# Zero-Temperature Relaxation in Spin-Polarized Fermi Systems

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We describe the effect of zero-temperature attenuation, which has been recently observed in spin dynamics of Fermi liquids, on various processes in helium and ferromagnetic systems. After a brief review of theoretical and experimental data on zero-temperature attenuation in transverse spin dynamics of helium systems, we discuss coupling between longitudinal and transverse processes, the Castaing instability in $^3He$ and $^3He-{}^4He$ mixtures, and applications to pure ferromagnetic metals.

## 1. Introduction

One of the recent developments in physics of Fermi liquids was a discovery of peculiar zero-temperature attenuation in transverse spin dynamics of spin-polarized Fermi liquids. In contrast to all other dissipative processes in pure Fermi liquids, the transverse relaxation time $\tau_\perp$ and the coefficient of transverse spin diffusion $D_\perp$ do not increase with decreasing temperature as $1/T^2$, but saturate and remain finite even at $T \to 0$. By transverse dynamics we mean the dynamics of components of magnetization perpendicular to its equilibrium direction. The transverse processes are excited, for example, by inhomogeneous tipping of spins in NMR experiments. Longitudinal processes in exchange systems, *i.e.,* processes which do not change the direction of polarization, do not exhibit any zero-temperature attenuation irrespective of spin polarization.

The zero-temperature attenuation in transverse dynamics was predicted first on the basis of general conservation law and symmetry arguments[1,2]. This prediction was confirmed by

direct transport calculations for degenerate Fermi gases[3–5] and, later, dense Fermi liquids[6]. The temperature saturation of transverse diffusion and relaxation has been observed in low-temperature spin dynamics experiments in spin-polarized liquid $^3He\uparrow$[7] and $^3He\uparrow - ^4He$ mixtures[8].

The transverse zero-temperature relaxation time is $\tau_\perp(T=0) \sim (Nv_F\sigma)^{-1}(T_F/\beta H)^2$ for a system of fermions with Fermi velocity (temperature) $v_F$ ($T_F$), magnetic moment $\beta$, effective cross-section $\sigma$, and density $N$ in the external magnetic field $H$. Since the usual temperature-driven relaxation time is $\tau_\perp(H=0) \sim (Nv_F\sigma)^{-1}(T_F/T)^2$, the transition from the temperature-driven to polarization-driven transverse attenuation occurs at the temperature $T_a \sim \beta H$ when the phase space between the spin-up and spin-down Fermi spheres is comparable to the thermal smearing of the Fermi spheres.

The reason for such an unusual behavior is that the transverse relaxation and spin diffusion at low temperatures are determined by collisionless decay of magnons. Spin polarization of the Fermi liquid opens phase space between the spin-up and spin-down Fermi spheres necessary to allow these decay processes for magnons with finite $\mathbf{k}$ (inhomogeneously tipped spins) even at $T=0$. Mathematically, the zero-temperature attenuation can be described by a pole contribution in the transverse component of the interaction function, and is, in this sense, similar to the Landau damping in collisionless plasma[5].

Below we will briefly describe theoretical and experimental aspects of this phenomenon, and discuss its consequences. We are interested in both helium and electron systems. In spin-polarized helium Fermi liquids, the zero-temperature transverse attenuation can affect other dynamic processes via the magnetic dipole-dipole interaction and non-linear coupling. Electron Fermi liquids with large degree of spin polarization exist in ferromagnetic metals. In itinerant ferromagnets, the manifestations of the zero-temperature transverse attenuation are similar to those in helium systems (with spin-lattice coupling to longitudinal modes). In Heisenberg ferromagnetic metals, the analogy is less direct: the Fermi-liquid zero-temperature transverse attenuation affects ferromagnetic properties only via exchange coupling of localized ferromagnetic spins to spins of conduction electrons.

In the next Section we give a simple theory of the zero-temperature transverse attenuation. In Sec. 3 we highlight experimental aspects of this phenomenon in helium systems. Then, in Sec.4, we describe the transfer of the zero-temperature attenuation into longitudinal channels by means of magnetic dipole interaction. Sec. 5 deals with Castaing instability in spin dynamics in an inhomogeneous setting. The last Section contains applications to pure ferromagnetic metals.

## 2. Theory

Usually, the conservation laws restrict all low-energy relaxation processes in Fermi liquids to a thin layer (with a relative thickness $T/T_F$) near the Fermi sphere where the occupation numbers $n$ change gradually from 1 to 0. Everywhere else there are either no particles (no "initial" states $n_{in}$), or all states are completely occupied (no space for "final" states $n_{fin}$). The probability of relaxation scattering processes for the fermions, which is proportional to $n_{in}\left(1-n_{fin}\right)$, acquires the factor $(T/T_F)^2$ and is very small. As a result, the relaxation time increases at low temperatures as $(T_F/T)^2$. In spin-polarized Fermi systems the situation is different: if the collision flip the spin of spin-up particle in the region *between* spin-up and spin-down Fermi spheres, this particle can easily change its energy since all spin-down states in this area are unoccupied.

Mathematically this means that the collision integral of the form

$$\int d^3p_1\, d^3p_2\, d^3p_3\, d^3p_4 W \delta\left(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4-\hbar\omega-2\beta_1 H\right)\delta\left(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4\right) \qquad (1)$$
$$\times\left[n_{1\uparrow}n_{2\uparrow}\left(1-n_{3\uparrow}\right)\left(1-n_{4\downarrow}\right)+n_{1\uparrow}n_{2\downarrow}\left(1-n_{3\downarrow}\right)\left(1-n_{4\downarrow}\right)\right]$$

does not go to zero at $T\to 0$ as $(T/T_F)^2$, but remains finite and is proportional, at small polarization $\beta H/T_F$, to $(\beta H/T_F)^2$.

This mechanism of zero-temperature attenuation requires a spin flip during collision and exists in exchange systems only in transverse spin channel, *i.e.*, for processes with changes in direction of magnetization such as spin waves, spin echo, and other NMR effects. The

attenuation for exchange longitudinal processes - processes without changes in direction of magnetization - involves similar collision integrals, but with equal numbers of up and down arrows, and vanishes at $T \to 0$ as $(T/T_F)^2$.

In general, there should be no dissipative collisions at $T = 0$. In Fermi liquids at $T = 0$ all incoherent processes, including the transverse ones, should disappear, and the interaction should be described by the Landau interaction function, *i.e.,* coherent molecular field. This seems to contradict the existence of zero-temperature attenuation. This contradiction is resolved if one notes that the microscopic equation for the transverse component of the Landau interaction function contains the integrals of the form[5]

$$\sigma \cdot \sigma' \int \frac{d^3p'}{(2\pi)^3} \left[ \frac{1 - n'_{\downarrow} - n'_{1\uparrow}}{p^2 + p_1^2 - p'^2 - p_1'^2 - i0\,\mathrm{sign}\,(p' - p_{F\downarrow})} - \mathrm{P}\frac{1}{p^2 + p_1^2 - p'^2 - p_1'^2} \right]$$

The imaginary (pole) part of this interaction function reproduces the integral (1). Therefore, the zero-temperature transverse attenuation can be interpreted as the imaginary (pole) part of the interaction function. In this sense, the zero-temperature attenuation is a direct analog of the Landau damping in collisionless plasma. Needless to say, this pole part disappears in the absence of polarization or for longitudinal processes.

The above simple theory is directly applicable to low-density Fermi liquids such as the $^3He$ component of $^3He \uparrow - ^4He$ mixtures, or to dense Fermi liquids at low spin polarization. The situation in dense highly polarized Fermi liquids is more complicated. Here the molecular fields acting on slightly tilted spin-ups and spin-downs are different because of the large distance between spin-up and spin-down Fermi surfaces. [This effect is analogous to the well-known particle-hole anisotropy away from the Fermi surface]. Then the microscopic equations of transverse spin dynamics, *i.e.* the equations for (small) transverse components of slightly tilted spins, have the form of two separate equations for tilted spin-ups and spin-downs with different molecular fields. It is not clear how these equations translate into macroscopic equations of spin dynamics, and what are the necessary modifications of the Leggett equation of macroscopic spin dynamics.

The Leggett equation of Fermi-liquid spin dynamics is a closed equation in macroscopic

magnetic moment $\mathbf{M}$. This equation in its original form cannot be applied to highly-polarized Fermi liquids: the molecular field term in the effective magnetic field inevitably involves integration of the magnetization distribution $\mathbf{m}$ with the Fermi-liquid interaction function $f$ between spin-up and spin-down Fermi spheres

$$\int f(\mathbf{p},\mathbf{p}')\,\mathbf{m}(\mathbf{p}')\left[n_{\uparrow}^{(0)}(\mathbf{p}')-n_{\downarrow}^{(0)}(\mathbf{p}')\right]d^3p'/(2\pi\hbar)^3 \tag{2}$$

Integral (2) can be written via the macroscopic magnetic moment

$$\mathbf{M}=\int \mathbf{m}(\mathbf{p}')\left[n_{\uparrow}^{(0)}(\mathbf{p}')-n_{\downarrow}^{(0)}(\mathbf{p}')\right]d^3p'/(2\pi\hbar)^3 \tag{3}$$

only if the interaction function is constant between the Fermi spheres. This is true either for dilute Fermi gases[2], or, as in the original Leggett derivation, at very low polarization when the Fermi spheres almost coincide.

### 3. Experiment

Recent experiments at Nottingham[8,9] have used the techniques of pulsed nuclear magnetic resonance to measure both transverse and longitudinal spin diffusion in a saturated ($x_3 = 6.4\%$) solution of $^3He$ in $^4He$. The active region of the experimental cell consisted of a 1 $mm$ diameter Stycast tube, 20 $mm$ in length around which an $R.F.$ coil (3 $mm$ radius, two turns of 0.6 $mm$ diameter $Cu$-wire) was positioned. A main field of 8.8 $T$ and a uniform gradient of 80 $mT/m$ were applied to the cell, in a direction normal to the axis of the tube. The polarization of the saturated solution in such a field was a few per cent and the Leggett spin rotation parameter $\mu M_0$ had a value of about 4 at the lowest temperatures.

In order to measure the transverse spin diffusion coefficient a $\theta - t_1 - 180°$ $R.F.$ pulse sequence was applied to the $^3$He spin system resulting in a spin-echo at time $2t_1$. The presence of transverse spin diffusion causes this echo signal to decay with inter-pulse time $t_1$. The height $h$ and phase $\phi$ of the spin-echo was fitted to the Leggett-Rice equations

$$(1+\mu^2M_0^2\cos^2\theta)\ln(h(t_1))+\frac{\mu^2M_0^2}{2}\sin^2\theta(h^2(t_1)-1)=-\frac{2}{3}\gamma^2G^2D_{\perp}t_1^3$$

$$\phi = -\mu M_0 \cos\theta \ln(h(t_1))$$

to obtain values for the spin-rotation parameter, $\mu M_0$, and the transverse spin diffusion coefficient $D_\perp$.

Longitudinal spin diffusion was measured using a technique similar to that used by Nunes et al[10]. By applying a $\theta = 180°$ *R.F.* pulse, the magnetization in the active region of the cell can be inverted. A longitudinal magnetization gradient will thus be set up between this region and the remainder of the cell. This will result in the diffusion of spins into this region to recover equilibrium. The recovery of the magnetization can be characterized by harmonics in the wavenumber of the spin current, $k$, as:

$$M_{final} - M_z(t) = \sum_k c_k e^{-D_\parallel k^2 t}.$$

The magnetization is sampled at times, $t$, after the application of the initial 180° pulse using a $21° - 1ms - 180°$ pulse sequence. The resulting recovery profile can then be used to find the longitudinal spin diffusion current, $D_\parallel$.

The measured transverse and longitudinal spin diffusion coefficients are plotted on the same graph in Figure 1. A clear deviation of $D_\perp$ from $D_\parallel$ can be seen at temperatures below about 30 $mK$. The longitudinal spin diffusion follows the expected $T^{-2}$ dependence of a degenerate Fermi liquid ($T_F = 417\ mK$ for a 6.4% solution of $^3$He in $^4$He) whereas the transverse spin diffusion approaches a constant value as $T \to 0\ K$. The results for $D_\perp$ have been fitted to the theory[6] in the low spin polarization approximation. A value for the anisotropy temperature of $T_a = (19 \pm 3)\ mK$ was obtained. A fit of the theory to earlier measurements of the transverse spin diffusion coefficient in an $x_3 = 3.8\%$ mixture yields a value of $T_a = (13 \pm 2)\ mK$ for this concentration.

Similar results in pure $^3He$ have been obtained by Wei *et al*[7] using the same pulsed NMR spin echo technique. In this case, the anisotropy temperature $T_a = 16\ mK$.

## 4. Dipole effects and longitudinal attenuation

Since the transverse attenuation is the only zero-temperature relaxation mechanism in *pure* exchange Fermi liquids for low-frequency long-wave processes, it is interesting to inquire whether this dissipation mechanism is coupled to and affects longitudinal Fermi-liquid processes.

There are two general mechanisms that couple longitudinal and transverse processes in helium: the magnetic dipole-dipole interaction and the non-linearity of equations of motion. We will look only at dipole coupling, which is quite strong in highly polarized systems[12], though the non-linear coupling also leads to interesting effects especially near the spin-wave (Castaing) instability.

The dipole interaction transfers the zero-temperature transverse attenuation into the longitudinal channel by two different mechanisms[11]. First, in spin-polarized systems with magnetic dipole-dipole interaction the spin-flip processes of the type (1) with dipole vertex $W$ are allowed in the longitudinal channel and enter the collision integral directly. Second, the dipole interaction couples the longitudinal modes to (attenuating) transverse spin waves. Then the collision integral (1) enters the longitudinal processes with transverse exchange vertex $W$ and dipole interaction in the coupling constant.

As a result of both, direct and indirect dipole processes, the effective zero-temperature attenuation in the longitudinal channel $\tau_{eff}\,(T=0)$ should differ from $\tau_{\perp}\,(T=0)$ by an extra coupling factor $(E_d/T_F)^2$ where the characteristic dipole energy is $\beta^2 Z^2 m^{3/2} T_F^{3/2}/\hbar^3$, and $Z$ is the microscopic parameter which describes the difference between the (pole terms for) Fermi liquids and gases. The transition from temperature-driven to polarization-driven zero-temperature *sound* attenuation should occur for longitudinal sound in sub-$\mu K$ region, *i.e.,* at considerably lower temperature than the recently observed anisotropy temperature $T_a$ at which the transverse attenuation loses its $1/T^2$ dependence. [For liquid $^3He\uparrow$ this corresponds to the temperatures below the superfluid transition when the theory of normal Fermi liquids cannot be applied directly. Thus the unmodified results can only be applied

to liquid $^3He\uparrow - ^4He$ mixtures].

In order to avoid separate independent calculations of attenuation for different hydrodynamic and high-frequency longitudinal modes in $^3He\uparrow$ and $^3He\uparrow - ^4He$ mixtures, we calculated (zero-) sound attenuation in a generic polarized Fermi liquid. This allowed us to extract the effective mode-independent zero-temperature relaxation time $\tau_{eff}(T=0)$ and viscosity $\eta_{eff}(T=0) = \rho v_F^2 \tau_{eff}\left(1 + F_1^{(s)}/3\right)/5$. The effective relaxation time could be used in conjunction with standard hydrodynamic and *hf* equations[2,13] for polarized for $^3He\uparrow$ and $^3He\uparrow - ^4He$ mixtures giving the attenuation of all sound and *hf* modes in terms of effective $\eta_{eff}$ and $\tau_{eff}$.

Although the effective zero-temperature longitudinal relaxation parameters are quite small because of the weakness of dipole interaction, these parameters provide the real zero-temperature cut-offs for longitudinal relaxation and transport. Since liquid helium, in contrast to electron systems, does not have any impurities, one may expect to observe these limiting cut-offs at ultra-low temperatures in highly polarized $^3He\uparrow$ or $^3He\uparrow - ^4He$ mixtures.

### A. Dipole collision integral and sound attenuation

Dipole interaction leads to spin-flip collisions even for longitudinal processes such as sound propagation. As a result, one can find zero-temperature terms with the spin structure (1) in the collision integral with the scattering probability

$$W(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4) = \left(\frac{E_d}{T_F}\right)^2 \left[\frac{(\mathbf{p}_1-\mathbf{p}_3)_z^3 (\mathbf{p}_1-\mathbf{p}_3)_+}{(\mathbf{p}_1-\mathbf{p}_3)^4} - \frac{(\mathbf{p}_1-\mathbf{p}_4)_z^3 (\mathbf{p}_1-\mathbf{p}_4)_+}{(\mathbf{p}_1-\mathbf{p}_4)^4}\right]^2$$

where the $z$-axis is chosen along the magnetic field (spin polarization), and the dipole energy is equal to

$$E_d = \beta^2 Z^2 m^{3/2} T_F^{3/2}/\hbar^3$$

The resulting sound attenuation is[11]

$$\mathrm{Im}\ \omega = \frac{E_d^2}{16\pi^5 \hbar T_F} \left(\frac{\beta H}{T_F}\right)^2 I\left(s\cos\theta\right) \tag{4}$$

where $s = \omega/kv_F$ is the (dimensionless) sound velocity, and the function $I\left(s\cos\theta\right)$ is plotted in Figure 2 for several values of $s$.

### B. Coupling between sound and spin waves

In Fermi liquids with exchange interaction between particles, longitudinal and transverse processes are decoupled. Weak magnetic dipole-dipole interaction couples longitudinal and transverse processes. As a result, the zero-temperature attenuation in transverse channels can lead to zero-temperature dissipation even for ordinary longitudinal processes.

In spin-polarized Fermi liquids, sound propagation in the absence of dipole interaction is described by a set of two coupled equations for densities $n_\uparrow$ and $n_\downarrow$ of spin-up and spin-down particles. The coupling of longitudinal dynamic equations for $n_\uparrow$, $n_\downarrow$ to the transverse equation of motion for the mixed spin component of the density matrix $n_{\uparrow\downarrow}$ is provided by magnetic dipole - dipole interaction with the Hamiltonian[14,15]

$$\frac{4}{3}\pi\beta^2 \left[\frac{3\left(\sigma\cdot\mathbf{q}\right)\left(\sigma'\cdot\mathbf{q}\right)}{q^2} - \left(\sigma\cdot\sigma'\right)\right],\ \mathbf{q} = \mathbf{p} - \mathbf{p}' \tag{5}$$

This Hamiltonian is responsible for two effects. First, it causes demagnetizing factors which, in an elliptical sample, are equivalent to the demagnetizing field $\mathbf{H}_d$. The integration of dipolar interaction, necessary for the calculation of demagnetizing field, is not trivial because of the divergence at small wave vectors. It is possible to show[16–18] that the demagnetizing field in spherical samples is, with good accuracy, equal to

$$\mathbf{H}_d = 4\pi\left(\mathbf{M}\cdot\mathbf{H}/H - \mathbf{M}/3\right)Z^2, \quad \mathbf{M} = \left(\beta/2\right)Tr_\sigma \int \sigma\widehat{n}_\sigma d\Gamma \tag{6}$$

This equation for $\mathbf{H}_d$ includes both the equilibrium contribution with $\mathbf{M}_0$ and the non-equilibrium part with $\delta\mathbf{M}$.

Second, the dipole interaction changes the effective Landau interaction function (molecular field):

$$\delta f_{\alpha\beta,\gamma\delta}\left(\mathbf{p},\mathbf{p}'\right)=\frac{4}{3}\pi Z^2\beta^2\left[\frac{3\left(\sigma_{\beta\gamma}\cdot\mathbf{q}\right)\left(\sigma_{\alpha\delta}\cdot\mathbf{q}\right)}{q^2}-\left(\sigma_{\beta\gamma}\cdot\sigma_{\alpha\delta}\right)\right] \tag{7}$$

where $Z$ is the usual renormalization coefficient in the pole part of the single-particle Green's function for Fermi liquids. [Note, that Eq.(7) contains only one of the diagrams for the vertex $\Gamma^{\omega}$. The other diagram is already included into the term with $\delta\mathbf{M}$ in the demagnetizing field $\mathbf{H}_d$ (6)]. The substitution of the dipole terms (6), (7) into the commutator in the equations of motion,

$$\left[\widehat{n},\widehat{\epsilon}\right],\ \epsilon_{\alpha\gamma}=\epsilon^{(0)}_{\alpha\gamma}-\beta\sigma_{\alpha\gamma}\cdot\delta\mathbf{H}_d+\int f_{\alpha\beta,\gamma\delta}\left(\mathbf{p},\mathbf{p}'\right)\delta n'_{\delta\beta}\,d\Gamma', \tag{8}$$

results in coupling of longitudinal and transverse equations.

As a result of this coupling, the sound waves acquire the zero-temperature attenuation[11]

$$\mathrm{Im}\,\omega=\frac{\hbar^2\left(kv_F\right)^2}{32\pi^2\tau_{\perp}}\left[\frac{F_0^{(s)}}{F_0^{(s)}-F_0^{(a)}}\right]^2\frac{E_d^2}{T_F^4}\left(\frac{k_z^2}{k^2}\frac{k^2-k_z^2}{k^2}\Gamma_1\left(s\right)+\frac{4k_z^4-3k^2k_z^2+k^4}{3k^4}\Gamma_2\left(s\right)\right) \tag{9}$$

where

$$\Gamma_1\left(s\right)=2s^2\left(s^2-1\right)\frac{w\left(s^2-3\right)-1/3}{w\left(s^2-1\right)-1}\left[2\frac{w\left(3s^2-1\right)-1}{1+F_0^{(a)}}+s^2-3w\left(s^2-1\right)^2-\frac{7}{5}\right], \tag{10}$$

$$\Gamma_2\left(s\right)=2s^2\left(s^2-1\right)\frac{w\left(s^2+3\right)-1/3}{w\left(s^2-1\right)-1}\left[w\left(s^4-1\right)-\frac{s^2}{3}-\frac{1}{5}\right],$$

$$w\left(s\right)=\frac{s}{2}\ln\frac{s+1}{s-1}-1,$$

and in the single-harmonic approximation $w\left(s\right)=1/F_0^{(s)}$.

The most important difference from (4) is the $k^2$-dependence of the attenuation (9) originating from the $\mathbf{k}\cdot\mathbf{v}$ factor in the coupling coefficient. The calculation was performed for low frequencies, $kv_F\ll\Omega_0$. At higher frequencies, the factor $\left(kv_F\right)^2$ should be substituted by the square of the Larmor frequency $\Omega_0$.

### C. Effective relaxation and viscosity

The above expressions for sound attenuation allow us to get the values of effective relaxation time and viscosity. Comparing Eqs.(4) and (9) with the standard expressions for (zero-)

sound attenuation in Fermi liquids, we immediately get[11]

$$\frac{1}{\tau_{eff}} = \frac{\mathrm{Im}\,\omega}{\xi(s)} = \frac{E_d^2}{16\pi^5\hbar T_F}\left(\frac{\beta H}{T_F}\right)^2 \frac{I(s\cos\theta)}{\xi(s)}, \tag{11}$$
$$\xi(s) = = s^2 \frac{w^2(s^2-1)(3s^2+1)+2w(s^2-1)-1}{w(s^2-1)-1},$$
$$w(s) = \frac{s}{2}\ln\frac{s+1}{s-1} - 1$$

for direct processes, and

$$\frac{1}{\tau_{eff}} = \frac{\hbar^2(kv_F)^2}{32\pi^2\tau_\perp \xi(s)}\left[\frac{F_0^{(s)}}{F_0^{(s)}-F_0^{(a)}}\right]^2 \frac{E_d^2}{T_F^4}\left(\frac{k_z^2}{k^2}\frac{k^2-k_z^2}{k^2}\Gamma_1(s) + \frac{4k_z^4-3k^2k_z^2+k^4}{3k^4}\Gamma_2(s)\right) \tag{12}$$

for indirect ones.

The high-frequency attenuation can be obtained by the method similar to that used in calculation of sound attenuation in Fermi liquids[19]. The analysis of the non-vanishing at $T=0$ collision operator of the type (1) shows that this integral is similar to those studied in[19,3,6] and should reduce to the form

$$\gamma_{lf}\left(1+\left(\frac{\omega}{\Omega_0/2}\right)^2\right) \tag{13}$$

(in dense Fermi liquids the Larmor frequency $\Omega_0$ experiences the usual Fermi-liquid renormalization), where $\gamma_{lf}$ determines the low-frequency sound attenuation in Fermi liquids,

$$\gamma_{lf} = \mathrm{Im}\,k = \frac{\xi(s)}{s\tau_{eff}v_F} \tag{14}$$

The effective field-driven viscosity at $T=0$,

$$\eta_{eff} = \frac{1}{5}\rho v_F^2 \tau_{eff}\left(1+F_1^{(s)}/3\right), \tag{15}$$

depends on the angle between the velocity gradient ($\mathbf{k}$) and the direction of polarization $z$. This anisotropy of the fluid dynamics in spin-polarized systems with dipole interaction is quite natural.

## 5. Castaing instability

### A. Castaing instability in spin dynamics

Studies of instabilities and non-linear effects help towards an understanding of spin dynamics in Fermi liquids. One of the most important spin-waves instabilities - the so-called Castaing instability - occurs in spin dynamics of spin-polarized Fermi liquids in the presence of a gradient of magnetic field and/or polarization.

At low spin polarization, the transverse spin dynamics in polarized Fermi liquids is governed by the Leggett equation (see, *e.g.,* review[2]):

$$\frac{\partial \mathbf{M}}{\partial t} + [\gamma \mathbf{B} \times \mathbf{M}] = \frac{\partial}{\partial x_k}[\frac{D_\perp}{1+\mu^2 M_0^2}(\frac{\partial \mathbf{M}}{\partial x_k} + \mu[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x_k}])] \tag{16}$$

If the magnetization gradients are small, the last term can be linearized in small deviations from equilibrium $\delta\mathbf{M}$ as $\mu[\mathbf{M}_0 \times \partial\delta\mathbf{M}/\partial x_k]$, and the spin excitations are weakly attenuated circularly polarized spin waves with the spectrum

$$\omega = \omega_0 + \frac{D_\perp k^2}{1+\mu^2 M_0^2}(i - \mu M_0) \tag{17}$$

Castaing[20] noticed that if the gradient in the magnetization $\nabla M$ is not negligible, the linearized last term in Eq.(16) is $\mu[\mathbf{M}_0 \times \partial\delta\mathbf{M}/\partial x_k] + \mu[\delta\mathbf{M} \times \nabla\mathbf{M}_0]$, and the excitation spectrum changes from (17) to

$$\omega = \omega_0 + \frac{D_\perp}{1+\mu^2 M_0^2}(i - \mu M_0)(k^2 - \mu \mathbf{k}\cdot\nabla\mathbf{M}). \tag{18}$$

For a sufficiently large gradient (or sufficiently small $\mathbf{k}$), the last bracket and, therefore, the imaginary part of the spectrum change sign. Instead of attenuation, the perturbation increases with time resulting in instability starting from

$$k_c = \mu \mathbf{n}\cdot\nabla\mathbf{M}, \;\; \mathbf{n} = \mathbf{k}/k \tag{19}$$

The non-linearity of the Leggett equation of spin dynamics, which is responsible for the Castaing instability, leads to a highly inhomogeneous final stationary distribution of

magnetization (magnetic domains) even in slightly inhomogeneous magnetic field[21]. Under certain conditions, the domain wall could become very wide[22]. Then the difference between longitudinal and transverse relaxation disappears, and the overall relaxation is determined by the shortest of the two, *i.e.*, by the field-driven zero-temperature transverse attenuation.

### B. Observation of the spin-wave instability

This instability is very general, and can be observed in helium systems in different configurations. We will illustrate it on the example of experiments[9] in saturated $^3He - {}^4He$ mixtures.

We observed an oscillating signal that could be induced by the application of a single *R.F.* tipping pulse of angle $\theta$ applied to a small region of the helium in the middle of the 1 $mm$ tube[23]. Large magnetization gradients were induced in the helium at the edges of this small region. We approximate these gradients as $|\nabla M| \sim M_0(1-\cos\theta)/\Delta x$, where $\Delta x$ is the distance over which they extend. A typical NMR signal produced by a $\theta = 105°$ pulse is plotted in Fig.3.

This long-lived ringing, which we interpret as a sign of instability, was observed only when the tipping angle exceeded some critical value $\theta_c \simeq 70°$. The frequencies of the oscillations were determined by Fourier transforming the signals; the frequency shift $\delta\omega$ away from the Larmor frequency increased with tipping angle. By substituting the expression for the magnetization gradient into the spectrum Eq.(18), we find that the frequency, $\delta\omega = \omega - \omega_0$, depends upon tipping angle as $\delta\omega \propto \cos\theta - \cos\theta_c$, where $\theta_c$ is the critical angle; i.e. the angle for which the last bracket of the spectrum is equal to zero, Fig. 4. Our estimate gives the value $k_c \sim 600$ cm$^{-1}$. This implies that the large magnetization gradient is over a distance of order 0.05 cm; this is consistent with the scale of our experimental setup.

These ringing signals possess several features which support an explanation in terms of an instability. There is a cutoff in tipping angle $\theta_c$ below which no signals were observed. This, together with the fact that there was no ringing signal at higher temperatures when

$\mu M_0$ is small, confirms the threshold nature of the phenomenon. The long-time scale of the signals and the initial increase in amplitude (Fig.3) are also characteristic of an instability. The frequencies of the oscillations scale as the cosine of the tipping angle, $\cos\theta - \cos\theta_c$. The presence of two frequency peaks on the Fourier analysis of the spectrum suggests that the signals are coming from regions either side of the *R.F.* coil where the magnetization gradients are slightly different. No such signals were observed during experiments on solutions with lower $^3He$ concentrations. Here $\mu M_0$ is of the same magnitude but negative, so that the instability propagates in the opposite direction, away from the receiving NMR coil.

Similar instabilities have been observed by Nunes[10] and Dmitriev et al[24]. The ringing continued for extremely long times, leading to conclusions about the existence of a metastable state (precessing spin domains)[21] after the instability develops. In our experiment we did not see such a long-time behavior due to the different setup. Only a small fraction of the spins in the lower chamber was tipped and the longitudinal spin diffusion coefficient $D_{||}$ was large, so that the instability was quickly suppressed by diffusion of up-spins into the coil.

### C. Dipole effects in Castaing instability

The non-linear coupling between longitudinal and transverse channels is enhanced close to the instability in spin dynamics (see, *e.g.,* Ref.[23] and references therein). We analyzed[11] the dipole effects near the onset of Castaing instability. Without the dipole effects, the instability occurs at $k_c$, Eq.(19). The dipole interaction makes the instability anisotropic by adding terms of the form $k_z^2$, $\mu k_z \nabla_z M$, $(\mu\nabla M)^2$, and $(\mu\nabla_z M)^2$ to Eq.(19). However, these terms contain a small factor $E_d/T_F$ (we will not give here the cumbersome coefficients). These anisotropic corrections do not have any fixed sign so that it is impossible to say whether the onset of instability occurs earlier in certain directions.

Though this instability exists in transverse spin dynamics, one of its features is that $\mu$ in Eq.(19) is proportional not to the transverse relaxation time $\tau_\perp$, but to the longitudinal time $\tau_{||}$, $\mu = \Omega_i \tau_{||}/M$. Since $\tau_{||} \propto 1/T^2$, the onset of instability $k^2 = \mu k_i \nabla_i M$ happens,

with decreasing temperature, at larger and larger wave vectors. The usual derivation of the instability condition assumes that the gradient of the longitudinal magnetization leads to a large longitudinal diffusion current and not to longitudinal oscillations, *i.e.*, that $1/\tau_{||} \gg kv_F$. These two conditions, taken together, limit the temperature range in which the instability can be observed to

$$T_F \gg T \gg T_F \left(\alpha a/L\right)^{1/4}/x^{1/3} \tag{20}$$

where $\alpha$ is the degree of spin polarization, $x$ is the molar density of the Fermi liquid, and $L$ is the spatial scale of the polarization gradient.

The dipole coupling between longitudinal and transverse channels leads to a substitution of $\tau_{||}$ by $\tau_{eff}$ and lifts this temperature limitation. At zero temperature, the instability occurs at $k^2 = \mu_{eff} k_i \nabla_i M$, $\mu_{eff} = \Omega_i \tau_{eff}/M$ under the condition $1/\tau_{eff} \gg kv_F$ . The compatibility of these equations requires high polarization with small gradient,

$$E_d \gg T_F \left(a/\alpha^3 L\right)^{1/4},$$

As a result, the instability exists even at zero temperature, but occurs at extremely small values of $k$.

## 6. Application to pure ferromagnetic metals

Ferromagnetic metals can be roughly separated into two groups: itinerant ferromagnets with ferromagnetism of conduction electrons, and metals with ferromagnetic ordering of inner, localized electrons (with Heisenberg interaction). The zero-temperature transverse attenuation in the former systems seems similar to helium systems, while in the latter group such effects appear only as a result of exchange coupling between the localized ferromagnetic electrons and the Fermi liquid of conduction electrons. In addition, the spin-lattice relaxation, absent in helium, presents a strong coupling mechanism between longitudinal and transverse channels for both types of ferromagnetic systems.

### A. Itinerant magnetism

The theory of transverse spin dynamics in electron Fermi liquid in itinerant ferromagnets should be similar to that in spin-polarized helium. To a large extent this is correct, especially well below the transition temperature. Close to the transition temperature the Fermi-liquid description is not applicable (see,*e.g.*,[25]). It is well-known[26] that the spin wave spectrum in ferromagnetic metals is similar to the spectrum of Silin spin waves in Fermi liquid . Careful analysis of the spectrum[26] shows that this spectrum contains the zero-temperature attenuation: the expression for the spectrum includes the integral *between* the spin-up and spin-down Fermi spheres,

$$\int .... [n_\uparrow - n_\downarrow] \, d\Gamma,$$

which, as any integral not localized near the Fermi surface, should contain a large imaginary part. However, this integration deep into the Fermi spheres makes the derivation[26] not self consistent; a consistent derivation should be based on the microscopic equations[6].

Apart from the zero-temperature attenuation, these equations have another interesting feature, namely, the spin-up - spin-down asymmetry. This effect is similar to a well-known particle-hole asymmetry in Fermi liquids away from the Fermi sphere. In itinerant ferromagnets the radii of the Fermi spheres for spin-up and spin-down particles differ by a large margin resulting in different molecular fields (Landau Fermi liquid functions) for quasiparticles near these Fermi surfaces. This means that the frequencies of *inhomogeneous* precession in the effective field for tipped spin-up and spin-down particles are different.

In general, the transfer of the microscopic equations[6] from Fermi liquids polarized by an external magnetic field to ferromagnetic Fermi liquids is rather straightforward, and we will not dwell on this here. Instead, we will mention another interesting aspect of microscopic equations. The spin-up - spin-down anisotropy of the effective field can give credence and microscopic justification[27] to the concept of reaction field suggested by Onsager[28] for segnetoelectric systems (this concept for ferromagnetic systems was discussed in[29]).

### B. Heisenberg systems

The above zero-temperature dissipation mechanism is inherent to Fermi liquids and, in its original form, does not exist in solid-state magnetic system of localized spins with Heisenberg interaction $J$. However, this unique Fermi-liquid dissipation mechanism should lead to some residual attenuation of magnons in pure ferromagnetic metals with Heisenberg interaction. We want to emphasize that in this section we are interested not in itinerant magnetism, for which the manifestation of Fermi-liquid effects is natural, but in an exchange magnetic system of localized electrons.

The effect is fairly straightforward, and is based on exchange coupling of localized ferromagnetic spins (*e.g.,* $3d$ electrons) to conduction (*e.g.,* $4s$) electrons. This exchange coupling results in small polarization (not to exceed several per cent) of conduction electrons of the order $J_1 \langle S \rangle / T_F$, where $J_1$ is the exchange coupling constant between localized ferromagnetic electrons with spins $\mathbf{S}$ and spins of conduction electrons $\sigma$. Polarization of spins of conduction electrons ensures the propagation of Silin spin waves in this system with finite zero-temperature attenuation $\tau_\perp (T=0) \sim (N v_F \sigma)^{-1} (T_F / J_1 \langle S \rangle)^2$. The exchange coupling between these attenuating Silin spin waves and ferromagnetic Heisenberg magnons transfers the zero-temperature attenuation to the magnon system resulting in the effective relaxation time $\tau_\perp^* \sim \tau_\perp (J_1/J)^2$. The competing processes that lead to the magnon attenuation are, obviously, scattering on impurities and spin-lattice processes studied long ago (see, *e.g.,*[30]). The former processes are small in pure metals, while the latter are suppressed at low temperatures.

The equilibrium energy of conduction electrons has the form

$$\varepsilon_i^e = \varepsilon_i^o - \beta_1^e \sigma_i \cdot \mathbf{H} - J_1 \sigma_i \cdot \langle \mathbf{S} \rangle / 2 \tag{21}$$

while the Hamiltonian of localized electrons is

$$\varepsilon_i^l = -\beta^l \mathbf{S}_i \cdot \mathbf{H} - \frac{1}{2} J_0 \langle \sigma \rangle \cdot \mathbf{S}_i - J \sum_{\mathbf{a}} (\mathbf{S}_{i+a_x} + \mathbf{S}_{i+a_y} + \mathbf{S}_{i+a_z}) \cdot \mathbf{S}_i \tag{22}$$

The effective parameters for conduction electrons are already renormalized by their Fermi-liquid interaction, $\beta_1^e = \beta^e / \left(1 + F_0^{(a)}\right)$, $J_1 = J_0 / \left(1 + F_0^{(a)}\right)$, while the averages

$$\langle \mathbf{S} \rangle = \sum \mathbf{S}_i N_i, \ \langle \sigma \rangle = \sum \sigma_i n_i = p_F m (\beta_1^e H + 1/2 J_1 < S_z >) / \pi^2 \hbar^3 \tag{23}$$

The Fermi-liquid term in the energy of conduction electrons has the usual form,

$$\delta \varepsilon^e_{\alpha\beta} = -\frac{1}{2} J \sigma_{\alpha\beta} \cdot \delta \mathbf{S} + \int f_{\alpha\beta\alpha'\beta'}(\mathbf{p}, \mathbf{p}') \delta n_{\beta'\alpha'}(\mathbf{p}') d\Gamma' \tag{24}$$

with the Landau Fermi-liquid function

$$\frac{p_F m}{\pi^2 \hbar^3} f_{\alpha\beta\alpha'\beta'}(\mathbf{p}, \mathbf{p}') = F^{(s)}(\mathbf{p}, \mathbf{p}') \delta_{\alpha\beta} \delta_{\alpha'\beta'} + F^{(a)}(\mathbf{p}, \mathbf{p}') \sigma_{\alpha\beta} \cdot \sigma_{\alpha'\beta'} \tag{25}$$

Often, in ferromagnetic systems $J_0 \langle S_z \rangle \gg J_0 \langle \sigma_z \rangle$, $\beta H$, and $\Omega_0 \gg \omega_0$. In this approximation, the analysis of the coupled equations of motion for localized and delocalized spins $\mathbf{S}$ and $\sigma$ with the Hamiltonian (21)-(25) yields, after some algebra, the following expression for the attenuation of ferromagnetic magnons[31]:

$$\mathrm{Im}\, \omega = -\frac{\beta^l H}{6 J_0 \langle S_z \rangle} \frac{\langle \sigma_z \rangle}{\langle S_z \rangle} \frac{k^2 v_F^2 \tau_\perp (1 + F_0^{(a)})(1 + F_1^{(a)}/3)}{1 + \left[\tau_\perp \Omega_0 \left(1 + F_1^{(a)}/3\right) / \left(1 + F_0^{(a)}\right)\right]^2}$$

This equation is valid only for $\beta^l H \gg J \langle S_z \rangle k^2 a^2$ and formally yields zero at $H = 0$. In smaller fields the attenuation does not vanish, but becomes proportional to $k^4$ in accordance with the general result[32].

The strength of the effect depends on the exchange interaction $J_0 \mathbf{S} \cdot \sigma$ between spins of ferromagnetic and conduction electrons. In free atoms the $s - d$ exchange is of the scale of $1\ eV$. In metals, screening weakens this exchange by about one or two orders of magnitude. There is also an enhancement factor which is related to the Kondo-like logarithmic divergence of the effective field. The localized electrons create the (transverse) coherent exchange field for conduction electrons which is equal to[33]

$$L_{coh} = \frac{i\pi}{2\hbar V} \left[\delta S_+ (n_\uparrow - n_\downarrow) - \delta \sigma^+ \langle S_z \rangle N\right] \times \left[4 t_2 + \int \frac{d^3 p'}{(2\pi\hbar)^3} P \frac{1}{\epsilon' - \epsilon} \left(2 N t_1 t_2 + t_2^2 (n_\uparrow - n_\downarrow + N)\right)\right]$$

The exchange field for localized electrons is similar. Here $t_1$ and $t_2$ are the bare direct and exchange interaction constants, and $N$ is the density of localized spins. If the polarization of conduction electrons is low, the direct interaction $t_1$ disappears from the results. The above integral, as other similar integrals in the theory of metals, diverges logarithmically. After introducing the usual cut-off, we get the following renormalization of the bare interaction:

$$J_0 = t_2 \left( 1 + 8\pi t_2 \nu_F \ln \frac{T_F(1 + F_0^{(a)})}{\beta^e H + t_2 \langle S_z \rangle / 2} \right)$$

where $\nu_F$ is the density of states on the Fermi surface. As a result of this large logarithmic enhancement of the interaction, $J_0$ can reach several hundred $K$ and the polarization of conduction electrons can exceed one per cent. Then the zero-temperature attenuation for conduction electrons $\tau_\perp$ can become shorter than $10^{-10}$ sec, and $\tau^*$ can reach $10^{-7}$ sec.

## 7. Conclusions

The zero-temperature transverse attenuation in spin-polarized Fermi liquids, which was observed recently in spin dynamics of $^3He \uparrow$ and $^3He \uparrow - ^4He$ mixtures, is the only low-frequency dissipative process in Fermi liquids at $T = 0$. This effect can have much broader implications than a simple low-temperature saturation of transverse transport parameters in polarized helium systems. We highlighted several of such effects.

The dipole coupling between longitudinal and transverse spin dynamics processes in spin-polarized Fermi liquids leads to the transfer of zero-temperature transverse attenuation into longitudinal channels. This transfer is responsible for the zero-temperature dipole contribution to the sound attenuation in a generic Fermi liquid described by the effective mode-independent longitudinal relaxation time and viscosity. These effective parameters provide the low-temperature limit for dissipation of various hydrodynamic and high-frequency modes in helium systems.

The zero-temperature attenuation processes have interesting implications for ferromagnetic metals. Of course, the direct manifestations of this Fermi-liquid anomaly can be observed in itinerant ferromagnets. Here the most interesting effect is, probably, not the

zero-temperature attenuation itself, but a pronounced spin-up - spin-down asymmetry of the effective field which could manifest itself in the formation of a peculiar Onsager reaction field.

In metals with ferromagnetism of localized Heisenberg spins, the effects of the zero-temperature Fermi-liquid interaction are indirect. In this case, the exchange coupling of localized and conduction electrons results in low residual polarization of spins of conduction electrons. This, in turn, leads to the propagation of Silin spin waves with small zero-temperature attenuation in the system of conduction electrons. The coupling of these spin waves to the spin waves in the system of localized Heisenberg electrons transfers the zero-temperature attenuation to ferromagnetic magnons. This mechanism is responsible for the residual attenuation of ferromagnetic magnons in pure ferromagnetic metals.

Another important peculiarity of spin dynamics in spin-polarized Fermi liquids is the spin-wave instability in inhomogeneous setting (Castaing instability). We presented and analyzed experimental data confirming the existence of this instability, and discussed some further experimental options. As a result of dipole transfer of zero-temperature attenuation into longitudinal channels, the Castaing instability does not disappear at ultra-low temperatures, though its observation would require a relatively large experimental installation. In addition, the dipole interaction makes all the processes, including the instability in spin-polarized Fermi liquids, highly anisotropic.

The non-linearity of the Leggett equation of spin dynamics, which is responsible for the Castaing instability, results in a highly inhomogeneous final distribution of magnetization even in almost homogeneous magnetic field. In these conditions, the difference between longitudinal and transverse relaxation disappears, and the overall relaxation is determined by the shortest of the two. Then the overall low-temperature relaxation is similar to the field-driven zero-temperature transverse attenuation in homogeneous systems.

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FIGURE CAPTIONS.

Figure 1. Temperature dependence of transverse (circles) and longitudinal (rhombs) diffusion coefficients, $D_{\perp}$ and $D_{||}$.

Figure 2. $I(s,x)$ as a function of $x = \cos\theta$, Eq.(4), for four values of $s$, $s = 2;\ 3;\ 3.47;\ 5$.

Figure 3. The ringing signals observed following a $\theta = 105^0$ pulse

Figure 4. The frequency shift as a function of the tipping angle. The line is a fit $\delta f = A(\cos\theta - \cos\theta_c)$.