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Low-temperature transport and quantum interference of ballistic particles in systems with random rough walls

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We present a simple theory of transport processes and quantum interference and localization effects in films and channels with rough walls.*

1. INTRODUCTION

Transport of particles and waves near random rough walls is a very old and important problem. Though it is clear that transport coefficients for transport along the walls should be expressed via the statistic characteristics of the walls, namely, via the correlation function of surface inhomogeneities, it is not obvious how to get such a description. Recently [1] we suggested a simple quantitative way of describing transport of ballistic particles along thin films and narrow channels with random rough walls (cf. [2, 3]). The same method provided a simple theoretical description of quantum interference and localization effects when the main dephasing effects are associated with the scattering by random walls.

The approach is based on the use of an explicit canonical coordinate transformation that flattens the walls by stretching the film in accordance with its exact local thickness. As a result of this canonical transformation the *bulk* Hamiltonian acquires additional random terms that reflect the initial roughness of the walls. This random bulk distortion can often be treated in the frame of the standard perturbative semi-classical bulk transport equation in the same way as any other bulk distortion, *e.g.*, impurities. This approach is valid when the amplitude of surface inhomogeneities l is small in comparison with their size (correlation radius) R and the film thickness L, $l \ll R,\ L$, and the particle wave vector along the film (channel) $q = 2\pi/\lambda$ is such that $l/L \ll q^2R^2,\ q^3RL^2$ *or* $1 \ll qR$. For the motion across the film we can solve the quantum problem with quantized discrete levels as well as the (semi-)classical one.

Our approach allowed us to calculate the transport coefficient and to study quantum interference effects caused by scattering by random rough walls.

2. RESULTS

2.1. Transport

We calculated conductivity, mobility, and diffusion in films as a function of the correlation function of surface roughness, particle density, temperature, and film thickness.

We solved the transport problem for an arbitrary correlation function of surface inhomogeneities. For simplicity, we will give the results only for the Gaussian correlations $\varsigma(s) = \langle \xi(\mathbf{r})\,\xi(\mathbf{r}+\mathbf{s})\rangle = l^2 \exp\left(-s^2/2R^2\right)$. The mean free path along the walls always have the scale

$$\mathcal{L} \sim L\left(LR/l^2\right) f\left(R/\lambda\right) \qquad (1)$$

and the problem reduces to the calculation of the dimensionless function $f(\lambda/R)$ for different situations. The factor LR/l^2 gives the number of wall collisions necessary to dephase the particle, while the function $f(\lambda/R)$ describes the effectiveness of individual collisions. Of course, $f(x) \to \infty$ in both limits $x \to \infty$ (flat walls) and $x \to 0$ (quantum reflection).

Fig.1 gives the function $f(x)$ for degenerate fermions in relatively thick films (classical motion). In the opposite limit of very thin films with discrete quantized states for the motion across the film, the parametrization is more complicated:

$$\mathcal{L} \sim L\frac{LR}{l^2}\frac{L}{Rz}f\left(z, \frac{R}{L}\right),\ z = \frac{2NL^2}{\pi} \qquad (2)$$

where N is the $2D$ density of fermions in the film. The function $f(z, R/L)$ for $R/L = .05$ is given in

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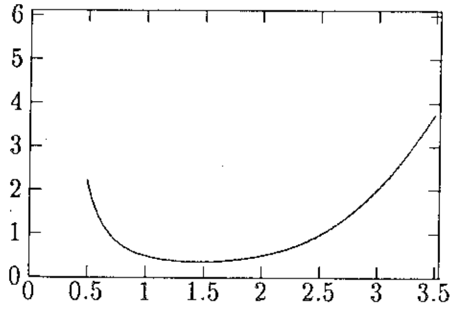

Figure 1: $f(x)$, Eq.(1)

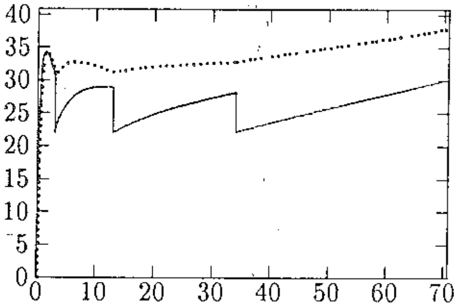

Figure 2: $f(z, R/L)$ 2 at $R/L = 0.05$

Fig.2. The discontinuities occur at the densities N at which new (higher) levels start to fill up. The discontinuities become less pronounced with an increase in R/L when the probability of transitions between discrete levels decreases (the dashed curve in Fig.2 corresponds to the same R/L but with artificially frozen interlevel transitions). The discontinuities smooth out at higher temperatures.

2.2. Quantum interference and localization

Our explicit results on particle transport allowed simple analysis of quantum interference and localization effects caused by scattering by rough walls. Since we were using a bulk-like formalism, we were able to reproduce in a very simple way the already known results [4, 5, 6] and to get some new ones.

These results strongly depend on dimensionality of the system. In thick (classical) $3D$ films the main effect is quantum correction to conductivity which is obtained, as usual, from the diffusion coefficient:

$$\Delta\sigma/\sigma = -8\pi^2\hbar^2/m^2D^2 \qquad (3)$$

In thinner films, the quantization of motion across the film becomes important, and the quantum corrections increase with decreasing interlevel transitions (increasing separation between levels). When the transition probabilities become negligible, the motion becomes $2D$ motion on each level j. Then the quantum corrections diverge, and one observes a wall-induced localization on the length (cf. [4])

$$\mathcal{R}_j \sim \mathcal{L}_j \exp\left(\pi q \mathcal{L}_j/2\right) \qquad (4)$$

The transport problem in $2D$ channels (strips) with rough walls and the expressions for the transport coefficients are similar to those in $3D$ films. However, in this case the localization (4) occurs even for wide channels. For narrower channels, the motion across the channels becomes quantized and gradually acquires the $1D$ features. In the end, for very narrow channels, the conductance becomes quantized [5] while the density of states is the same as in any $1D$ system in random potential [1].

3. CONCLUSIONS

We developed a simple uniform approach to ballistic transport between random rough walls. We calculated the explicit dependence of conductivity and diffusion in thin films and narrow channels on statistical properties of the walls, density and wavelength of particles, thickness of the film, temperature, *etc.* for a wide variety of physical situations. The transport results revealed an existence of a new mesoscopic transport scale, $\left(L^2R/l^2\right) f\left(R/\lambda\right)$. The simplicity of transport calculations leads to transparent results for quantum interference and localization in systems of different dimensionality. Two main differences between boundary-restricted and bulk-dominated processes are a peculiar operator form of an effective perturbation and a possible large correlation radius of inhomogeneities R.

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