Spin Dynamics in Spin-Polarized Fermi Systems

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We present a simple description of transverse dynamics of dilute Fermi systems in terms of statistical quasiparticles. We give explicit expressions for the quasiparticle interaction function and collision integral at arbitrary temperatures, and discuss the implications of this approach in classical and degenerate regions, including the quantum gases. The results provide an unexpected explanation for the zero-temperature attenuation in transverse spin dynamics and peculiar I_2-terms in molecular field as imaginary (pole) and real (principal) parts of the quasiparticle interaction function.

1. INTRODUCTION

Recently there was a noticeable interest in transverse spin dynamics of spin-polarized Fermi liquids and gases (see references in review[1]). Spin dynamics experiments (spin diffusion and echo, spin-wave resonances, *etc.)* are described using either Landau-Silin-Leggett equations for Fermi liquids, or kinetic equation for dilute polarized gases.[1-5] Despite serious attempts, a uniform description of gases in a wide temperature range is not completely satisfactory. The problems are associated less with an extension of the quasiparticle (qp) approach to high temperatures than with a limited applicability of the microscopic and phenomenological Landau theories to *transverse* dynamics. The Fermi liquid theory exhibits several anomalies that do not exist for longitudinal processes and are suppressed at low polarizations.

Straightforward application of the Landau theory to transverse dynamics at high polarization is questionable because of unavoidable integration of distribution functions between widely separated Fermi surfaces for spin-up and spin-down particles (see[6,7] and references therein). Because of strong

attenuation away from the Fermi surface, such integration is forbidden in the Fermi liquid theory. Experimentally, this leads to a profound difference between relaxation times, $\tau_\parallel$ and $\tau_\perp$, and spin diffusion coefficients, $D_\parallel$ and $D_\perp$, for longitudinal and transverse components of magnetization even without the Leggett-Rice effect. The transverse coefficients do not increase with $T \to 0$ as $1/T^2$, but remain finite even at $T = 0$. This anomaly[8,1] was confirmed recently in calculations for gases[5] and experiments with ^3He and $^3He - {}^4He$.[9,10]

Other anomalies are caused by spin-up - spin-down asymmetry. This effect is similar to the particle-hole asymmetry away from the Fermi surface: separation between spin-up and spin-down Fermi spheres makes the molecular fields for spin-ups and spin-downs distinctly different. This results in a split of phenomenological equation of transverse dynamics in polarized Fermi liquids into two coupled equations on two different energy shells $\omega = \epsilon_{\uparrow\uparrow}(\mathbf{p} - \mathbf{k}/2)$ and $\omega = \epsilon_{\downarrow\downarrow}(\mathbf{p} + \mathbf{k}/2)$.[11] Another anomaly is more technical. The Landau interaction function in non-polarized systems can be expressed either as a limit Γ^ω of the full vertex, or via the *irreducible* vertex $\Gamma^{(1)}$[12]. At high polarization, the interaction function can be expressed only via the *irreducible* vertex and not as any limit of the full vertex.[11,6]

On the other hand, spin dynamics in classical gases is described by a standard Boltzmann equation and does not exhibit any signs of low-temperature anomalies. To understand the transition from Fermi liquid to classical description, we need a uniform theory for the whole temperature range. This problem is addressed below. We will describe spin dynamics in terms of "statistical qp" and give expressions for the qp interaction function and collision integral in the whole temperature range.

2. STATISTICAL QUASIPARTICLES: INTERACTION FUNCTION AND COLLISION INTEGRAL

Diagrammatic kinetic equation for $n_{\uparrow\downarrow}$ in the second order in the interaction can be transformed to the Fermi-liquid form

$$\partial_t \widehat{n} + (i/\hbar)[\widehat{n}, \widehat{\epsilon}] = \widehat{\mathcal{L}}\{\widehat{n}\}, \quad \widehat{\epsilon}(\mathbf{p}, \mathbf{r}) = \widehat{\epsilon}_0 + \int \widehat{f}(\mathbf{p}, \mathbf{p}_1)\, \delta\widehat{n}(\mathbf{p}_1, \mathbf{r})\, d\Gamma' \quad (1)$$

with a rather unusual expressions for the *transverse* component of (Landau) interaction function $f(\mathbf{p}, \mathbf{p}_1)\, \overrightarrow{\sigma} \cdot \overrightarrow{\sigma}_1$,

$$f(\mathbf{p}, \mathbf{p}_1) = -T(\mathbf{p} - \mathbf{p}_1) - \frac{1}{4}\int \frac{d^3 p'}{(2\pi\hbar)^3}\{T(\mathbf{p}_1 - \mathbf{p}')\, T(\mathbf{p}_1' - \mathbf{p}_1) \times \quad (2)$$

$$\left[\frac{1 - n'}{\epsilon + \epsilon_1 - \epsilon' - \epsilon_1' - i0 \cdot \operatorname{sign}(p' - p_\downarrow)} - P\frac{1}{\epsilon + \epsilon_1 - \epsilon' - \epsilon_1'} \right]$$

$$- \left[T^2 \left(\mathbf{p}_1' - \mathbf{p}' \right) - T \left(\mathbf{p}_1' - \mathbf{p}_1 \right) T \left(\mathbf{p}_1' - \mathbf{p}' \right) \right] \times$$

$$[\frac{n_{\downarrow\downarrow}' - n_{1\downarrow\downarrow}'}{\epsilon + \epsilon_1 - \epsilon' - \epsilon_1' - i0 \cdot \text{sign} \left(p' - p_\downarrow \right)} +$$

$$\frac{n_{\uparrow\uparrow}' - n_{1\uparrow\uparrow}'}{\epsilon + \epsilon_1 - \epsilon' - \epsilon_1' - i0 \cdot \text{sign} \left(p' - p_\uparrow \right)}] \}$$

and the transverse collision integral $\mathcal{L}_{\uparrow\downarrow} = \mathcal{L}_{\text{coll}} =$

$$\int \frac{d^3 p_1}{(2\pi\hbar)^3} \frac{d^3 p'}{(2\pi\hbar)^3} \{ \delta \left(\epsilon + \epsilon_1 - \epsilon' - \epsilon_1' \right) T \left(\mathbf{p}_1 - \mathbf{p}' \right) T \left(\mathbf{p}_1' - \mathbf{p}_1 \right) \times$$

$$\{ [n_{\uparrow\downarrow} \left(\mathbf{p}_1 \right) \left(n - 1 \right) - n_{\uparrow\downarrow} \left(\mathbf{p} \right) \left(n_1 - 1 \right)] \left(1 - n' \right) +$$

$$2 \left[n_{\uparrow\downarrow} \left(\mathbf{p}_1 \right) - n_{\uparrow\downarrow} \left(\mathbf{p} \right) \right] \left[n_{\uparrow\uparrow}' \left(n_{1\downarrow\downarrow}'^{(0)} - n_{1\downarrow\downarrow}' \right) + n_{\downarrow\downarrow}'^{(0)} \left(1 - n_{\downarrow\downarrow}' \right) \right] \} +$$

$$\delta \left(\epsilon + \epsilon_1' - \epsilon' - \epsilon_1 \right) \left[T^2 \left(\mathbf{p}_1' - \mathbf{p}' \right) - T \left(\mathbf{p}_1' - \mathbf{p}_1 \right) T \left(\mathbf{p}_1' - \mathbf{p}' \right) \right] \times$$

$$\{ 2 \left[n_{\uparrow\downarrow} \left(\mathbf{p} \right) - n_{\uparrow\downarrow} \left(\mathbf{p}_1 \right) \right] \left[n_{1\uparrow\uparrow}' \left(n_{\uparrow\uparrow}' - n_{\uparrow\uparrow}'^{(0)} \right) + n_{1\downarrow\downarrow}' \left(n_{\downarrow\downarrow}' - n_{\downarrow\downarrow}'^{(0)} \right) \right] +$$

$$n_{\uparrow\downarrow} \left(\mathbf{p}_1 \right) \left[\left(n_1' - n' \right) \left(n - 1 \right) + 2 n_{\uparrow\uparrow}' \left(1 - n_{\uparrow\uparrow}'^{(0)} \right) + 2 n_{\downarrow\downarrow}' \left(1 - n_{\downarrow\downarrow}'^{(0)} \right) \right] -$$

$$n_{\uparrow\downarrow} \left(\mathbf{p} \right) \left[\left(n' - n_1' \right) \left(n_1 - 1 \right) + 2 n_{\uparrow\uparrow}' \left(1 - n_{\uparrow\uparrow}'^{(0)} \right) + 2 n_{\downarrow\downarrow}' \left(1 - n_{\downarrow\downarrow}'^{(0)} \right) \right] \} (3)$$

there $T \left(\mathbf{p}_i - \mathbf{p}_j \right)$ is the scattering T-matrix, $n_{\uparrow\uparrow}$ are the spin components of the single-particle distribution, $n = n_{\uparrow\uparrow} + n_{\downarrow\downarrow}$, and $n^{(0)} = n \left(T = 0 \right)$. These expressions can be simplified for quantum gases where $T \left(\mathbf{p}, \mathbf{p}_1 \right) = -8\pi\hbar^2 a/m$ (a is the s-wave scattering length):

$$f \left(\mathbf{p}, \mathbf{p}_1 \right) = \frac{8\pi\hbar^2 a}{m} + \frac{16\pi^2 a^2 \hbar^4}{m^2} \int \frac{d^3 p'}{(2\pi\hbar)^3} [P \frac{1}{\epsilon + \epsilon_1 - \epsilon' - \epsilon_1'}$$

$$- \frac{1 - n_{\downarrow\downarrow}' - n_{1\uparrow\uparrow}'}{\epsilon + \epsilon_1 - \epsilon' - \epsilon_1' - i0 \cdot \text{sign} \left(p' - p_\downarrow \right)}], \qquad (4)$$

$$\mathcal{L}_{\text{coll}} = \frac{32\pi^3 \hbar^3 a^2}{m^2} \int \frac{d^3 p_1}{(2\pi\hbar)^3} \frac{d^3 p'}{(2\pi\hbar)^3} \delta \left(\epsilon + \epsilon_1 - \epsilon' - \epsilon_1' \right) \times$$

$$[n_{\uparrow\downarrow} \left(\mathbf{p}_1 \right) \left(n - 1 \right) - n_{\uparrow\downarrow} \left(\mathbf{p} \right) \left(n_1 - 1 \right)] \left(1 - n' \right) +$$

$$2 \left[n_{\uparrow\downarrow} \left(\mathbf{p}_1 \right) - n_{\uparrow\downarrow} \left(\mathbf{p} \right) \right] \left[n_{\uparrow\uparrow}' \left(n_{1\downarrow\downarrow}'^{(0)} - n_{1\downarrow\downarrow}' \right) + n_{\downarrow\downarrow}'^{(0)} \left(1 - n_{\downarrow\downarrow}' \right) \right] (5)$$

Eqs.(2) − (5) provide uniform description of spin dynamics, and contain both low-temperature anomalies and high-temperature classical expressions.

3. QUANTUM, DEGENERATE, AND CLASSICAL GASES

Gas is characterized by two density parameters, Nr_0^3 and $N\lambda^3$ (r_0 is the interaction radius, λ is the de Broglie wavelength). Expansion in $Nr_0^3 \ll 1$ corresponds to account for two-, three-, and multi-particle collisions; in practice, one rarely goes beyond the two-particle correlations. Parameter $N\lambda^3$ describes the distribution function rather than the density: in Boltzmann gases $N\lambda^3 << 1$, while for degenerate gases $N\lambda^3 \sim 1$.

The ratio $r_0/\lambda \sim pr_0/\hbar \sim \left(Nr_0^3/N\lambda^3\right)^{1/3}$ depends on degeneracy of the system. In Boltzmann gases, $r_0/\lambda \sim (mT)^{1/2}\left(r_0/\hbar\right)$ does not depend on density and is large at $T \gg \hbar^2/mr_0^2$. At $T \ll \hbar^2/mr_0^2$ one can reach the "quantum" region with $r_0/\lambda << 1$ for Boltzmann gases $T \gg T_F$. In degenerate gases, $r_0/\lambda \sim p_F r_0/\hbar \sim N^{1/3}r_0 << 1$ contains density and is small. We will use the term "quantum gases" for all gases for which $r_0/\lambda << 1$, irrespective of their degeneracy. All degenerate gases are "quantum" as far as the density is low, $Nr_0^3 \ll 1$, while Boltzmann gases are "quantum" only at $\hbar^2 n^{2/3}/m \ll T \ll \hbar^2/mr_0^2$. This "quantum" domain is a natural transition between the Boltzmann and degenerate regions when many quantum effects and the expansion in $Nr_0^3 << 1$ and $r_0/\lambda << 1$ are similar irrespective of degeneracy. Another appealing feature is that these are the gases of long-wave $r_0/\lambda << 1$ "slow" $pr_0/\hbar << 1$ particles for which collisions reduce to the s-wave scattering with the momentum-independent amplitude $-a$.

One should be careful in applying $(2) - (5)$ at different temperatures. In degenerate gases, the density expansion in Nr_0^3 coincides with the interaction and momentum expansion for the $T-$matrix in $p_F r_0/\hbar \sim N^{1/3}r_0$. Since the density expansion in Nr_0^3 is truncated, the accuracy requires not to go beyond the s-wave scattering for the T-matrix, $i.e.$, to use Eqs.(4), (5) rather than (2), (3). In Boltzmann gases, one can use exact T-matrix. However, higher order density terms in Eqs.(2), (3) constitute expansion in $N\lambda^3$. These terms are legitimate for degenerate gases when $N\lambda^3 \gg Nr_0^3$, and higher order terms in $N\lambda^3$ can be kept within the lowest order in Nr_0^3. In "classical" Boltzmann gases $N\lambda^3 \ll Nr_0^3$, and the interaction function and collision integral (2), (3) should contain only in the lowest orders in the distribution functions (this effectively makes the interaction function equal to zero). In "quantum" Boltzmann gases with $N\lambda^3 \gg Nr_0^3$ higher order terms in $N\lambda^3$ can be retained. In quantum gases, the one can keep all higher order density terms in the interaction function, and can restrict oneself to the s-wave approximation for the T-matrix (this is a must for degenerate gases).

The meaning of Eqs.(2) $-$ (5) is different at different temperatures. In the degenerate case, the imaginary (pole) part in the interaction function describes the zero-temperature attenuation (finite diffusion and relaxation

at $T = 0$). [This pole term is similar to the Landau damping in collisionless plasma]. At $T = 0$ the interaction function (4) coincides with the irreducible vertex of Ref.,[11] while its real part coincides with the results of direct calculation in.[14] The collision integral (3), (5) at $T = 0$ is zero. At finite temperatures there are two different T^2-contributions to dissipation: finite-temperature terms in Im f (4), and the collision integral (5); this explains the temperature dependence of attenuation in Ref.[13].

The real part of f-function (2), (4) is the I_2 term which has been repeatedly discussed for Boltzmann gases. The first-order density term with $P\frac{1}{\epsilon}$ in molecular field was suggested for Boltzmann gases in Ref.[15] on the basis of the expansion in $\widehat{U}$. It soon became clear[16,17] that the first-order density I_2- term in[15] should disappear as a result of the renormalization from the interaction $\widehat{U}$ to the $\widehat{T}$-matrix (this cancellation can be seen in the square brackets in Eq.(4)). The silent consensus was that the terms with $P\frac{1}{\epsilon}$ are always cancelled for Boltzmann gases, though this has never been checked in higher orders. The second-order I_2-term (2), (4) exists and is the same *at all temperatures*. Its contribution to kinetic equation (1) contains the product of *three* distribution functions and should be used very cautiously for classical gases since standard Boltzmann equation allows the product of maximum two distributions. This I_2-term can be used in the classical Boltzmann temperature range only if its $P\frac{1}{\epsilon}$ structure is significant and unique. At lower temperatures, including the quantum region for Boltzmann gases, the unrestricted use of this term does not cause any problems.

4. DISCUSSION. BEYOND THE SECOND ORDER

The combination of the interaction function (2), (4) with the collision integral (3), (5) extends the Fermi liquid approach (1) to all temperatures and provides the description in the frame of transverse statistical qp. The zero-temperature attenuation and the I_2-terms are simply the imaginary (pole) and real (principal) parts of the qp interaction function which coincides at $T = 0$ with the exact *irreducible* vertex.[11] This picture holds up to the second order in the interaction. Unfortunately, there is no consistent diagrammatic derivation of the transverse transport equation in the third order. What is known concerns only microscopic calculations at $T = 0$ and classical gases.

In the Fermi liquid domain, the spin-up - spin-down asymmetry manifests itself in the temporal non-locality of the interaction. The equation in the Green's function $G_{\uparrow\downarrow}(\omega, \mathbf{p}; t, \mathbf{r})$ can be reduced to the transport equation in qp distribution $n_{\uparrow\downarrow}(\mathbf{p}; t, \mathbf{r})$ only if the shape of the δ-type temporal peak in $G_{\uparrow\downarrow}(\omega, \mathbf{p})$ is preserved in dynamics. Starting from the third order in the

interaction,[11] the δ-peak splits into two on different energy shells (roughly speaking, this corresponds to different molecular fields for tilted spin-ups and spin-downs). The spin-up - spin-down asymmetry makes equations of motion for two peaks not identical except for homogeneous conditions when the equations can be brought together by going to the rotating reference frame (for inhomogeneous precession the rotation frequency depends on coordinates and momenta). Then the transverse transport equation is a set of *two* coupled equations in some partial transverse densities.[11] This problem arises only for transverse spin dynamics; in longitudinal dynamics the transition from microscopic description to statistical qp is fairly standard.

The problems for high-temperature classical Boltzmann gases are different, though the attempts to go beyond the Boltzmann equation also demonstrated the importance of temporal non-local and dissipative off-shell terms.[18] The results indicate that it is still possible to get a single closed kinetic equation in the distribution function, though the results obtained so far concern only longitudinal dynamics where this fact is hardly surprising.

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