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Zero-Temperature Relaxation in Pure Fermi Liquids and Ferromagnetic Metals

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We discuss the effect of the zero-temperature transverse attenuation in spin-polarized Fermi liquids on related phenomena in helium and electron systems. In helium Fermi liquids, the magnetic dipole-dipole interaction leads to a transfer of transverse attenuation to longitudinal processes resulting in finite sound attenuation and effective viscosity even at zero temperature. In Heisenberg ferromagnetic metals, this Fermi-liquid effect affects the attenuation of ferromagnetic magnons as a result of exchange coupling between spins of ferromagnetic and conduction electrons.

1. INTRODUCTION

In contrast to all other dissipative processes in pure Fermi liquids, the transverse relaxation time $\tau_\perp$ and transverse spin diffusion $D_\perp$ in spin-polarized Fermi liquids do not increase with decreasing temperature as $1/T^2$, but saturate and remain finite at $T \to 0$. Longitudinal processes in exchange systems, *i.e.* processes which do not change the direction of polarization, do not exhibit any zero-temperature attenuation irrespective of spin polarization. The low-temperature saturation of transverse diffusion and relaxation was predicted on the basis of conservation law and symmetry arguments[1,2] and confirmed by transport calculations for degenerate Fermi gases[3–5] and dense Fermi liquids.[6] The effect was observed in low-temperature spin dynamics experiments in liquid $^3He\uparrow$[7] and $^3He\uparrow - {}^4He$ mixtures.[8]

The transverse zero-temperature relaxation time is $\tau_\perp(T=0) \sim (Nv_F d^2)^{-1}(T_F/\beta H)^2$ for fermions with Fermi velocity (temperature) v_F (T_F), magnetic moment β, cross-section d^2, and density N. Since the usual relaxation time is $\tau_\perp(H=0) \sim (Nv_F d^2)^{-1}(T_F/T)^2$, the transition from the temperature-driven to polarization-driven attenuation occurs at $T \sim \beta H$ when the space between the spin-up and spin-down Fermi spheres is comparable to the thermal smearing of the Fermi surface.

The zero-temperature attenuation can be described by a pole contribution in the mean field, and is, in this sense, similar to the Landau damping in plasma.[5] This pole term does not contribute to other observables outside transverse dynamics for which spin polarization opens phase space between the spin-up and spin-down Fermi spheres necessary for collisionless decay of magnons with finite $\mathbf{k}$ at $T = 0$. By now, the existence of the zero-temperature attenuation in spin-polarized Fermi liquids is well established.

Transverse spin dynamics is coupled to other processes thus transferring temperature saturation. In helium, the magnetic dipole and non-linear couplings transfer the zero-temperature attenuation into longitudinal channels. In Heisenberg ferromagnets, the zero-temperature transverse attenuation affects ferromagnetic properties via exchange spin coupling of ferromagnetic and conduction electrons.

2. DIPOLE COUPLING TO LONGITUDINAL DYNAMICS IN HELIUM

The transverse attenuation is the sole zero-temperature relaxation in *pure exchange* Fermi liquids. It can be transferred to longitudinal channels by magnetic dipole or non-linear coupling. We will study the former though the latter also leads to interesting effects especially near the Castaing instability.

There are two dipole mechanisms. The magnetic dipole interaction permits longitudinal spin-flip processes and leads at $T = 0$ to the finite longitudinal collision integral with dipole vertex. Second, the dipole interaction couples longitudinal modes to attenuating spin waves. We will consider the first mechanism. The second one, which has been described in Ref.,[9] is weaker except for high polarization.

The resulting longitudinal attenuation τ_{eff} is smaller than $\tau_{\perp}(T = 0)$ by a factor $(E_d/T_F)^2$ where $E_d = \beta^2 Z^2 m^{3/2} T_F^{3/2}/\hbar^3$, and Z is the microscopic parameter in the quasiparticle pole term. The transition from the temperature-driven to this effective attenuation occurs in sub-μK region, well below the saturation of transverse spin diffusion. This effect slightly resembles the temperature saturation of the dipole relaxation time T_1.[10] For $^3He\uparrow$ this temperature is below the superfluid transition when the theory of normal Fermi liquids cannot be applied directly. The results can be applied, without modifications, to $^3He\uparrow - {}^4He$ mixtures.

In order to avoid separate calculations for various modes in $^3He\uparrow$ and $^3He\uparrow - {}^4He$, we calculated the sound attenuation in a generic polarized

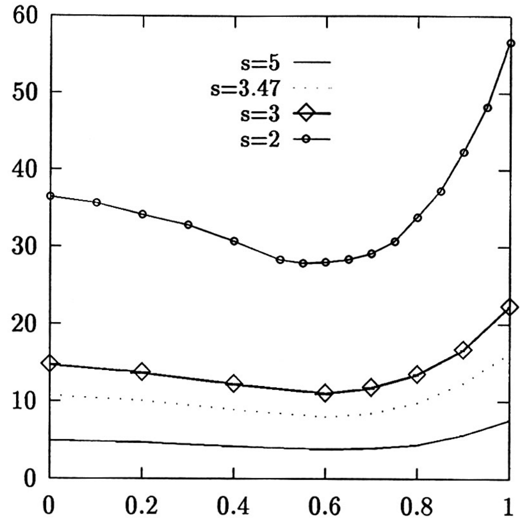

Fig. 1. $I(s,x)$ as a function of $x = cos\theta$ for $s = 2; 3; 3.47; 5$

Fermi liquid at $T = 0$ and extracted the effective mode-independent relaxation time τ_{eff} and viscosity η_{eff}. These parameters can be used in conjunction with standard hydrodynamic or *hf* description[2,11] of $^3He\uparrow$ and $^3He\uparrow - ^4He$. Although the dipole interaction in helium is weak and the effective time τ_{eff} is long, it provides the zero-temperature cut-off for longitudinal relaxation and transport. Since helium does not have any impurities, these limiting cut-offs can be observed at ultra-low temperatures.

The dipole interaction enters the non-vanishing at $T = 0$ collision integral via the scattering probability

$$W_{\mathbf{p}_1\mathbf{p}_2}^{\mathbf{p}_3\mathbf{p}_4} = \left(\frac{E_d}{T_F}\right)^2 \left[\frac{(\mathbf{p}_1-\mathbf{p}_3)_z^3(\mathbf{p}_1-\mathbf{p}_3)_+}{(\mathbf{p}_1-\mathbf{p}_3)^4} - \frac{(\mathbf{p}_1-\mathbf{p}_4)_z^3(\mathbf{p}_1-\mathbf{p}_4)_+}{(\mathbf{p}_1-\mathbf{p}_4)^4}\right]^2$$

The resulting sound attenuation is (for more details see[12])

$$Im\,\omega = \frac{E_d^2}{16\pi^5\hbar T_F}\left(\frac{\beta H}{T_F}\right)^2 I\left(s\cos\theta\right) \tag{1}$$

where $sv_F = \omega/k$ is the sound velocity, θ is the angle between $\mathbf{H}$ and $\mathbf{k}$, and the function $I\left(s\cos\theta\right)$ is plotted in Figure 1.

The most important difference between (1) and the attenuation resulting from the coupling between sound and spin waves is the k^2-dependence of the attenuation[9] originating from the $\mathbf{k}\cdot\mathbf{v}$ factor in the coupling coefficient at $kv_F \ll \Omega_0$. At higher frequencies, the factor $(kv_F)^2$ in Ref.[9] should be replaced by the square of the Larmor frequency Ω_0. At very high frequencies, the attenuation can be obtained by the method of Refs.[4,12] The dipole anisotropy of the fluid dynamics in spin-polarized systems is not surprising.

The sound attenuation provides the relaxation time and viscosity:

$$\begin{aligned}
\frac{1}{\tau_{eff}} &= \frac{E_d^2}{16\pi^5\hbar T_F}\left(\frac{\beta H}{T_F}\right)^2\frac{I(s\cos\theta)}{\xi(s)}, \\
\eta_{eff} &= \frac{1}{5}\rho v_F^2\tau_{eff}\left(1+\frac{1}{3}F_1^{(s)}\right),\ w(s)=\frac{s}{2}\ln\frac{s+1}{s-1}-1, \\
\xi(s) &= s^2\frac{w^2(s^2-1)(3s^2+1)+1}{w(s^2-1)-1}+2
\end{aligned}$$

3. ATTENUATION OF MAGNONS IN HEISENBERG FERROMAGNETIC METALS

The transverse zero-temperature dissipation is inherent to Fermi liquids and, in its original form, does not exist in Heisenberg system of localized spins. We will show that this unique Fermi-liquid mechanism still leads to residual attenuation of magnons in pure ferromagnetic metals. Note, that we are interested not in itinerant magnetism, which always exhibits Fermi-liquid features, but in magnetic system of localized electrons.

The effect is based on exchange coupling $J_1\vec{S}\cdot\vec{\sigma}$ between spins $\vec{S}$ and $\vec{\sigma}$ of ferromagnetic (*e.g.*, $3d$) and conduction (*e.g.*, $4s$) electrons which leads to a small polarization of conduction electrons of the order of $J_1\langle S\rangle/T_F$. Polarization of conduction electrons ensures the propagation of Silin spin waves in this system with finite attenuation $\tau_\perp(T=0)\sim\left(Nv_Fd^2\right)^{-1}\left(T_F/J_1\langle S\rangle\right)^2$. The exchange coupling between these attenuating Silin spin waves and ferromagnetic Heisenberg magnons transfers the zero-temperature attenuation to the magnon system. By the order of magnitude $\tau_\perp^*\sim\tau_\perp\left(J_1/J\right)^2$ where J is the Heisenberg exchange between localized electrons. The competing processes are, obviously, scattering on impurities and spin-lattice processes studied long ago (see, *e.g.*,[14]). The former processes are small in pure metals, while the latter are suppressed at low temperatures.

The Hamiltonians of conduction and localized electrons have the form

$$\begin{aligned}
\varepsilon^e &= \varepsilon^o-\beta_1^e\vec{\sigma}\cdot\vec{H}-J_1\vec{\sigma}\cdot\left\langle\vec{S}\right\rangle/2+\delta\varepsilon, \\
\varepsilon_i^l &= -\beta^l\vec{S}_i\cdot\vec{H}-\frac{1}{2}J_0\langle\vec{\sigma}\rangle\cdot\vec{S}_i-J\sum_{\mathbf{a}}\vec{S}_{\mathbf{i+a}}\cdot\vec{S}_{\mathbf{i}}
\end{aligned}$$

The parameters for conduction electrons already contain the Fermi-liquid renormalization, $\beta_1^e=\beta^e/\left(1+F_0^{(a)}\right)$, $J_1=J_0/\left(1+F_0^{(a)}\right)$. The averages

$$\left\langle\vec{S}\right\rangle=\sum\vec{S}_iN_i,\ \langle\vec{\sigma}\rangle=\sum\vec{\sigma}_in_i=p_Fm\left(\beta_1^e\vec{H}+1/2J_1\left\langle\vec{S}\right\rangle\right)/\pi^2\hbar^3,$$

The Fermi-liquid term for conduction electrons is

$$\delta\varepsilon^e_{\alpha\beta} = -\frac{1}{2}J_1\vec{\sigma}_{\alpha\beta}\cdot\delta\vec{S} + \int f_{\alpha\beta\alpha'\beta'}(\mathbf{p},\mathbf{p}')\delta n_{\beta'\alpha'}(\mathbf{p}')d\Gamma',$$

$$\frac{p_F m}{\pi^2\hbar^3} f_{\alpha\beta\alpha'\beta'}(\mathbf{p},\mathbf{p}') = F_s(\mathbf{p},\mathbf{p}')\delta_{\alpha\beta}\delta_{\alpha'\beta'} + F_a(\mathbf{p},\mathbf{p}')\vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\alpha'\beta'}$$

The equation of motion for the circular component of magnetization $\delta\sigma^+$ is

$$\begin{aligned} iL^+_{coll} &= \left(\omega - \mathbf{k}\cdot\mathbf{v} - \frac{2}{\hbar}\left(\beta^e_1 H + \frac{1}{2}J_1\langle S_z\rangle\right)\right)\delta\sigma^+ + \\ &\quad \left(\mathbf{k}\cdot\mathbf{v}\delta_+ + \frac{2}{\hbar}(n_\uparrow - n_\downarrow)\right)\left(\frac{1}{4}J_1\delta S_+ - \frac{\pi^2\hbar^3}{p_F m}\int F_a(\mathbf{p},\mathbf{p}')\delta\sigma^{+\prime}d\Gamma'\right) \\ \int L^{coll}_{\downarrow\uparrow}d\Gamma &= 0,\ \tau_\perp\int \mathbf{v}L^{coll}_{\downarrow\uparrow}d\Gamma = -\int \mathbf{v}\delta\sigma^+ d\Gamma, \end{aligned}$$

where $n_{\uparrow,\downarrow}$ are the equilibrium distribution functions for spin-up and spin-down particles, $\delta_+ = \delta(\varepsilon - \varepsilon_{0\uparrow}) + \delta(\varepsilon - \varepsilon_{0\downarrow})$.

We look for the solution in the form $\delta\sigma^+ = n_0 + n_1\mathbf{k}\cdot\mathbf{v}$, $\delta S_+ = S_0$. Then the eigenvalue equation reduces to a set of three coupled equations,

$$\begin{aligned} 0 &= \left(\omega - \omega_0 - J\langle S_z\rangle k^2a^2/\hbar\right)S_0 + \frac{p_F m}{2\pi^2\hbar^3}\Omega_0 J_1\langle S_z\rangle n_0, \\ 0 &= (\omega - \Omega_0)n_0 - k^2v_F^2(1 + F_1^{(a)}/3)n_1/3 + J_0 S_0/2\hbar, \\ 0 &= (\omega - \Omega_0\frac{1 + F_1^{(a)}/3}{1 + F_0^{(a)}} + \frac{i}{\tau_\perp})n_1 - (1 + F_0^{(a)})n_0 + \frac{J_0(1 + F_0^{(a)})}{2\hbar\Omega_0}S_0, \end{aligned}$$

where the Larmor frequencies for conduction and localized electrons are $\hbar\Omega_0 = 2(\beta^e H + 1/2J_0\langle S_z\rangle)$ and $\hbar\omega_0 = (\beta^l H + 1/2J_0\langle\sigma_z\rangle)$.

In ferromagnetic metals $J_0\langle S_z\rangle \gg J_0\langle\sigma_z\rangle, \beta H$, and $\Omega_0 \gg \omega_0$. If also $\beta^l H \gg J\langle S_z\rangle k^2a^2$, the residual attenuation of ferromagnetic magnons is

$$Im\omega = -\frac{\beta^l H}{6J_0\langle S_z\rangle}\frac{\langle\sigma_z\rangle}{\langle S_z\rangle}\frac{k^2v_F^2\tau_\perp(1 + F_0^{(a)})(1 + F_1^{(a)}/3)}{1 + \left[\tau_\perp\Omega_0\left(1 + F_1^{(a)}/3\right)/\left(1 + F_0^{(a)}\right)\right]^2}$$

In lower fields the attenuation is proportional to k^4 as in Ref.[15]

The scale of the effect is determined by the exchange J_0 between spins of ferromagnetic and conduction electrons. In metals, the bare $s - d$ exchange $t_2 \sim 1\ eV$ is weakened by screening by one or two orders of magnitude, but is enhanced by the Kondo-like logarithmic renormalization. The transverse exchange field for conduction electrons is[16]

$$\begin{aligned} L_{coh} &= \frac{i\pi}{2\hbar V}\left[\delta S_+(n_\uparrow - n_\downarrow) - \delta\sigma^+\langle S_z\rangle\right] \times \\ &\quad \left[4t_2 + \int\frac{d^3p'}{(2\pi\hbar)^3}P\frac{1}{\epsilon' - \epsilon}\left(4t_1t_2 + t_2^2\left(n'_\uparrow + n'_\downarrow - 2\right)\right)\right] \end{aligned}$$

If the polarization of conduction electrons is low, the direct interaction t_1 disappears from the results. The integral is typical for the theory of metals and diverges logarithmically. The usual cut-off provides the logarithmic renormalization enhancement of the bare exchange t_2:

$$J_0 = 2\pi t_2 \left(1 + (t_2 \nu_F/4) \ln \left[T_F \left(1 + F_0^{(a)}\right) / (\beta^e H + \pi t_2 \langle S_z \rangle)\right]\right)$$

where ν_F is the density of states on the Fermi surface. As a result, J_0 can reach hundreds of K, and the polarization of conduction electrons can exceed one per cent. Then $\tau_\perp$ for conduction electrons can become shorter than 10^{-10} sec, and $\tau_\perp^*$ can reach 10^{-7} sec.

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