Your Name: _____

PHY203
Exam #1
Chapters 1-4
Fri., 9/30/22

Solutions

1. Staring at x=y=0, a person walks due West for 250 m, then due North for 225 m, then 30.0° North of East for 200 m. Assume the trip takes 5.00 minutes. Take East as the +x-direction and North as the +y-direction

a. Find the average speed of the trip. **10**

$$\text{speed}_{avg} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{250\text{m} + 225\text{m} + 200\text{m}}{300\text{ s}} = 2.25\ \frac{\text{m}}{\text{s}}$$

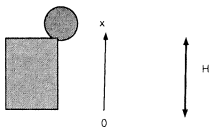
b. Find the average velocity of the trip in vector notation. **20**

$$\vec{vel}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\Delta \vec{r} = -250\text{m}\,\hat{i} + 225\text{m}\,\hat{j} + 200\text{m}(\cos 30.0^\circ\,\hat{i} + \sin 30^\circ\,\hat{j})$$

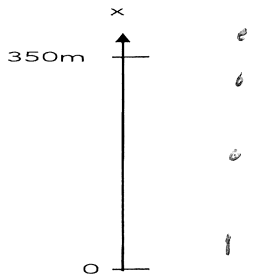
$$= -76.8\text{m}\,\hat{i} + 325\text{m}\,\hat{j}$$

$$\vec{v}_{avg} = \frac{-76.8\text{m}\,\hat{i} + 325\text{m}\,\hat{j}}{300\text{ s}} = (-0.256\,\hat{i} + 1.08\,\hat{j})\ \frac{\text{m}}{\text{s}}$$



2. A ball is thrown straight down from a cliff of height H=350 m with an initial speed of 8.50 m/s starting at t=0. Among other things, we want to find the time it takes the ball to hit the ground. (Ignore air resistance.)

a. Produce a motion diagram of the ball: **5**



b. Fill out the table of known quantities for the ball: **5**

Parameter	Known Value
x_0	350m
x_f	0
v_0	$-8.50\ m/s$
v_f	
a	$-9.81\ m/s^2$
t	

c. Find the time it takes the ball to hit the ground. **15**

$$A: \quad 0 = 350 - 8.50t - \frac{1}{2}gt^2$$

$$4.905t^2 + 8.50t - 350 = 0$$

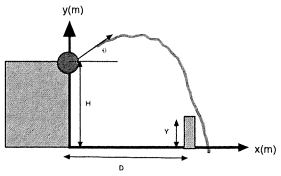
$$t = \frac{-8.50 \pm \sqrt{8.50^2 + 4 \cdot 4.905 \cdot 350}}{9.81}$$

$$= 7.639$$

d. Find the velocity just before the ball hits the ground (magnitude and sign). **5**

$$v = -8.50 - g(7.63)$$

$$= -83.3\ m/s$$



3. A cannonball is shot from a cliff of height H = 120 m at a castle wall, as shown above. The castle wall is Y = 65.0 m high and a horizontal distance D = 225 m from the cannon. The initial horizontal component of velocity of the ball is 35.0 m/s in magnitude. Take y = 0 at ground level. Assume the ball just grazes the top of the wall. Among other things, we want to find the initial velocity of the cannonball. (Ignore air resistance.)

a. Make a sketch of the trajectory of the ball from start until it hits the ground. **5**

b. Fill out the tables of known values. (Take the "final" position as when the ball grazes the wall.) **5**

Parameter	Known Value
x_0	0
x_f	225 m
v_{x0}	35.0 m/s
v_{xf}	35.0 m/s
a_x	0
t	

Parameter	Known Value
y_0	120 m
y_f	65.0 m
v_{y0}	
v_{yf}	
a_y	$-9.81\ m/s^2$
t	

c. Find the initial velocity of the cannonball and write it in <u>vector notation</u> using the coordinate system above. **20**

x: A: $225 = 0 + 35.0t + 0,\ t = 6.43\ s$

y: A: $65.0 = 120 + v_{y0}(6.43) - \frac{1}{2}g(6.43)^2$

$v_{y0} = 23.0\ m/s$

$\vec{v_0} = (35.0\hat{i} + 23.0\hat{j})\ m/s$

d. Find the velocity and acceleration of the cannonball at its highest point and write them in vector notation. **10**

$\vec{a} = -9.81\hat{j}\ m/s^2$

$\vec{v} = \vec{v_{0x}} = 35.0\hat{i}\ m/s$

1. Staring at x=y=0, a person walks due West for 300 m, then due North for 275 m, then 30.0° North of East for 250 m. Assume the trip takes 5.00 minutes. Take East as the +x-direction and North as the +y-direction

a. Find the average speed of the trip. **10**

$$\text{speed}_{avg} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{300\text{m} + 275\text{m} + 250\text{m}}{300\text{s}}$$

$$= 2.75\ \text{m/s}$$

b. Find the average velocity of the trip in vector notation. **20**

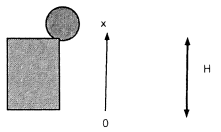
$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\Delta\vec{r} = -300\text{m}\,\hat{i} + 275\text{m}\,\hat{j} + 250\text{m}(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) - 0$$

$$= -83.5\text{m}\,\hat{i} + 400\text{m}\,\hat{j}$$

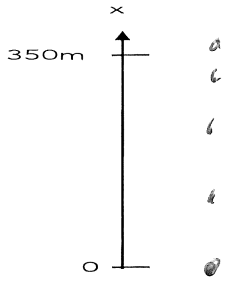
$$\vec{v}_{avg} = \frac{-83.5\text{m}\,\hat{i} + 400\text{m}\,\hat{j}}{300\text{s}}$$

$$= (-0.278\,\hat{i} + 1.33\,\hat{j})\ \text{m/s}$$



2. A ball is thrown straight down from a cliff of height H=400 m with an initial speed of 9.50 m/s starting at t=0. Among other things, we want to find the time it takes the ball to hit the ground. (Ignore air resistance.)

a. Produce a motion diagram of the ball: **5**



b. Fill out the table of known quantities for the ball: **5**

Parameter	Known Value
x_0	400
x_f	0
v_0	$-9.50\ m/s$
v_f	
a	$-9.81\ m/s^2$
t	

c. Find the time it takes the ball to hit the ground. **15**

$$A: \quad 0 = 400 - 9.50t - \frac{1}{2}gt^2$$

$$4.905t^2 + 9.50t - 400 = 0$$

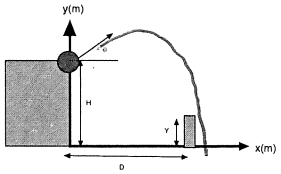
$$t = \frac{-9.50 \pm \sqrt{9.50^2 + 4 \cdot 4.905 \cdot 400}}{9.81}$$

$$= 8.11\ s$$

d. Find the velocity just before the ball hits the ground (magnitude and sign). **5**

$$v = -9.50 - g(8.11)$$

$$= -89.1\ m/s$$



3. A cannonball is shot from a cliff of height H = 140 m at a castle wall, as shown above. The castle wall is Y = 65.0 m high and a horizontal distance D = 275 m from the cannon. The initial horizontal component of velocity of the ball is 40.0 m/s in magnitude. Take y = 0 at ground level. Assume the ball just grazes the top of the wall. Among other things, we want to find the initial velocity of the cannonball. (Ignore air resistance.)

a. Make a sketch of the trajectory of the ball from start until it hits the ground. **5**

b. Fill out the tables of known values. (Take the "final" position as when the ball grazes the wall.) **5**

Parameter	Known Value
x_0	0
x_f	275 m
v_{x0}	40.0 m/s
v_{xf}	40.0 m/s
a_x	0
t	

Parameter	Known Value
y_0	140 m
y_f	65.0 m
v_{y0}	
v_{yf}	
a_y	$-9.8\ m/s^2$
t	

c. Find the initial velocity of the cannonball and write it in <u>vector notation</u> using the coordinate system above. **20**

x: A: $275 = 0 + 40.0t + 0$, $t = 6.88$ s

y: A': $65.0 = 140. + v_{y0}(6.88) - \frac{1}{2}g(6.88)^2$

$v_{y0} = 22.8$ m/s

$\vec{v}_0 = (40.0\,\hat{i} + 22.8\,\hat{j})$ m/s

d. Find the velocity and acceleration of the cannonball at its highest point and write them in vector notation. **10**

$\vec{a} = -9.81\,\hat{j}\ m/s^2$

$\vec{v} = \vec{v}_0 = 40.0\,\hat{i}$ m/s