Zero-Temperature Attenuation and Transverse Spin Dynamics in Fermi Liquids. III. Low Spin Polarizations

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This is the third in a series of papers on microscopic theory of transverse dynamics in spin-polarized Fermi liquids. In the lowest order in polarization our exact general theory of Ref. 1 reproduces the conventional Landau-Silin-Leggett theory of Fermi liquids. The next term in polarization expansion contains a zero-temperature attenuation with the magnitude that does not depend on polarization. This attenuation results in a finite relaxation time in transverse spin dynamics at zero temperature and is responsible for anomalous temperature behavior of spin diffusion. The zero-temperature attenuation is determined by two angular harmonics of the derivatives of the mass operators and the irreducible vertex in off-shell directions, and cannot be expressed via standard Fermi liquid harmonics. At high polarizations, the parameters of transverse spin dynamics are calculated as an expansion in polarization. The expansion involves complex values of interaction function and energy on a set of isoenergetic surfaces with the radii between the Fermi momenta for up and down spins, $p_\downarrow$ and $p_\uparrow$. The results explain recent experimental data on spin diffusion in spin-polarized liquid $^3He\uparrow$ and $^3He\uparrow$-^4He mixtures. The comparison with experimental data indicates that the superfluid transition temperature for ^3He in ^3He-^4He mixtures may be much lower than the current estimates.

1. INTRODUCTION

It has been known for a long time that a straightforward extension of the phenomenologic Landau theory of Fermi liquids fails in the case of *transverse* (off-diagonal in spin) phenomena in spin-polarized Fermi liquids with *high* degrees of spin polarization. The reason is the contribution of off-

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shell terms and spatial and temporal non-locality of the many-body Fermi liquid interaction. These effects are negligible for non-polarized systems and for longitudinal dynamics in polarized systems, but become important for transverse spin dynamics in highly polarized Fermi systems. However, the undeniable success of the Leggett-Silin theory of spin dynamics in weakly polarized Fermi liquids has, to some extent, masked a lack of understanding of spin dynamics in highly polarized systems when the difference of Fermi energies for up and down spins is comparable to the Fermi energy itself.

In two previous papers of this series[1,2] (referred below as I and II) we developed a general non-local microscopic theory of transverse dynamics in binary or spin polarized Fermi liquids,[1] and applied this exact theory to low density Fermi liquids.[2] Two main features of our exact non-local equations at $T = 0$ are the existence of a peculiar zero-temperature attenuation in transverse spin dynamics and the strong retardation which results in doubling of equations in spin densities. We obtained closed microscopic expressions for all relevant parameters including the zero-temperature attenuation and the corresponding relaxation time. The results explain the intrinsic inconsistency of numerous semi-phenomenologic attempts to generalize the Landau theory and make it applicable to highly polarized Fermi liquids.

Cumbersome and not very transparent general equations of Ref. 1 express the characteristics of transverse spin dynamics through exact irreducible vertex functions and self energies of a Fermi liquid. Though these equations lead to several general qualitative conclusions applicable to all Fermi liquids, the vertex functions and self energies cannot be calculated explicitly for most of the systems even without spin polarization. Therefore, practical applications of the general results of Ref. I should involve either model calculations or approximations which can be justified only in some limiting cases. In II we simplified the general expressions of Ref. I for the case of dilute Fermi liquids, and obtained simple explicit expressions for the parameters of transverse spin dynamics at arbitrary spin polarizations as an expansion in density of the Fermi liquid. We also demonstrated how and with what accuracy the general expressions of Ref. I reduce to the conventional equations of spin dynamics in nearly ideal Fermi gases.

Another natural limiting case is the case of low spin polarization when one should recover the well-known Landau-Silin-Leggett result.[3-5] This is discussed in the next Section. In Sec. 3 we go beyond the Silin-Leggett limit, and calculate explicitly the spectrum, zero-temperature attenuation, and relaxation time at low polarizations. In Sec. 4 we develop a general polarization expansion for transverse spin dynamics. In Sec. 5 we extend the results to finite temperatures, and compare our results for low polarizations with the recent experimental data on ^{3}He↑ and ^{3}He↑-^{4}He mixtures.

2. LOW SPIN POLARIZATIONS

In this Section we will analyze the exact non-local microscopic equations of Ref. 1 at vanishingly low spin polarizations, and reduce them to the Landau-Silin-Leggett equations.

In the beginning we want to make two comments. In what follows we use as thermodynamic variables the external magnetic field **H** and the chemical potential μ. This implies that we are dealing with a Fermi liquid in which spins are polarized by the so-called brute force technique, i.e., by the external magnetic field. However, practically all of the results can be applied to spin-polarized Fermi liquids in which the spin polarization is caused by any other means (see, e.g., review[6]). In these cases the adequate thermodynamic variables are the chemical potentials for particles with up and down spins, $\mu_\uparrow$ and $\mu_\downarrow$, rather than H and μ.

Second, we are interested in low-frequency (aside from the Larmor frequency, $\Omega_0 = 2\beta H$, which may be arbitrary) and long-wave dynamics. More precisely, we assume that kv_0 (v_0 is the Fermi velocity) and $\delta\omega = \omega - \Omega_0$ are much smaller than the characteristic internal molecular field Ω_i. [An oversimplified estimate for Ω_i is $\Omega_i \sim -(4\pi a_{\mathrm{eff}}/m^*)(N_\uparrow - N_\downarrow)$ where α_{eff} is some effective scattering length, $N_{\uparrow,\downarrow}$ are the densities of spin-ups and spin-downs, m^* is the effective mass, and we assume $\hbar = 1$ throughout this paper. For more accurate expressions for Ω_i in weakly polarized or dilute Fermi systems see Ref. 6. At present, it is not clear how to introduce a similar quantity in dense highly polarized quantum liquids; see the Summary.] This "hydrodynamic" regime for transverse spin dynamics covers spin diffusion, low-frequency spin rotation and long-wave spin waves. Below we will expand all the equations in kv/Ω_i and $\delta\omega/\Omega_i$. This is a standard procedure in spin dynamics which usually does not cause any problems. At low spin polarization we will also perform an expansion in polarization. Since Ω_i is proportional to spin polarization Δ, it vanishes when the polarization Δ goes to zero. Therefore, though the polarization and "hydrodynamic" expansions are formally independent from each other, these two types of expansion should be coordinated by keeping $kv/\Omega_i(\Delta) \ll 1$. This means that with lowering polarizations our results should be restricted to more and more homogeneous systems, exactly in the same way as in the standard Silin-Leggett theory. Not accidentally, this will also take care of a formal singularity at $\Delta \to 0$ (inherent to spin dynamics in all Fermi liquids,[3-6]) in the spectrum of spin waves which is proportional to $(kv)^2/\Omega_i$.

Let us start from the equations of Ref. 1 and relevant notations. In I we have shown that the strong temporal non-locality does not allow to derive a closed Landau-like mean field equation for a mixed (off-diagonal)

spin component of single-particle (or quasi-particle) distribution $n_{\downarrow\uparrow}(\mathbf{p})$. Instead, the exact equation in the mixed spin component of the single-particle Green's function $G_{\uparrow\downarrow}(P)$ $(P = (p_0, \mathbf{p})$ is the 4-momentum) reduces to a set of two coupled non-local equations in some partial transverse *pseudo*-densities, $\delta n_\uparrow(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\uparrow)}(\mathbf{p})$ and $\delta n_\downarrow(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\downarrow)}(\mathbf{p})$. The *pseudo*-densities, taken separately, do not correspond to any observables, and are, generically, the momentum distributions for transverse magnetic moments $\delta n_{\uparrow\downarrow}(\mathbf{p}) = \mathrm{Tr}_\sigma \hat{\sigma}^+ \delta \hat{n}_\sigma(\mathbf{p})$ originating from slightly tilted spins of spin-up and spin-down particles. These two coupled equations in partial transverse *pseudo*-densities, $\delta n_\uparrow(\mathbf{p})$ and $\delta n_\downarrow(\mathbf{p})$, are equivalent to the Dyson equation in $\delta G_{\uparrow\downarrow}(P)$ at $T = 0$, Eqs. (I.34):

$$W_\downarrow(K; \mathbf{p}) \, \delta n_\downarrow(\mathbf{p}) = \tfrac{1}{2} \int \left[F_{\downarrow\downarrow}(K; \mathbf{p}, \mathbf{p}') \, \delta n_\downarrow(\mathbf{p}') \right.$$

$$\left. + F_{\downarrow\uparrow}(K; \mathbf{p}, \mathbf{p}') \, \delta n_\uparrow(\mathbf{p}') \right] (\theta_\downarrow(\mathbf{p}') - \theta_\uparrow(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3}$$

$$W_\uparrow(K; \mathbf{p}) \, \delta n_\uparrow(\mathbf{p}) = \tfrac{1}{2} \int \left[F_{\uparrow\downarrow}(K; \mathbf{p}, \mathbf{p}') \, \delta n_\downarrow(\mathbf{p}') \right. \qquad (1)$$

$$\left. + F_{\uparrow\uparrow}(K; \mathbf{p}, \mathbf{p}') \, \delta n_\uparrow(\mathbf{p}') \right] (\theta_\downarrow(\mathbf{p}') - \theta_\uparrow(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3}$$

The transverse *pseudo*-densities, $\delta n_\uparrow(\mathbf{p})$ and $\delta n_\downarrow(\mathbf{p})$, are defined via the small off-diagonal component of the Green's function $\delta G_{\uparrow\downarrow}(P)$ at two different frequencies p_0 as

$$\delta n_\uparrow(\mathbf{p}) = g(P_\uparrow), \, \delta n_\downarrow(\mathbf{p}) = g(P_\downarrow),$$

$$\delta G_{\uparrow\downarrow}(P; t, \mathbf{r}) = g(P) \left[G_{\downarrow\downarrow}\left(P + \frac{K}{2} \right) - G_{\uparrow\uparrow}\left(P - \frac{K}{2} \right) \right] \exp(i\mathbf{kr} - i\omega t) \quad (2)$$

where the 4-vector $K = (\delta\omega, \mathbf{k})$ characterizes temporal and spatial non-localities of the theory, the energy components p_0 of the 4-vectors $P_\uparrow$ and $P_\downarrow$ are determined by the single-particle energy spectra $\varepsilon_\uparrow$ and $\varepsilon_\downarrow$:

$$P_\downarrow = \left(\varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2} \right) - \mu - \frac{\delta\omega}{2} - \beta H, \mathbf{p} \right), \, P_\uparrow = \left(\varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2} \right) - \mu + \frac{\delta\omega}{2} + \beta H, \mathbf{p} \right),$$

$$\delta\omega = \omega - 2\beta H, \, \theta_\downarrow(\mathbf{p}) = \theta\left(\left| \mathbf{p} + \frac{\mathbf{k}}{2} \right| - p_\downarrow \right), \, \theta_\uparrow(\mathbf{p}) = \theta\left(\left| \mathbf{p} - \frac{\mathbf{k}}{2} \right| - p_\uparrow \right) \quad (3)$$

and $\mathbf{p}_{\uparrow,\downarrow}$ are Fermi momenta for up and down spins. Note, that in I and II the functions $\theta_{\uparrow,\downarrow}$ are defined in a slightly different way, namely, as

$\theta(\varepsilon_{\uparrow,\downarrow}(\mathbf{p} \mp \mathbf{k}/2) - \mu)$. The definition (3) is more consistent with the results of Appendix C in I. For most of the purposes, including the calculations in I and II, these two definitions are equivalent. Below, the difference is important.

The functions $W_{\uparrow,\downarrow}$ in the l.h.s. of Eqs. (1) play the role of generalized Liouville operators. They are expressed through the single-particle energies, $\varepsilon_\uparrow$ and $\varepsilon_\downarrow$, and some *pseudo*-energies, $\tilde{\varepsilon}_\uparrow$ and $\tilde{\varepsilon}_\downarrow$:

$$W_\downarrow(K; \mathbf{p}) = \left[\delta\omega + 2\beta H - \varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) + \tilde{\varepsilon}_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) \right] Z_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)$$

$$W_\uparrow(K; \mathbf{p}) = \left[\delta\omega + 2\beta H - \tilde{\varepsilon}_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) + \varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) \right] Z_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right).$$

(4)

These energies and *pseudo*-energies are defined through the mass operators for *pure* spin-up and spin-down states of particles $\Sigma_{\uparrow\uparrow}(P)$ and $\Sigma_{\downarrow\downarrow}(P)$ as (I.37, 40)

$$\varepsilon_\uparrow(\mathbf{p}) = \frac{p^2}{2m} + \Sigma_{\uparrow\uparrow}(\varepsilon_\uparrow(\mathbf{p}) - \mu, \mathbf{p}) - \beta H,$$

$$\varepsilon_\downarrow(\mathbf{p}) = \frac{p^2}{2m} + \Sigma_{\downarrow\downarrow}(\varepsilon_\downarrow(\mathbf{p}) - \mu, \mathbf{p}) + \beta H$$

$$\tilde{\varepsilon}_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) = \frac{(\mathbf{p} - \mathbf{k}/2)^2}{2m} + \Sigma_{\uparrow\uparrow}\left(\varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right)\right.$$

$$\left. - \mu - \delta\omega + 2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta) H, \mathbf{p} - \frac{\mathbf{k}}{2}\right) - \beta H \qquad (5)$$

$$\tilde{\varepsilon}_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) = \frac{(\mathbf{p} + \mathbf{k}/2)^2}{2m} + \Sigma_{\downarrow\downarrow}\left(\varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\right.$$

$$\left. - \mu + \delta\omega - 2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta H, \mathbf{p} + \frac{\mathbf{k}}{2}\right) + \beta H$$

where the function $\beta_1(\mathbf{p}; \mathbf{k}, H)$ determines the difference in energies between spin-ups and spin-downs, and describes the longitudinal susceptibility of the Fermi liquid:

$$\beta_1(\mathbf{p}; \mathbf{k}, H) = \frac{1}{2H}\left[\varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) - \varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right)\right] \qquad (6)$$

The renormalization coefficients $Z_{\uparrow,\downarrow}$ in Eq. (4) are given, according to Eq. (I.29), as

$$1 - \frac{1}{Z_{\uparrow,\downarrow}(\mathbf{p})} = \frac{\partial\Sigma_{\uparrow\uparrow,\downarrow\downarrow}(p_0 = \varepsilon_{\uparrow,\downarrow}(\mathbf{p}), \mathbf{p})}{\partial p_0} \qquad (7)$$

The 4-component interaction function F_{ik} in the r.h.s. of Eq. (1) serves as a generalized Landau function for transverse phenomena, and is equal to (I.36)

$$F_{\downarrow\downarrow}(K; \mathbf{p}, \mathbf{p}') = Z_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) Z_\downarrow\left(\mathbf{p}' + \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_\downarrow, P_\downarrow')$$

$$F_{\downarrow\uparrow}(K; \mathbf{p}, \mathbf{p}') = Z_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) Z_\uparrow\left(\mathbf{p}' - \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_\downarrow, P_\uparrow')$$

$$F_{\uparrow\downarrow}(K; \mathbf{p}, \mathbf{p}') = Z_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) Z_\downarrow\left(\mathbf{p}' + \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_\uparrow, P_\downarrow')$$

$$F_{\uparrow\uparrow}(K; \mathbf{p}, \mathbf{p}') = Z_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) Z_\uparrow\left(\mathbf{p}' - \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_\uparrow, P_\uparrow')$$

(8)

where the generating function $\mathfrak{C}(K; P, P')$ is related to the mixed spin component of the *irreducible* vertex function $\tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}(K; P, P')$ via the integral Eq. (I.23)

$$\mathfrak{C}(K; P, P') = \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}(K; P, P') + \int \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}(K; P, Q)$$

$$\times \phi_r(Q + K/2, Q - K/2)\,\mathfrak{C}(K; Q, P')\frac{d^4Q}{(2\pi)^4} \qquad (9)$$

and ϕ_r is the regular part of the Green's functions defined as

$$G_{\downarrow\downarrow}\left(P + \frac{K}{2}\right) - G_{\uparrow\uparrow}\left(P - \frac{K}{2}\right)$$

$$= i\phi_r\left(P + \frac{K}{2}, P - \frac{K}{2}\right) \times \left[\delta\omega - \frac{\mathbf{pk}}{m} - \Sigma_{\downarrow\downarrow}\left(P + \frac{K}{2}\right) + \Sigma_{\uparrow\uparrow}\left(P - \frac{K}{2}\right)\right]$$

$$+ i\pi\left[\theta\left(\varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) - \mu\right) - \theta\left(\varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - \mu\right)\right]$$

$$\times\left[Z_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\delta\left(p_0 + \frac{\delta\omega}{2} + \beta H - \varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) + \mu\right)\right.$$

$$\left. + Z_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right)\delta\left(p_0 - \frac{\delta\omega}{2} - \beta H - \varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) + \mu\right)\right] \qquad (10)$$

Now let us reduce Eqs. (1) to the standard Silin-Leggett equation in case of vanishingly low polarization. If the magnetic field and polarization are low, then all the quantities with up and down arrow indices are close

to each other. What is more, the term $2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta) H$ in the argument of the mass operators (5) is small. As a result, the *pseudo*-energies (5) are nearly equal to real energies and to each other:

$$\tilde{\varepsilon}_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) \approx \varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) + [2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta) H - \delta\omega]\left[1 - Z_\uparrow^{-1}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right)\right]$$

$$\tilde{\varepsilon}_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) \approx \varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) - [2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta) H - \delta\omega]\left[1 - Z_\downarrow^{-1}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\right]$$

$$(11)$$

With the same accuracy, we can rewrite the non-local Liouville operators $W_{\uparrow,\downarrow}$ (4) as

$$W_\downarrow(K; \mathbf{p}) \approx \left[\delta\omega + 2(\beta - \beta_1(\mathbf{p}; 0, H)) H - \frac{\mathbf{k}\mathbf{v}_\downarrow + \mathbf{k}\mathbf{v}_\uparrow}{2}\right]$$

$$\times Z_\uparrow^{-1}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) Z_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)$$

$$(12)$$

$$W_\uparrow(K; \mathbf{p}) \approx \left[\delta\omega + 2(\beta - \beta_1(\mathbf{p}; 0, H)) H - \frac{\mathbf{k}\mathbf{v}_\downarrow + \mathbf{k}\mathbf{v}_\uparrow}{2}\right]$$

$$\times Z_\downarrow^{-1}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) Z_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right)$$

where we have used Eqs. (6), (7), (11). Two functions $W_{\uparrow,\downarrow}$ (12) differ only by the Z-factors. Since the terms in the square brackets are already small in polarization or $\mathbf{k}$, the arrow indices of the Z-factors and the wave vectors $\mathbf{k}$ in their arguments can be dropped making the functions $W_{\uparrow,\downarrow}$ identical to each other. Note, that in the linear approximation the sum of Fermi velocities for up and down spins $v_\uparrow + v_\downarrow$ is equivalent to $2v_0$ (v_0 is the Fermi velocity in non-polarized Fermi liquid of the same density).

The main source of polarization and field dependence of the r.h.s. of Eqs. (1) is associated with the θ-functions (3) in the integrand. In the lowest approximation in polarization, these θ-functions reduce to

$$\theta\left(\left|\mathbf{p} + \frac{\mathbf{k}}{2}\right| - p_\downarrow\right) - \theta\left(\left|\mathbf{p} - \frac{\mathbf{k}}{2}\right| - p_\uparrow\right)$$

$$\approx \theta\left(\varepsilon_\downarrow\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) - \mu\right) - \theta\left(\varepsilon_\uparrow\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - \mu\right)$$

$$(13)$$

$$\approx 2\beta_1(\mathbf{p}; \mathbf{k}, H) H\delta(\varepsilon_0(\mathbf{p}) - \mu)$$

$$\approx [2\beta_1(\mathbf{p}; 0, H) H + \mathbf{v} \cdot \mathbf{k}] \delta(\varepsilon_0(\mathbf{p}) - \mu)$$

where $\varepsilon_0(\mathbf{p})$ is the spectrum of quasi-particles without polarization. Since this term is already linear in polarization and $\mathbf{k}$, the polarization dependence of all other terms in the r.h.s. of Eqs. (1) can be neglected. In practical terms this means that we can omit arrow indices of the interaction functions, and consider all four components of F_{ik} equal to each other. For the same reason we can disregard the indices of the Z-factors (12) (and the wave vector $\mathbf{k}$ in their arguments). Then both Eqs. (1) become identical, and the set (1) collapses into a single equation, which, because of the δ-function (13) in the integrand, involves only a single Fermi surface $|\mathbf{p}| = p_0$:

$$[\delta\omega + 2(\beta - \beta_{10}) H - \mathbf{k} \cdot \mathbf{v}_0] \, \delta n$$
$$= [2\beta_{10} H + \mathbf{k} \cdot \mathbf{v}_0] \int F^{(a)}(\mathbf{p}_0, \mathbf{p}_0') \, \delta n(\mathbf{p}_0') \, do/4\pi \qquad (14)$$

with $\beta_{10} = \beta_1(p_0; 0, 0)$. The usual antisymmetric component of the Landau function $F^{(a)}$ is related to the above interaction functions F_{ik} and the vertex functions (8), (9) as

$$F^{(a)}(\mathbf{p}_0, \mathbf{p}_0') = \frac{4\pi p_0 m}{(2\pi)^3} Z(p_0)^2 \, \mathbb{C}(0; P_0, P_0) \qquad (15)$$

Eq. (14) is exactly the same as the Landau-Silin-Leggett equation.[3,4] This equation describes the Silin spin waves with the spectrum

$$\omega = \Omega_0 + \frac{1}{3} \frac{k^2 v_0^2}{\Omega_i} (1 + F_0^{(a)})$$
$$\Omega_i = \frac{(F_0^{(a)} - F_1^{(a)}/3) \, \Omega_0}{(1 + F_1^{(a)}/3)(1 + F_0^{(a)})} \qquad (16)$$

where $F_0^{(a)}$ and $F_1^{(a)}$ are the zeroth and first angular harmonics of the Landau function $F^{(a)}(\mathbf{p}_0, \mathbf{p}_0')$. Of course, with this accuracy, there is no zero-temperature attenuation, and the spectrum (16) is real.

3. LOW SPIN POLARIZATIONS: BEYOND THE LANDAU-SILIN-LEGGETT APPROXIMATION

In this Section we will calculate the main term in the zero-temperature attenuation at low polarizations. To do so, we have to go beyond the Landau-Silin-Leggett approximation for Eqs. (1). Since we are not interested in higher order polarization corrections to the *real* part of the spectrum (16), we should leave all the real parts of the vertex function and mass operators

in the same form as in Sec. 2, and add only the lowest in polarization imaginary contributions to the vertex and self-energies in Eqs. (1)–(9).

We will start from the r.h.s. of Eqs. (1). We will use a more accurate equation for the difference of the θ-functions, $\theta_\downarrow - \theta_\uparrow$, instead of a single δ-function (13):

$$\theta\left(\left|\mathbf{p}+\frac{\mathbf{k}}{2}\right|-p_\downarrow\right)-\theta\left(\left|\mathbf{p}-\frac{\mathbf{k}}{2}\right|-p_\uparrow\right)\approx\frac{1}{2}\left[\delta(p-p_1)+\delta(p-p_2)\right](p_2-p_1),$$

$$\left|\mathbf{p}_1+\frac{\mathbf{k}}{2}\right|=p_\downarrow,\quad\left|\mathbf{p}_2-\frac{\mathbf{k}}{2}\right|=p_\uparrow \tag{17}$$

Then the integration over $d\mathbf{p}'$ in Eqs. (1) is equivalent to substitution of $\mathbf{p}_1$ and $\mathbf{p}_2$ instead of $\mathbf{p}'$ in the integrand. In order to get a closed set of equations, we should consider Eqs. (1) only on the surfaces $|\mathbf{p}| = p_1$ and $|\mathbf{p}| = p_2$. As a result, the set (1) will reduce to a set of four equations in $\delta n_{\uparrow,\downarrow}(\mathbf{p}_{1,2})$. Each of these equations is an integral equation in angles; these equations can easily be solved by expansion of the functions F_{ik} in angular harmonics in the same way as it is done in a standard Silin-Leggett case. What is more, we will see that in the lowest approximation in polarization, these four equations still can be reduced to a single one.

The mass-operators $\Sigma_{\uparrow\uparrow}$ and $\Sigma_{\downarrow\downarrow}$ are real only on their Fermi surfaces, $|\mathbf{p}| = p_{\uparrow,\downarrow}$. Away from the Fermi surfaces the self-energies obtain imaginary parts which, close to the Fermi surface, are quadratic in the distance from the Fermi surface:

$$\text{Im}\,\Sigma_{\uparrow\uparrow}=\frac{1}{m^*}\,\sigma(p-p_\uparrow)^2\,\text{sign}(p-p_\uparrow),$$

$$\text{Im}\,\Sigma_{\downarrow\downarrow}=\frac{1}{m^*}\,\sigma(p-p_\downarrow)^2\,\text{sign}(p-p_\downarrow)$$

while the difference in coefficients σ for up and down spins in Eq. (17) is insignificant at low polarizations (we introduced the factor $1/m^*$ so that to keep the coefficient σ dimensionless). For dilute Fermi systems, according to Appendix B of II,

$$\sigma=a^2p_0^2/\pi \tag{18}$$

where a is the s-wave scattering length. These imaginary parts should be added to the energies and *pseudo*-energies (5).

If we keep only the lowest order polarization terms in real and imaginary parts of energies ε and *pseudo*-energies $\tilde{\varepsilon}$, the energies for up and down spins on the surfaces $|\mathbf{p}| = p_{1,2}$ can be written as

$$\varepsilon_\downarrow \left(\mathbf{p}_1 + \frac{\mathbf{k}}{2} \right) = \varepsilon_\downarrow(\mathbf{p}_\downarrow),$$

$$\varepsilon_\downarrow \left(\mathbf{p}_2 + \frac{\mathbf{k}}{2} \right) = \varepsilon_\downarrow(\mathbf{p}_\downarrow) + \mathbf{v}_\downarrow \cdot \mathbf{p}_\uparrow + \mathbf{k} \cdot \mathbf{v}_\downarrow$$

$$+ i\sigma \left[\frac{(p_\uparrow - p_\downarrow)^2}{m^*} + 2\mathbf{k} \cdot \mathbf{v}_0(p_\uparrow - p_\downarrow) + \frac{(\mathbf{k} \cdot \mathbf{v}_0)^2}{p_0 v_0} \right],$$

$$\varepsilon_\uparrow \left(\mathbf{p}_2 - \frac{\mathbf{k}}{2} \right) = \varepsilon_\uparrow(\mathbf{p}_\uparrow),$$

$$\varepsilon_\uparrow \left(\mathbf{p}_1 - \frac{\mathbf{k}}{2} \right) = \varepsilon_\uparrow(\mathbf{p}_\uparrow) + \mathbf{v}_\uparrow \cdot \mathbf{p}_\downarrow$$

$$- \mathbf{k} \cdot \mathbf{v}_\uparrow - i\sigma \left[\frac{(p_\uparrow - p_\downarrow)^2}{m^*} + 2\mathbf{k} \cdot \mathbf{v}_0(p_\uparrow - p_\downarrow) + \frac{(\mathbf{k} \cdot \mathbf{v}_0)^2}{p_0 v_0} \right]$$

Similar expressions can be obtained for *pseudo*-energies $\tilde{\varepsilon}$ using Eqs. (5), (11). Then the generalized Liouville operators $W_{\downarrow,\uparrow}$ (4), (13) on the surfaces $|\mathbf{p}| = p_{1,2}$ become equal to each other:

$$W_{\downarrow,\uparrow}(K; \mathbf{p}_{1,2}) \approx \delta\omega + 2(\beta - \beta_{10}) H - \mathbf{k} \cdot \mathbf{v}_0$$

$$+ \frac{i\sigma}{2T_0} [4\beta_{10}H(\beta_{10}H + \mathbf{k} \cdot \mathbf{v}_0) + (\mathbf{k} \cdot \mathbf{v}_0)^2] \qquad (19)$$

where T_0 is the Fermi energy $p_0^2/2m^*$, and we took into account that $(p_0/m^*)(p_\uparrow - p_\downarrow) \approx 2\beta_{10} H$.

Similar parametrization should be done in the r.h.s. of Eqs. (1). Here we can keep the Silin-Leggett form for the real parts of all functions, except for the θ-functions for which we should use Eq. (17) instead of Eq. (13). Since the imaginary part of the mass operators is quadratic in distance for the frequency from the mass surface, the imaginary part of its derivative on the mass surface, and, therefore, the imaginary parts of the Z-factors (7), are equal to zero. Therefore, in the lowest approximation in polarization, the sole source of imaginary terms in the r.h.s. of Eqs. (1) is the renormalized vertex function $\mathfrak{C}$ (8), (9). These imaginary terms are linear in $|\mathbf{p}_1 - \mathbf{k}/2| - p_\uparrow$ and $|\mathbf{p}_2 + \mathbf{k}/2| - p_\downarrow$. Then the introduction of imaginary parts into the r.h.s. of Eqs. (1) is equivalent to substitution

$$F_{ik}(\mathbf{p}, \mathbf{p}') \to F_{ik}(\mathbf{p}, \mathbf{p}') + \frac{2i\pi^2}{p_0^2 m^* Z^2(p_0)} \gamma(\mathbf{p}, \mathbf{p}') \left[p_\uparrow - p_\downarrow + p_0' \frac{\mathbf{k}}{\mathbf{p}_0} \right] \quad (20)$$

with the same function γ irrespective of the arrow indices of the functions F. In the low density limit (see Appendix A of II),

$$\gamma(\mathbf{p}_0, \mathbf{p}_0') \equiv \gamma(\theta) = a^2 p_0^2/2\pi \, \cos(\theta/2) \qquad (21)$$

Finally, after the integration over momenta with the help of Eq. (17), the set of Eqs. (1), (19), (20) reduces to a single equation in $\delta n = [\delta n_\downarrow(p_1) + \delta n_\uparrow(p_2)]/2$ which differs from Eq. (14) only by small imaginary terms:

$$\left[\delta\omega + 2(\beta - \beta_{10}) H - \mathbf{k} \cdot \mathbf{v}_0 + \frac{i\sigma}{2T_0} (4\beta_{10} H(\beta_{10} H + \mathbf{k} \cdot \mathbf{v}_0) + (\mathbf{k} \cdot \mathbf{v}_0)^2)\right] \delta n$$

$$= [2\beta_{10} H + \mathbf{k} \cdot \mathbf{v}_0] \int \left[F^{(a)}(\mathbf{p}_0, \mathbf{p}_0') + i \frac{2\beta_{10} H + \mathbf{k} \cdot \mathbf{v}_0'}{2T_0} \gamma(\mathbf{p}_0, \mathbf{p}_0')\right] \delta n \frac{do'}{4\pi}$$

The integration over the angles in the r.h.s. of this equation is performed by introducing the angular harmonics of the functions $F^{(a)}$, γ, and $\delta n = n_0 + \mathbf{k} \cdot \mathbf{v}_0 n_1$:

$$\left[\delta\omega + 2(\beta - \beta_{10}) H - \mathbf{k} \cdot \mathbf{v}_0 + \frac{i\sigma}{2T_0} (4\beta_{10} H(\beta_{10} H + \mathbf{k} \cdot \mathbf{v}_0)\right.$$

$$\left. + (\mathbf{k} \cdot \mathbf{v}_0)^2)\right] (n_0 + \mathbf{k} \cdot \mathbf{v}_0 n_1)$$

$$= [2\beta_{10} H + \mathbf{k} \cdot \mathbf{v}_0] \left[\left(F_0^{(a)} + i \frac{\beta_{10} H}{T_0} \gamma_0 + i \frac{\mathbf{k} \cdot \mathbf{v}_0}{2T_0} \frac{\gamma_1}{3}\right) n_0\right.$$

$$+ \frac{1}{3} \mathbf{k} \cdot \mathbf{v}_0 \left(F_1^{(a)} + i \frac{\beta_{10} H}{T_0} \gamma_1\right) n_1$$

$$\left. + \frac{i}{3} k^2 v_0^2 \left(\gamma_2 \left(\frac{3(\mathbf{k} \cdot \mathbf{v}_0)^2}{k^2 v_0^2} - 1\right) + \gamma_0\right) n_1\right] \tag{22}$$

Eq. (22) is solved in a standard way. First, we integrate Eq. (22) over the angles:

$$\left[\delta\omega + 2(\beta - \beta_{10}) H + \frac{i\sigma}{2T_0} \left(4\beta_{10}^2 H^2 + \frac{1}{3} k^2 v_0^2\right)\right] n_0$$

$$- \frac{1}{3} k^2 v_0^2 \left(1 - 2i\sigma \frac{\beta_{10} H}{T_0}\right) n_1$$

$$= 2\beta_{10} H \left(F_0^{(a)} + i \frac{\beta_{10} H}{T_0} \gamma_0\right) n_0 + \frac{i}{9} \frac{k^2 v_0^2}{2T_0} \gamma_1 n_0$$

$$+ \frac{1}{3} k^2 v_0^2 \left(\frac{F_1^{(a)}}{3} + i \frac{\beta_{10} H}{T_0} \left(\frac{\gamma_1}{3} + \gamma_0\right)\right) n_1 \tag{23}$$

In I we showed, using the method similar to Ref. 7, that at $k = 0$ the eigenvalue of Eqs. (1) is always $\delta\omega = 0$. This means that

$$2(\beta - \beta_{10}) H + i\sigma \frac{2\beta_{10}H}{T_0} \beta_{10}H = 2\beta_{10}H \left(F_0^{(a)} + i \frac{\beta_{10}H}{T_0} \gamma_0 \right)$$

The real part of this equation is equivalent to a standard expression for the magnetic susceptibility of a Fermi liquid:

$$\beta_{10} = \frac{\beta}{1 + F_0^{(a)}} \tag{24}$$

while the imaginary part is simply

$$\sigma = \gamma_0 \tag{25}$$

In the case of low density Fermi systems, this equation can be easily confirmed with the help of Eqs. (18), (21) for σ and γ. As a result, Eq. (23) reduces to

$$\left(\delta\omega + \frac{i}{3} \left(\gamma_0 - \frac{\gamma_1}{3} \right) \frac{k^2 v_0^2}{2T_0} \right) n_0$$

$$- \frac{1}{3} k^2 v_0^2 \left[1 + \frac{1}{3} F_1^{(a)} - \frac{i\Omega_0}{2T_0(1 + F_0^{(a)})} \left(\gamma_0 - \frac{\gamma_1}{3} \right) \right] n_1 = 0 \tag{26}$$

where $\Omega_0 = 2\beta H$ is the Larmor frequency.

The second equation can be obtained by multiplying Eq. (22) by $\mathbf{k} \cdot \mathbf{v}_0$ and integrating over angles:

$$- \frac{1}{3} k^2 v_0^2 \left(1 + F_0^{(a)} - \frac{i\Omega_0(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})} \right) n_0$$

$$+ \frac{1}{3} k^2 v_0^2 \frac{\Omega_0}{1 + F_0^{(a)}} \left[F_0^{(a)} - \frac{1}{3} F_1^{(a)} + \frac{i\Omega_0(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})} \right] n_1 = 0 \tag{27}$$

The eigenvalue of Eqs. (26), (27) is equal to

$$\delta\omega = \frac{1}{3} k^2 v_0^2 \frac{1 + F_0^{(a)}}{\Omega_0} \left[1 + F_0^{(a)} - \frac{i\Omega_0(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})} \right]$$

$$\times \frac{1 + F_1^{(a)}/3 - i\Omega_0(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)})}{F_0^{(0)} - F_1^{(a)}/3 + i\Omega_0(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)})}$$

$$- \frac{i}{3} \frac{k^2 v_0^2}{2T_0} \left(\gamma_0 - \frac{\gamma_1}{3} \right) \tag{28}$$

or, since $\Omega_0/T_0 \ll 1$,

$$
\delta\omega = \frac{1}{3} \frac{k^2 v_0^2}{\Omega_0} \frac{(1 + F_0^{(a)})^2 (1 + F_1^{(a)}/3)}{F_0^{(a)} - F_1^{(a)}/3}
$$

$$
\times \left[1 - \frac{i\Omega_0(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})} \left\{ \frac{1}{1 + F_0^{(a)}} + \frac{1}{1 + F_1^{(a)}/3} \right. \right.
$$

$$
\left. \left. + \frac{1}{F_0^{(a)} - F_1^{(a)}/3} + \frac{(F_0^{(a)} - F_1^{(a)}/3)}{(1 + F_0^{(a)})(1 + F_1^{(a)}/3)} \right\} \right]
$$

$$
\equiv \frac{1}{3} \frac{k^2 v_0^2}{\Omega_i} (1 + F_0^{(a)}) \left[1 - \frac{i}{\Omega_i \tau_{\perp 0}} \right]
$$

where the term in the square brackets in Eq. (29) determines the zero-temperature attenuation of the Silin spin waves (16), and $\tau_{\perp 0}$ is the characteristic relaxation time for the zero-temperature attenuation,

$$
\tau_{\perp 0}^{-1} = \frac{\Omega_0^2(\gamma_0 - \gamma_1/3)(1 + 2F_0^{(a)} - F_1^{(a)}/3)}{2T_0(1 + F_0^{(a)})^2 (1 + F_1^{(a)}/3)^2} \tag{29}
$$

We will use Eqs. (28), (29) in Sec. 5 for comparison with experimental data.

In stable systems the imaginary part of the frequency $\delta\omega$ should be negative and $\tau_{\perp 0}$ - positive. Since the interaction always results in attenuation of single-particle states, the coefficients $\sigma = \gamma_0$ are positive. The standard stability condition with respect to a ferromagnetic transition has the form $1 + F_0^{(a)} > 0$. Then, if $1 + 2F_0^{(a)} - F_1^{(a)}/3 > 0$, the stability of the polarized Fermi liquid means that $\gamma_0 - \gamma_1/3 > 0$. In essence, this condition is equivalent to the natural requirement of positive transport cross-section for particles in the transverse channel. A more interesting question is what happens if $1 + 2F_0^{(a)} - F_1^{(a)}/3$ changes sign.

4. ARBITRARY POLARIZATIONS: POLARIZATION EXPANSION

In I we gave a formal solution of spin dynamics equations (1) at arbitrary polarizations, and determined the spectrum of spin waves including the zero-temperature attenuation. However, these expressions are extremely cumbersome, and are not convenient for practical applications. Below we will outline another method of solving Eqs. (1) which is equivalent to an expansion in polarization. This method may be useful for numerical and model calculations for spin-polarized Fermi liquids.

One of the main differences between Eqs. (1) for polarized Fermi liquids and Eq. (14) for vanishingly small polarizations is the presence θ-functions in the kernel of the integral equations instead of δ-functions (13). This makes integral equations (1) not only equations in angles, as for non-polarized Fermi liquids, but also integral equations in absolute values of momenta. There is no simple general procedure of solving such equations. Therefore, in contrast to low polarizations, the general results cannot be expressed via angular harmonics of some interaction functions.

There are two ways to deal with this difficulty and to solve Eqs. (1) in form of an expansion in polarization. The first one is to continue the expansion (13) as an expansion in derivatives of the δ-functions,

$$\theta\left(\left|\mathbf{p}+\frac{\mathbf{k}}{2}\right|-p_\downarrow\right)-\theta\left(\left|\mathbf{p}-\frac{\mathbf{k}}{2}\right|-p_\uparrow\right)$$

$$=\frac{1}{2p}\mathbf{k}\cdot\mathbf{p}\sum_{n=1}^{\infty}\frac{(p_\uparrow-p_\downarrow)^n}{2^n n!}\delta^{(n)}(p-p_0)[1+(-1)^n]$$

$$+\sum_{n=1}^{\infty}\frac{(p_\uparrow-p_\downarrow)^n}{2^n n!}\delta^{(n-1)}(p-p_0)[1-(-1)^n] \tag{30}$$

Then the integration in the r.h.s. of Eqs. (1) over dp' will reduce these equations to a large set of angular integral equations in $\delta n_{\uparrow,\downarrow}^{(s)}(\mathbf{p}_0)$ with a series of derivatives of the interaction functions $F_{ik}^{(s)}(\mathbf{p}_0, \mathbf{p}_0')$ in the kernel. As a result, the spectrum of spin waves can be represented as a series in polarization with the coefficients that contain all the derivatives of the interaction functions up to the corresponding order.

We use an equivalent, but slightly different method in the spirit of Eq. (17):

$$\theta\left(\left|\mathbf{p}+\frac{\mathbf{k}}{2}\right|-p_\downarrow\right)-\theta\left(\left|\mathbf{p}-\frac{\mathbf{k}}{2}\right|-p_\uparrow\right)$$

$$=\theta(p-p_\downarrow)-\theta(p-p_\uparrow)+\frac{1}{2p}\mathbf{k}\cdot\mathbf{p}[\delta(p-p_\downarrow)+\delta(p-p_\uparrow)] \tag{31}$$

We split the interval $p_\uparrow - p_\downarrow$ into small segments δp, and, instead of exact integration in the r.h.s. of Eqs. (1), perform a Newton-Cotes expansion with coefficients a_s. This is equivalent to changing the momentum integration between $p_\downarrow$ and $p_\uparrow$ into a summation over a set of intermediate values of momenta p_s with the distances δp between them. Then the set (1) reduces to a set of angular equations in $\delta n_{\uparrow,\downarrow}(\mathbf{p}_s)$.

The eigenvectors of Eqs. (1) with the k^2-accuracy can be written as

$$\delta n_{\uparrow,\downarrow}(\mathbf{p}_s)=1+\mathbf{k}\cdot\mathbf{p}_s\,\delta n_{\uparrow,\downarrow}^{(1)}(p_s) \tag{32}$$

The functions $\delta n_{\uparrow,\downarrow}^{(1)}(\mathbf{p}_s)$ satisfy the equations

$$W_\Downarrow^{(1)}(K_0; p_s)\, \delta n_\Downarrow^{(1)}(p_s) - \frac{\delta p}{2} \sum_{l,\Uparrow} a_l \frac{p_l^3}{p_s} F_{\Downarrow\Uparrow}^{(1)}(K_0; p_s, p_l)\, \delta n_\Uparrow^{(1)}(p_l)$$

$$= -W_\Downarrow^{(k),(1)}(K_0; p_s) + \frac{\delta p}{2} \sum_{l,\Uparrow} a_l p_l^2 F_{\Downarrow\Uparrow}^{(k),(1)}(K_0; p_s, p_l)$$

$$+ \frac{1}{p_s} (F_{\Downarrow\Uparrow}^{(0)}(K_0; p_s, p_\downarrow) + F_{\Downarrow\Uparrow}^{(0)}(K_0; p_s, p_\uparrow)), \qquad (33)$$

where the upper indices indicate the harmonics of the functions W, F in angular Legendre polynomials, the double arrow indices $\{\Downarrow, \Uparrow\}$ take the values $\uparrow$ and $\downarrow$, and

$$W_{\Downarrow s} = W_\Downarrow(\mathbf{p}_s), \quad F_{\Downarrow\Uparrow sl} = F_{\Downarrow\Uparrow}(\mathbf{p}_s, \mathbf{p}_l'), \quad F_{\Downarrow\Uparrow s\downarrow} = F_{\Downarrow\Uparrow}(\mathbf{p}_s, \mathbf{p}_\downarrow'),$$

$$K_0 = (2\beta H, 0), \quad \mathbf{p} \cdot \mathbf{k}\, W_\Downarrow^{(k)}(\mathbf{p}) = \mathbf{k} \cdot \frac{\partial W_\Downarrow(K_0; \mathbf{p})}{\partial \mathbf{k}},$$

$$\mathbf{p} \cdot \mathbf{k}\, F_{\Downarrow\Uparrow}^{(k)}(\mathbf{p}, \mathbf{p}') = \mathbf{k} \cdot \frac{\partial F_{\Downarrow\Uparrow}(K_0; \mathbf{p}, \mathbf{p}')}{\partial \mathbf{k}}$$

The solutions of two equations (33) can be written in the matrix form as

$$\delta n_{\Downarrow s}^{(1)} = \sum_{l,\Uparrow} [\hat{W}_{\Downarrow s}^{(1)} \delta_{\Downarrow\Uparrow} - \delta p \hat{F}_{\Downarrow\Uparrow sl}^{(1)} a_l p_l^3 / 2 p_s]^{-1}$$

$$\times \left[-\hat{W}_{\Downarrow s}^{(k),(1)} + \frac{\delta p}{2} \sum_{\Downarrow,n} \hat{F}_{\Downarrow\Uparrow sl}^{(k),(1)} a_n p_n^2 + \frac{1}{2p_l} \sum_\Downarrow (\hat{F}_{\Downarrow\Uparrow\downarrow l}^{(0)} + \hat{F}_{\Downarrow\Uparrow\downarrow l}^{(0)}) \right] \qquad (34)$$

The substitution of Eqs. (32), (34) into Eqs. (1) leads, after some straightforward, though cumbersome algebra, to the spectrum of spin waves in the form $\omega = \Omega_0 + \alpha k^2$ with

$$\alpha = \frac{Z_{eff}\, \delta p}{N_\uparrow - N_\downarrow} \sum_{\Downarrow,\Uparrow,s} a_s p_s^2 \left\{ \frac{1}{12} (W_\Downarrow^{(p),(2)}(p_s) - W_\Downarrow^{(p),(0)}(p_s))\, \delta n_\Downarrow^{(1)}(p_s) \right.$$

$$+ F_{\Downarrow\Uparrow}^{(k),(1)}(K_0; p_s, p_\uparrow)\, p_s + F_{\Downarrow\Uparrow}^{(1)}(K_0; p_s, p_\uparrow)(\delta n_\Downarrow^{(1)}(p_\uparrow)$$

$$+ \delta n_\Uparrow^{(1)}(p_\uparrow))\, p_\uparrow + F_{\Downarrow\Uparrow}^{(k),(1)}(K_0; p_s, p_\downarrow)\, p_s$$

$$+ F_{\Downarrow\Uparrow}^{(1)}(K_0; p_s, p_\downarrow)(\delta n_\Downarrow^{(1)}(p_\downarrow) + \delta n_\Uparrow^{(1)}(p_\downarrow))\, p_\downarrow$$

$$+ \frac{\delta p}{8} \sum_l a_l p_l^2 \left[\frac{2}{3} F_{\Downarrow\Uparrow}^{(k_\alpha k_\beta),(0)}(K_0; p_s, p_l) - \frac{2}{3} F_{\Downarrow\Uparrow}^{(k_\alpha k_\beta),(2)}(K_0; p_s, p_l) \right. \qquad (35)$$

$$\left. \left. + F_{\Downarrow\Uparrow}^{(k_\alpha k_\beta),(1)}(K_0; p_s, p_l) + F_{\Downarrow\Uparrow}^{(k),(1)}(K_0; p_s, p_l)(\delta n_\Downarrow^{(1)}(p_l) + \delta n_\Uparrow^{(1)}(p_l)) \right] \right\};$$

$$Z_{eff}^{-1} = 1 - \frac{\delta p/2}{N_\uparrow - N_\downarrow} \sum_{\Downarrow,\Uparrow,s} a_s p_s^2 \left[\Sigma_{\Downarrow\Downarrow}^{(p_0),(0)} \left(P_{\Downarrow s} + \frac{K_0}{2} \right) \right.$$

$$\left. + \delta p \sum_l a_l p_l^2 F_{\Downarrow\Uparrow}^{(k_0),(0)}(K_0; p_s, p_l) \right]$$

The real part of the spectrum α' and the zero-temperature attenuation α'' are expressed via the zeroth, first and the second angular harmonics of the functions W and F on a set of isoenergetic surfaces with the radii between $p_\downarrow$ and $p_\uparrow$. The number of these intermediate isoenergetic surfaces is determined by a desired order in polarization expansion: each additional order in polarization means a decrease in δp and involves an extra iso-energetic surface. For example, in the zeroth (Silin-Leggett) order one needs only the values of all functions on the surface with the radius p_0, in the next order (as in Sec. 3)—on two surfaces with the radii $p_\downarrow$ and $p_\uparrow$, the next order involves the surfaces $p_\downarrow$, p_0, and $p_\uparrow$, and so on. Of course, this expansion is equivalent to the expansion in the derivatives of the functions W and F based on Eq. (30).

The energies and the scattering amplitudes are real only on the proper Fermi surfaces, and become complex away from these surfaces. The zero-temperature attenuation, i.e., Im α in Eq. (35), has two major sources:

(i) Non-zero imaginary parts in single-particle energies $\varepsilon(\mathbf{p})$ and *pseudo*-energies $\tilde{\varepsilon}(\mathbf{p})$ between the Fermi spheres $p_\downarrow$ and $p_\uparrow$ result not only in non-zero Im $W_{\downarrow,\uparrow}$ on all intermediate isoenergetic surfaces p_i, but also in imaginary parts of energy components of the 4-vectors $P_\Downarrow$ in generalized Landau functions $F_{\Downarrow\Uparrow}(K_0; \mathbf{p}, \mathbf{p}')$ (8) and renormalization functions $Z_\Downarrow(\mathbf{p})$ (7)

(ii) Derivatives of vertex functions in off-shell directions that reflect spatial non-localities similar to those in Ref. 8.

Generally, when polarizations and densities are not very low, the imaginary terms in α are of the same order as the real ones, and spin waves are strongly damped. This means that the main parameter of the transverse spin dynamics $\Omega_i \tau_\perp (T=0) \sim 1$. As a result, the transverse spin dynamics reduces to spin diffusion of transverse magnetic moment with a simultaneous rotation with a similar speed.

5. EXTENSION TO FINITE TEMPERATURES AND COMPARISON WITH EXPERIMENTAL DATA

In this Section we will discuss recent experiments.[9,10] Since the degree of spin polarization in both experiments was very low, we can apply the results of Sec. 3.

At low spin polarizations the zero-temperature attenuation is small, and the transverse relaxation time should be long, $\Omega_i \tau_{\perp 0} \gg 1$. The relaxation time $\tau_{\perp 0}$ for the zero-temperature attenuation is given by Eq. (29). The corresponding limiting value of the transverse spin diffusion coefficient is

$$D_{\perp 0} \equiv D_\perp(T=0) = \tfrac{1}{3} v_0^2 \tau_{\perp 0}(1 + F_0^{(a)}) \qquad (36)$$

This transverse relaxation time $\tau_{\perp 0}$ (29) can be used either for kinetic equation in τ-approximation, or for the macroscopic Leggett equations of spin dynamics at $T = 0$.

The results for the spectrum, $\tau_{\perp 0}$, and $D_{\perp 0}$ are based on *exact* equations of Ref. 1 at $T = 0$. Comparison with experimental data requires the information at finite temperatures. At finite temperatures our results should be modified in two ways. First, the logarithmic singularity in the particle-hole channel at small energy transfer, which is extremely important for the Fermi liquid theory and is responsible for the form of our Landau-like equations, obtains a finite temperature cutoff and smears out. However, at low temperatures, $T \ll T_0$, the peak is still sharp, and the form of the equations is to a large extent preserved. This change can be easily incorporated into the theory of Ref. 1. The second change concerns an appearance of the temperature attenuation that manifests itself in a shift of all the poles away from the real axis. This involves a collision integral in the transport equation. Technically this means that one has to start from the finite temperature diagrammatic equations like, for example, Keldysh or Kadanoff-Baym techniques rather than from the Dyson equation as in Ref. 1. This can be done consistently only for very dilute Fermi gases as in Refs. 12–14. Similar calculations at higher densities are still to be done.

In our derivation we use the fact that both the temperature-driven and polarization-driven attenuations are small at low temperature and polarization, and the sources of these attenuations can be included in the spin dynamics equation independently from each other. What is more, at low polarization the temperature-driven transverse relaxation time should be equal to the well-known[15,16] longitudinal relaxation time. Then the temperature-driven relaxation $\tau_{\perp T}$ can be incorporated into Eq. (22) by adding the following collision integral $I(n)$ to the r.h.s.:

$$I(n) = -\frac{i}{\tau_{\perp T}} (1 + F_1^{(a)}/3)(\delta n(\mathbf{p}) - \overline{\delta n(\mathbf{p})} - 3\overline{\delta n(\mathbf{p}) \cos \phi} \cos \phi)$$

$$= -\frac{i}{\tau_{\perp T}} (1 + F_1^{(a)}/3) \, \mathbf{k} \cdot \mathbf{v} n_1(p) \tag{37}$$

Of course, this term does not show up in the r.h.s. of Eq. (26), and appears only in Eq. (27). Finally, the spectrum obtains the form

$$\delta \omega = \frac{1}{3} \frac{k^2 v_0^2}{\Omega_i} (1 + F_0^{(a)}) \left[1 - \frac{i}{\Omega_i} \left(\frac{1}{\tau_{\perp 0}} + \frac{1}{\tau_{\perp \tau}} \right) \right] \tag{38}$$

where one should use as $\tau_{\perp T}$ a standard ("longitudinal") spin diffusion time in non-polarized Fermi liquid:[15,16]

$$\tau_{\perp T} = \frac{32\pi^2}{m^3 T^2} \left\langle W(\theta, \phi) \frac{(1 - \cos \theta)(1 - \cos \phi)}{\cos \theta/2} \right\rangle^{-1} C_D \tag{39}$$

Here $W(\theta, \phi)$ is the (quasi-)particles' scattering probability on the Fermi surface, C_D is the Brooker-Sykes coefficient for spin diffusion, and $\langle ... \rangle$ means the angular average. In dilute systems $W = 8\pi^3 a^2/m^2$. It is convenient to introduce an effective scattering length a^* and a dimensionless function $s(\theta, \phi)$ instead of the scattering probability $W(\theta, \phi)$,

$$W(\theta, \phi) = \frac{64\pi^3 a^{*2}}{3m^2} \frac{s(\theta, \phi)}{\langle s(\theta, \phi)(1 - \cos\theta)(1 - \cos\phi)/\cos(\theta/2)\rangle} \tag{40}$$

so that for dilute systems $a^* = a$, and $s(\theta, \phi) = 1$. Then the relaxation time Eq. (39) obtains the form similar to that of dilute systems[6]

$$\tau_{\perp T} = \frac{3C_D}{8\pi m a^{*2} T^2} \tag{41}$$

The effective transverse relaxation time is a combination of $\tau_{\perp 0}$ (29) and $\tau_{\perp T}$ (41)

$$\frac{1}{\tau_{\perp \text{eff}}} = \frac{1}{\tau_{\perp 0}} + \frac{1}{\tau_{\perp T}}, \quad \tau_{\perp \text{eff}}(T) = \frac{\tau_{\perp 0} T_a^2}{T_a^2 + T^2} \tag{42}$$

where T_a is the temperature at which the finite-temperature attenuation $\tau_{\perp T}$ gives place to the zero-temperature attenuation $\tau_{\perp 0}$. According to Eqs. (29), (41), (42),

$$T_a^2 = \frac{3C_D \Omega_0^2 (\gamma_0 - \gamma_1/3)}{8\pi a^{*2} p_0^2 (1 + F_0^{(a)})^2} \frac{(1 + 2F_0^{(a)} - F_1^{(a)}/3)}{(1 + F_1^{(a)}/3)^2}$$

Both the function $W(\theta, \phi)$ (i.e., the effective scattering length a^{*2}) and the harmonics $\gamma_{0,1}$ are determined by the exact vertex function, and should be related to each other [γ_0 determines the attenuation of particles near the Fermi surface associated with the scattering of particles on the Fermi surface with the probability $W(\theta, \phi)$; the link between γ_1 and $W(\theta, \phi)/a^{*2}$ is less obvious]. At present the exact relation is unknown, except for dilute systems, and we are forced to consider them as independent parameters.

In case of dilute Fermi gases, $\gamma_0 - \gamma_1/3 = 4a^2 p_0^2/3\pi$, $1 \gg F_0^{(a)} \gg F_1^{(a)}$, $a^* = a$, and T_a reduces to

$$T_a = \Omega_0 C_D^{1/2}/\sqrt{2}\,\pi \tag{43}$$

with $C_D = 0.8$ (see [6]). The expression (43) is very similar to the result of Refs. 12–14, $T_a = \Omega_0/2\pi$; the difference is explained by the fact that the results of Refs. 12–14 are based on a variational solution of the transport equation, while Eq. (43) is based on the exact transport results[1,2,6] in the

low-density low-polarization limit. Following the results at low densities, we can parametrize the coefficients $\gamma_{0,1}$ by introducing another effective length a_γ, so that in the low density limit $a^* = a_\gamma = a$:

$$\gamma_0 - \frac{\gamma_1}{3} = \frac{4p_0^2 a_\gamma^2}{3\pi}$$

Then

$$T_a = \frac{C_D^{1/2}\Omega_0 a_\gamma}{\sqrt{2}\,\pi a^*(1 + F_0^{(a)})} \frac{(1 + 2F_0^{(a)} - F_1^{(a)}/3)^{1/2}}{(1 + F_1^{(a)}/3)} \tag{44}$$

The existence of the zero-temperature relaxation (29) and the transition from the temperature-driven attenuation (37), (41) to the field-driven attenuation (29) at $T = T_a$ (44) have been recently confirmed in experiments[9] on spin-polarized ^{3}He in magnetic field $8T$. The corresponding degree of spin polarization is extremely low, and our equations can be applied without any additional assumptions.

According to experimental data, the transition temperature T_a is 16.4 ± 2.2 mK. This is much larger than an extrapolation[9] $T_a^{(0)} = 6.5$ mK from the variational dilute-gas results[12-14] with an heuristic substitute $\Omega_0 \to \Omega_0/(1 + F_0^{(a)})$ which was supposed to take care of the anomalously high magnetic susceptibility of ^{3}He where $1/(1 + F_0^{(a)}) \approx 3.4$, $T_a^{(0)} = \Omega_0/2\pi(1 + F_0^{(a)})$. The missing factor should be recovered from Eq. (44) which yields a very reasonable numerical value for the combination of previously unknown parameters:

$$\frac{C_D^{1/2}a_\gamma}{a^*} \frac{(1 + 2F_0^{(a)} - F_1^{(a)}/3)^{1/2}}{1 + F_1^{(a)}/3} = 1.72 \pm 0.23$$

The second experiment[10] concerns the measurements in spin-polarized ^{3}He↑ − ^{4}He mixtures with ^{3}He concentrations 0.5% ($T_0 \sim 76$ mK), 1% ($T_0 \sim 120$ mK), and 3.8% ($T_0 \sim 290$ mK) in an external magnetic field up to $8.8T$ ($\Omega_0 = 2\beta H \sim 13.7$ mK) at temperatures $T \geq 13$ mK. The experiment[10] displayed a relatively strong field dependence in spin dynamics at ^{3}He concentrations 3.8%. Here the explanation is somewhat different from the pure ^{3}He.

Our results cannot be applied directly at ^{3}He concentration 3.8% because of the following anomaly. According to an experimental observation[17], the parameter $F_0^{(a)} - F_1^{(a)}/3$ becomes zero at some critical concentration x_c between 3% and 5%. At this critical concentration, the spectrum of Silin spin waves (16) has a singularity and should be reexamined. This has been done in Ref. 18 in the frame of the Silin-Leggett approximation.

In this approximation, the spectrum $\delta\omega$ near the critical concentration becomes linear in the wave vector k even in a long-wave range instead of standard quadratic dependence (16). As we will see below, the existence of the zero-temperature attenuation modifies the results[18] and helps to preserve the quadratic form of the spectrum in a wide range of wavelengths.

Near the critical concentration x_c one cannot neglect the term with $\delta\omega$ in the l.h.s. of Eq. (27) in which we also include the term with the temperature-driven attenuation (38), (39) (cf. Ref. 18):

$$-\frac{1}{3}k^2 v_0^2 \left(1 + F_0^{(a)} - \frac{i\Omega_0(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})}\right) n_0 + \frac{1}{3}k^2 v_0^2$$

$$\times \left\{ \delta\omega + \frac{\Omega_0}{1 + F_0^{(a)}}\left[F_0^{(a)} - \frac{1}{3}F_1^{(a)} \right.\right.$$

$$\left.\left. + \frac{i\Omega_0(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})}\right] + \frac{i(1 + F_1^{(a)}/3)}{\tau_{\perp T}} \right\} n_1 = 0 \qquad (45)$$

Then the expression for the spectrum obtains the form which is somewhat different from Eq. (28):

$$\delta\omega + \frac{i}{3}\frac{k^2 v_0^2}{2T_0}\left(\gamma_0 - \frac{\gamma_1}{3}\right)$$

$$= \frac{1}{3}k^2 v_0^2 \frac{1 + F_0^{(a)}}{\Omega_0}\left[1 + F_0^{(a)} - \frac{i\Omega_0(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})}\right]$$

$$\times \frac{1 + F_1^{(a)}/3 - i\Omega_0(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)})}{\left(\begin{array}{c} F_0^{(a)} - F_1^{(a)}/3 + i\Omega_0(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)}) \\ + (\delta\omega + i(1 + F_1^{(a)}/3)/\tau_{\perp T})(1 + F_0^{(a)})/\Omega_0 \end{array}\right)} \qquad (46)$$

Everywhere, except the denominator, we can assume that we are exactly at the point x_c and neglect the difference between $F_0^{(a)}$ and $F_1^{(a)}/3$ as well as the small terms with attenuation γ. Then, instead of Eq. (29), the spectrum (46) reduces to

$$\delta\omega = \frac{k^2 v_0^2(1 + F_0^{(a)})^2/3}{\left(\begin{array}{c} \Omega_0(F_0^{(a)} - F_1^{(a)}/3)/(1 + F_0^{(a)}) + i\Omega_0^2(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)})^2 \\ + i(1 + F_1^{(a)}/3)/\tau_{\perp T} + \delta\omega \end{array}\right)} \qquad (47)$$

Near the critical point $|x - x_c| \ll 1$ the small difference $F_0^{(a)} - F_1^{(a)}/3$ in the denominator of Eq. (47) should be parametrized as

$$F_0^{(a)} - F_1^{(a)}/3 = (1 + F_0^{(a)})f\left[(x_{c0} - x) + k_H\frac{\Omega_0}{T_0} + k_T\frac{T^2}{T_0^2}\right], \qquad (48)$$

where x_{c0} is the critical concentration at $T = H = 0$, and the terms with $k_{H,T}$ describe field and temperature dependencies of the *real* parts of characteristic parameters [here these corrections are important because the principal term is anomalously small]. The critical concentration depends on temperature and external field as

$$x_c(T, H) = x_{c0} + k_H \Omega_0/T_0 + k_T T^2/T_0^2 \tag{49}$$

Finally, the spectrum (47) obtains the form

$$\delta\omega = \frac{k^2 v_0^2 (1 + F_0^{(a)})^2/3}{\Omega_0 f(x_c - x) + i\Omega_0^2(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)})^2 + i(1 + F_0^{(a)})/\tau_{\perp T} + \delta\omega} \tag{50}$$

In the long-wave limit,

$$kv_0 \ll \frac{1}{1 + F_0^{(a)}} \left| \Omega_0 f(x_c(T, H) - x) + \frac{i\Omega_0^2(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})^2} + \frac{i(1 + F_0^{(a)})}{\tau_{\perp T}} \right| \tag{51}$$

the spectrum is always quadratic,

$$\delta\omega = \frac{1}{3} \frac{k^2 v_0^2 (1 + F_0^{(a)})^2}{\Omega_0 f_1(x_c - x) + i\Omega_0^2(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)})^2 + i(1 + F_0^{(a)})/\tau_{\perp T}} \tag{52}$$

The spectrum becomes linear, as in Ref. 18,

$$\delta\omega = kv_0(1 + F_0^{(a)})/\sqrt{3}$$

only for short waves when Eq. (51) is invalid. In contrast to the prediction of Ref. 18, the presence of the zero-temperature attenuation makes the range (51), (52) finite even at critical concentration and $T = 0$ when Eq. (51) reduces to

$$kv_0 \ll \frac{\Omega_0^2(\gamma_0 - \gamma_1/3)}{2T_0(1 + F_0^{(a)})^3} \tag{53}$$

As one can see from Eq. (52), the spectrum near the critical point (49) is very sensitive to the magnetic field. At the critical point itself the spectrum is purely diffusive,

$$\delta\omega = -iD_{\text{eff}}^{(c)} k^2, \quad D_{\text{eff}}^{(c)} = \frac{1}{3} \frac{v_0^2(1 + F_0^{(a)})\tau_{\perp T}}{1 + \tau_{\perp T}\Omega_0^2(\gamma_0 - \gamma_1/3)/2T_0(1 + F_0^{(a)})^3} \tag{54}$$

Close to this point the effective diffusion coefficient D_{eff} (54) obtains a small spin-rotation part,

$$D_{\text{eff}} = D_{\text{eff}}^{(c)} \Big/ \left[1 - i \frac{(1 + F_0^{(a)})^{-1} \tau_{\perp T} \Omega_0 f(x_c(T, H) - x)}{1 + \tau_{\perp T} \Omega_0^2 (\gamma_0 - \gamma_1/3)/2 T_0 (1 + F_0^{(a)})^3} \right] \qquad (55)$$

Far away from this point the spectrum returns to the form (39).

In experimental conditions[10] the spin-rotation part was virtually unobservable. This means that the spin diffusion coefficient is described by Eq. (54). According to the experimental data, the temperature dependence of the diffusion coefficient is

$$DT^2 = \frac{AT^2}{T^2 + T_a^2} \qquad (56)$$

(with $T_a \approx 14 \, \text{mK}$), which is identical to Eq. (42). We will again write $\tau_{\perp T}$ in the form (41) and use the same parametrization for the coefficients $\gamma_0 - \gamma_1/3$ as above. Then the temperature T_a (55), (56) obtains the form

$$T_a = \frac{\Omega_0}{\pi} \left(\frac{C_D}{2} \right)^{1/2} \frac{a_\gamma}{a^*} \frac{1}{(1 + F_0^{(a)})^{3/2}}$$

Since $\Omega_0 \approx 14 \, \text{mK}$, the experimental result[10] $T_a \approx 14 \, \text{mK}$ means that

$$\frac{a_\gamma}{a^*} \frac{C_D^{1/2}}{(1 + F_0^{(a)})^{3/2}} \approx 4.5$$

This value seems to be quite feasible though relatively large.

The fact that the parameter $a_\gamma/a^*(1 + F_0^{(a)})^{3/2}$ is relatively large at $x = 3.8\%$ may be very important especially if one remembers that at this concentration $F_0^{(a)} - F_1^{(a)}/3 \approx 0$. Both facts signal a strong deviation from a simple extrapolation of a low concentration behavior for which $F_0^{(a)}$ is positive and increases with concentration proportionally to $x^{1/3}$ while $F_1^{(a)}$ is much smaller and is proportional to $x^{2/3}$.[6] Therefore, the most probable conclusion is that at concentrations $x > 2 \div 3\%$ the Fermi liquid harmonic $F_0^{(a)}$ in helium mixtures is effectively suppressed with respect to its low concentration value. This, in turn, means that the effective (attractive) interaction at relatively high concentrations is much weaker than its extrapolation from low concentrations when the s-wave scattering length $a \approx -1A$.[6,19] If this is true, all current attempts to observe ^{3}He superfluidity in ^{3}He-^{4}He mixtures become hopeless.

SUMMARY

In this paper we applied our exact microscopic theory[1] of transverse phenomena in spin-polarized Fermi liquids to Fermi systems with low spin polarizations such as liquid normal ^{3}He↑ or not very dilute ^{3}He↑-^{4}He liquid mixtures in any currently accessible magnetic fields. For the first time, the results take into account the inherent spatial and temporal non-locality of the fermion interaction, and attenuation of single-particle states away from the corresponding Fermi surfaces. We calculated explicitly the parameters of transverse spin dynamics as an expansion in polarization. The results are expressed via the angular harmonics of mass operators and interaction functions on a set of isoenergetic surfaces with the radii between $p_↓$ and $p_↑$.

The results display the sources of peculiar collisionless attenuation in *transverse* spin dynamics of polarized Fermi gases at zero temperature. This zero-temperature attenuation is important when the degree of spin polarization is of the order of or larger than the ratio T/T_0.

Our general microscopic results can be simplified considerably at low spin polarizations. As expected, the results in the lowest order in polarization coincide with the results of the standard Silin-Leggett approach which is based on the conventional Landau theory.[3–5] Within this approximation, a set of exact equations in partial transverse spin *pseudo*-densities collapse into a single standard equation in transverse spin density. Of course, within this accuracy, the zero-temperature attenuation vanishes.

The zero-temperature attenuation shows up in the next order in polarization. A simple expression (29) gives this attenuation through, apart from standard Fermi liquid harmonics, angular harmonics of the imaginary parts of derivatives of the mass operator and interaction function in off-shell directions at the Fermi surface. These harmonics cannot be expressed via any Landau parameters of the conventional Fermi liquid theory. Therefore, in the frame of phenomenologic theory of transverse spin dynamics, these quantities should be viewed as independent parameters. Our equations express these new parameters through the microscopic characteristics of the Fermi liquid. More compact relations between these parameters and the parameters of the standard temperature-driven attenuation remain unknown except for dilute systems.

Our results give a good quantitative description of recent experimental results on transverse spin dynamics in liquid ^{3}He↑ and ^{3}He↑-^{4}He mixtures. The equations reproduce correctly the observed temperature dependence of the transverse relaxation time and spin diffusion coefficient. Our description of the transition from the temperature-driven to the zero-temperature attenuation is in good numerical agreement with experimental data. The

data on spin diffusion in ^{3}He-^{4}He mixtures with 3.8% ^{3}He suggest that the superfluid transition for the ^{3}He component is strongly suppressed in comparison with estimates that are based on extrapolation from the low concentration limit.

In this paper we made the first attempt to incorporate the finite-temperature effects into our exact theory at $T=0$. This was done assuming that the *inverse* transverse relaxation time $\tau_\perp^{-1}$ is an analytic function of both the temperature and the external magnetic field and combining the first terms in the corresponding expansions. A more consistent study of the finite-temperature effects is yet to be done. On the other hand, the fit of experimental data[9,10] to Eq. (56) is not perfect. At present, it is not completely clear whether the reason is the quality of the experimental data, or the accuracy of the theory (in particular, the finite-temperature effects *or* the applicability of the macroscopic Leggett equation for the calculation of the spin diffusion coefficient from the spin-echo experiments).

The question of applicability of conventional macroscopic equations of transverse spin dynamics (the Leggett equations[4,6]) at high polarizations is not settled. It seems that the Leggett equations are applicable at low spin polarizations which correspond to the results of Sections 2 and 3. In this case the only difference from the conventional theory should be the presence of zero-temperature attenuation (36) and transverse spin diffusion (37) which renormalize the parameters in the Leggett equations and, to some extent, change their meaning.

At higher polarizations, the split of microscopic equations of spin dynamics into two equations in two partial transverse *pseudo*-densities[1] can change the situation. It is not clear whether the corresponding macroscopic equations will remain as equations in a single magnetic moment and a single spin current, or will also split into a set of two coupled equations in partial moments and currents. In the former case, the equation of motion will definitely keep the Leggett form.

The answer to this question is unknown. This problem is very important, since the macroscopic equations of spin dynamics are the sole basis of theoretical interpretation of experimental NMR data.

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