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BOUNDARY EFFECTS IN TRANSVERSE SPIN DYNAMICS OF SPIN-POLARIZED QUANTUM GASES

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Spin dynamics of spin-polarized quantum gases near non-magnetic walls is discussed. We consider boundary-induced line shifts and attenuation of spin-waves, and a possible "macroscopic" boundary condition for systems close to a Knudsen ballistic regime. By a coordinate transformation, we reduce the problem of collisions with a rough wall to an equivalent problem with a specular wall but with stochastic bulk perturbations. The boundary effects are described by a bulk-like transverse spin-diffusion coefficient inversely proportional to the correlation function of surface inhomogeneities. This leads to an effective macroscopic boundary condition applied for the spin-wave resonances. The situation is changed at low temperatures by an appearance of absorbed boundary layers which lead to additional exchange processes and renormalize the molecular field near the walls. The experimental implications for helium and hydrogen systems are discussed, including the signs of surface spin modes.

Spin dynamics of spin-polarized quantum gases is studied[1] at low temperatures and densities of gases when the mean free paths are rather long[2]. Even a ballistic regime seems to be accessible making the boundary scattering as or even more important than the bulk collisions.

We report the effects of scattering of particles by non-magnetic walls on spin waves in polarized quantum gases. We are interested in changes in spin-wave resonances caused by a scattering by rough walls and by exchanges with adsorbed

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particles. We are also looking for a way to describe the boundary scattering in terms of some effective "hydrodynamic" boundary condition for macroscopic equations of spin dynamics.

The collisions with rough walls are often described by a microscopic boundary condition[3] for a transport equation. Usually, it is difficult to solve the corresponding integro-differential equations and to extract information about the system from such an approach. Therefore, we have adopted another general method: we reduce our problem of elastic collisions with inhomogeneous non-magnetic walls to an equivalent problem of a specular reflection from smooth homogeneous walls but with some stochastic bulk imperfections.

We consider a layer of gas restricted by one rough wall $x = L - \xi(\mathbf{s})$ ($\mathbf{s}$ are the 2D coordinates y, z in the plane of the wall), and a perfect specular wall $x = 0$. The function $\xi(\mathbf{s})$ is random, $\langle\xi(\mathbf{s})\rangle = 0$, with the binary correlations $\xi^{(2)}(|\mathbf{s}_1-\mathbf{s}_2|) = \langle\xi(\mathbf{s}_1)\xi(\mathbf{s}_2)\rangle$. All the observables should be averaged over ξ.

The shift of inhomogeneity from the boundary to the bulk is achieved by a coordinate transformation:

$$X' = x\frac{L}{L-\xi(\mathbf{s})}; \quad Y' = y; \quad Z' = z \tag{1}$$

This transformation makes both walls, $X' = 0$ and $X' = L$, smooth, and we can apply the simplest boundary condition $\Psi(0) = \Psi(L) = 0$. But the transformation (1) changes the initial Hamiltonian $\hat{H}_0$ adding to it a stochastic bulk perturbation $\hat{V}$:

$$\hat{H}=\hat{H}_0+\hat{V}, \quad \hat{H}_0=\frac{p^2}{2m}, \quad \hat{V}=\frac{1}{2m}\left(\frac{2\xi}{L}p_x^{\,2}+xp_x\frac{\xi_y}{L}\, p_y+xp_x\frac{\xi_z}{L}p_z + h.c.\right) \tag{2}$$

where we keep only the first orders of ξ and its derivatives assuming that the roughness is small.

The perturbation $\hat{V}$ should be included into the collision integral L_{coll} for the Boltzmann equation:

$$\begin{aligned} L_{coll} &= \frac{p_\perp^{\,4}}{16\pi L^2 m}\int d\varphi\,\eta(p_\perp,\varphi-\phi)\,[m(p_x,p_\perp,\phi)-m(p_x,p_\perp,\varphi)] \\ &+ \frac{p_x^{\,4}}{4\pi L^2 m}\int d\varphi\,\xi_2(p_\perp,\varphi-\phi)\,[m(p_x,p_\perp,\phi)-m(p_x,p_\perp,\varphi)], \\ &\eta(p_\perp,\varphi-\phi) \equiv \xi^{(2)}(p_\perp,\varphi-\phi)\,[1-\cos(\varphi-\phi)]^2 \end{aligned} \tag{3}$$

Here p_x and $\mathbf{p}_\perp$ are the components of momenta perpendicular and parallel to the wall, φ and ϕ are the angles between $\mathbf{p}_\perp$ and some axis for scattered and incident particles, $m(p_x, p_\perp, \varphi) = \mathrm{Tr}_\sigma\, \sigma^+ \hat{n}_\sigma(\mathbf{p})$ is the momentum distribution of the transverse magnetic moment, $\hat{n}_\sigma(\mathbf{p})$ is the single-particle density matrix,

$\xi^{(2)}(p_\perp, \varphi-\phi)$ is the Fourier transform of $\xi^{(2)}(|\mathbf{s}_1-\mathbf{s}_2|)$. The collision operator (3) corresponds to a conservation of p_x and to a random scattering in the *yz*-plane.

The collisions (3) give rise to a spin diffusion parallel to the wall and affect the frequencies and attenuation of spin waves. The corresponding tensor of spin diffusion coefficients for transverse magnetic moment is

$$D_{xx} = 0, \quad D_{xy} = D_{yx} = D_{xz} = D_{zx} = 0$$
$$D_{zz} = D_{yy} = \frac{8\pi L^2}{\alpha m N}\int dp_x d^2p_\perp \frac{p_\perp^{\,2} M^{(0)}(p_x, p_\perp)}{p_\perp^{\,4}(\eta_0-\eta_1) + 4p_x^{\,4}(\xi_0-\xi_1)} \tag{4}$$

where η_k and ξ_k are the harmonics of η and $\xi^{(2)}$, $M^{(0)}(\mathbf{p}) = n_\uparrow(\mathbf{p}) - n_\downarrow(\mathbf{p})$ is the equilibrium distribution of magnetization, $\alpha = (N_\uparrow-N_\downarrow)/N$ is the degree of spin polarization, and $N_{\uparrow,\downarrow}$ are the numbers of spin-up and spin-down particles per unit volume. Using the Leggett equations for spin polarized Fermi gases[1] and Eq.(4) for the transverse spin diffusion coefficient, one can determine the surface-induced attenuation of spin waves:

$$\omega(\boldsymbol{k}) = \Omega_0 + \frac{\gamma k^2}{3\Omega_{int}} - \frac{i\gamma^2 q^2}{9\Omega^2_{int} D},$$
$$\gamma(\alpha, T) = \frac{\langle v^2\rangle_\uparrow N_\uparrow - \langle v^2\rangle_\downarrow N_\downarrow}{N_\uparrow - N_\downarrow}, \quad \langle v^t\rangle_{\uparrow,\downarrow} = \frac{1}{N_{\uparrow,\downarrow}}\int v^t n_{\uparrow,\downarrow} d\Gamma, \tag{5}$$
$$\Omega_{int} = -\frac{4\pi a\hbar}{m}(N_\uparrow - N_\downarrow), \quad \boldsymbol{k} = (k_x, \boldsymbol{q}),$$

where $\Omega_0 = 2\beta H$ is the Larmor frequency, $\langle v^t\rangle_{\uparrow,\downarrow}(T)$ are the velocities of up- and down spins averaged over the equilibrium distributions, the internal frequency Ω_{int} characterizes the molecular field, and *a* is the bulk *s*-wave scattering length. Eq.(5) can be used for quantum gases at all temperatures from the Boltzmann region down to the degenerate one.

The transverse spin diffusion coefficient (4) leads to a macroscopic-like boundary condition for spin dynamics[4]:

$$\boldsymbol{M}(x) + \Lambda\frac{\partial}{\partial x}\boldsymbol{M}(x) = 0,$$
$$\Lambda = -i\frac{\gamma q^2 L}{12k_x^{\,2}(\Omega_{int}+i/\tau_\perp)D}, \quad k_x\Lambda \ll 1, \tag{6}$$
$$\Lambda = -i\frac{12(\Omega_{int}+i/\tau_\perp)D}{\gamma q^2 L}, \quad k_x\Lambda \gg 1,$$

where $\tau_\perp$ is the bulk transverse relaxation time. The first of Eqs.(6) describes a strong diffusion relaxation of magnetic moment near the boundary, while the second one reflects the case of a nearly zero magnetic current through the wall.

The major changes occur with an onset of an adsorption of particles. Then one encounters two new types of exchange for adsorbed particles between themselves and with identical bulk particles. The former processes are suppressed when the surface mobility of adsorbed particles is low, while the latter depend on the binding energy of adsorbed particles and the range of interaction between bulk and surface particles. For a short-range interaction $U(r)$, the surface-bulk exchange is proportional to $U(R_0/\lambda)^2 \ll U$ ($R_0 \sim a$ is the size of the bound state, $\lambda \gg a$ is the particles wavelength). In the opposite case of a long-range interaction, the exchange integral is of the order of $U(E/U_0)^2 \ll U$ (U_0 is the depth of the bound state, $E \ll U_0$ is the particles' energy).

This exchange has a strong effect on the constant Λ (6). Since the density of adsorbed layers increases exponentially with decreasing temperature, $\rho = NR_0\exp(U_0/T)$, the formation of layers leads to an exponential decrease in Λ. If the exchanges within the boundary are insignificant, then Λ is real, and the main effect is a shift of spin-wave resonances while the linewidths are still determined by bulk collisions.

In the opposite case of strong interboundary exchanges, Λ becomes imaginary leading to an exponential increase in linewidths. This additional attenuation results not from some new dissipation mechanism, but from an effective dephasing between bulk and surface precessions with different exchange frequencies. Most probably, this situation corresponds to experimental data[2] confirming the existence of a strong surface molecular field. However, the available information is insufficient to make a definite conclusion on the existence of surface spin modes in experimental conditions[2].

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