\ \hat{j}$ always

$\vec{v} = v_{0x}\hat{i} = 55.1\hat{i}\ m/s$

1. A ball is thrown straight up from a cliff of height 200 m, with an initial speed of 45.0 m/s at t=0. Take "up" as the positive x-direction and x=0 at ground level.

a. Find the velocity and acceleration at the highest point (magnitudes and signs).

$$v = 0$$

$$a = -9.81 \text{ m/s}^2$$

b. Find the position of the ball at its highest point.

$$v^2 = v_0^2 + 2a\,\Delta x$$

$$0 = 45.0^2 + 2(-9.81)\,\Delta x$$

$$\Delta x = 103$$

$$x = 103 + 200 = 303 \text{ m.}$$

c. Find the time at which the ball is at a position one third of the way to the highest point (from ground level).

$$x = \frac{303}{3} = 101 \text{ m}$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$101 = 200 + 45.0t + \frac{1}{2}(-9.81)t^2$$

$$4.905t - 45.0t - 99 = 0$$

$$t = \frac{45.0 \pm \sqrt{45.0^2 + 4 \cdot 4.905 \cdot 99}}{9.81}$$

$$t = 11.0 \text{ s}$$

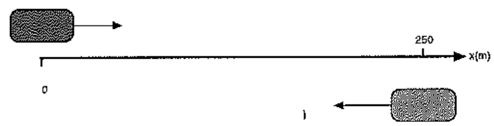
d. Find the velocity and acceleration (magnitudes and signs) of the ball when it is one third of the way to the highest point.

$$a = -9.81 \text{ m/s}^2 \text{ always}$$

$$v = v_0 + at$$

$$= 45.0 + (-9.81)(11)$$

$$= -63.0 \text{ m/s}$$



2. Two trains approach each other on parallel tracks. At a time of t=0 the trains are separated by 250 m. At t=0 the train on the left is at and starts to travel with a constant acceleration of 3.50 m/s^2 to the right. At t=0 the train on the right is at rest and begins to accelerate at a magnitude of 4.50 m/s^2 to the left. The train accelerates for 5.00 s then continues at a constant speed.

a. Using the coordinate system depicted above, write an equation of motion (x vs. t) for the train on the left:

x_l= $\frac{1}{2}(3,50)t^2 = 1,75t^2$

b. Using the coordinate system depicted above, write equations of motion (x vs. t) for the train on the right for t<5.00 s <u>and</u> t≥5.00s:

t<5.00 s x_r= $250 - \frac{1}{2}(4,50)t^2 = 250 - 2,25t^2$

$x(5) = 250 - 2,25(5)^2 = 194\,m$

$V(4) = -(4,50)(5) = -22,5\,m/s$

t≥5.00 s x_r =

$194 - (22,5)(t-5)$

c. Find the time at which the centers of the trains are side-by-side.

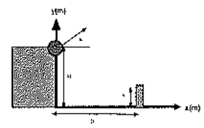
$1,75t^2 = 194 - (22,5)(t-5)$

$= 194 - 22.5t + 112,5$

$1,75t^2 + 22,5t - 306,5 = 0$

$t = \frac{-22,5 \pm \sqrt{22,5^2 + 4 \cdot 1,75 \cdot 306,5}}{3,50}$

$t = 8,28\,s$



3. A cannonball is shot from a cliff of height H = 275 m at a castle wall. The initial velocity of the ball is v_0 at an angle of θ with respect to the horizontal direction. The castle wall is 55.0 m high and a horizontal distance of D = 750 m from the base of the wall. Assume the initial vertical speed is 55.5 m/s and that the cannonball just grazes the top of the wall.

a. Find the time it takes the ball to reach the wall.

$$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$$

$$55.0 = 275 + 55.5t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 55.5t - 220 = 0$$

$$t = \frac{55.5 \pm \sqrt{55.5^2 + 4 \cdot 4.905 \cdot 220}}{9.81}$$

$$t = 14.4s$$

b. Find the initial velocity of the cannonball and <u>write it in vector notation</u> using the coordinate system above.

$$x: \quad 750 = v_{0x}(t) = v_{0x}(14.4)$$

$$v_{0x} = 52.0 \text{ m/s}$$

$$\vec{v_0} = (52.0\hat{i} + 55.5\hat{j}) \text{ m/s}$$

c. Find the y position of the cannonball at its highest point.

$$v^2 = v_{y0}^2 + 2a_y \Delta y$$

$$0 = 55.5^2 + 2(-9.81)\Delta y$$

$$\Delta y = 157 \text{m}$$

$$+ 275 \text{m}$$

$$y = 432 \text{m}$$

d. Find the velocity and acceleration of the cannonball in vector notation at its highest point.

$$\vec{a} = -9.81 \text{ m/s}^2 \hat{j} \text{ always}$$

$$\vec{v} = v_{0x}\hat{i} = 52.0\hat{i} \text{ m/s}$$