

## TRANSVERSE RELAXATION IN TWO-LEVEL FERMI LIQUIDS AND GASES

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An exact microscopic theory is developed for transverse dynamics in two-level Fermi systems like spin-polarized Fermi liquids. The origin of a transverse zero-temperature attenuation is a dephasing of interlevel transitions in inhomogeneous conditions. We analyze different sources of this relaxation using an extrapolation from low polarizations or densities. These sources include imaginary terms in the vertex and single-particles' energies and pseudo-energies away from the Fermi spheres, and derivatives of the vertex function in off-shell directions. The attenuation for dilute systems starts from a second order term in density. The main non-local contribution is of the next order in density, but is not negligible if the interaction range is large. At very low polarizations and densities, one recovers the already known results. The implications for liquid  $^3\text{He}\uparrow$  and  $^3\text{He}\uparrow$ - $^4\text{He}$  mixtures are discussed.

We study the transverse dynamics related to interlevel transitions of fermions with discrete energy levels associated with internal degrees of freedom. The phenomena related to interlevel transitions are common to many multilevel systems, such as the dynamics of transverse magnetization in spin-polarized systems. Below we dwell on two-level systems like spin-polarized Fermi liquids of spin-1/2 particles.

Often, the interlevel transitions are coherent at  $T \rightarrow 0$ , and give rise to a collective mode (spin wave). However, this mode may have an attenuation even at  $T = 0$ . We will illustrate

this using polarization and density expansions. The results are applicable to any two-level systems in which the bare gap between energy levels does not depend on particles' momenta.

We have shown recently<sup>1</sup> that the transverse dynamics of polarized Fermi liquids is governed by a set of two coupled equations in two partial transverse densities  $\delta n_{\downarrow} \equiv \delta n_{\uparrow\downarrow}^{(\downarrow)}(\mathbf{p})$  originating from slightly tilted spin-ups and spin-downs:

$$\left[ \omega \mp \epsilon_{\downarrow}(\mathbf{p} \pm \frac{\mathbf{k}}{2}) \pm \epsilon'_{\uparrow\downarrow}(\mathbf{p} \mp \frac{\mathbf{k}}{2}) \right] Z_{\downarrow}(\mathbf{p} \pm \frac{\mathbf{k}}{2}) \delta n_{\downarrow}(\mathbf{p}) = \frac{1}{2} \int \frac{d^3 \mathbf{p}'}{(2\pi)^3} (\theta_{\downarrow}(\mathbf{p}') - \theta_{\uparrow}(\mathbf{p}')) \sum_{\uparrow, \downarrow} F_{\uparrow\downarrow}(K; \mathbf{p}, \mathbf{p}') \delta n_{\downarrow}(\mathbf{p}'), \quad (1)$$

where the double-arrow indices  $\downarrow, \uparrow$  take two values  $\uparrow$  and  $\downarrow$ , the upper sign in  $\pm$  corresponds to  $\downarrow$ , the 4 operators  $F_{\uparrow\downarrow}$  play the role of generalized non-local transverse Landau functions, the pseudo-energies  $\epsilon'_{\uparrow\downarrow} \neq \epsilon_{\uparrow\downarrow}$ , and  $K = (\omega, \mathbf{k})$ . The definition of the functions  $F_{\uparrow\downarrow}$ ,  $\epsilon'_{\uparrow\downarrow}(\mathbf{p})$ ,  $\epsilon_{\uparrow\downarrow}(\mathbf{p})$  and  $Z_{\uparrow\downarrow}(\mathbf{p})$  via the exact Green's functions, irreducible vertex, and the mass operators coincide with Ref.1. The exact expression<sup>2</sup> for the eigenspectrum  $\omega = 2\beta H + \alpha k^2$  is too cumbersome to be given here.

The simplest explanation of the zero-temperature attenuation  $\text{Im}\alpha \equiv \alpha'' \neq 0$  is that the oscillations of  $\delta n_{\downarrow, \uparrow}$  feel different molecular fields. This causes a dephasing of oscillations and an inhomogeneous broadening. The immediate sources of the zero-temperature attenuation in Eqs.(1) are: i) imaginary parts in single-particle energies and pseudo-energies away from the Fermi spheres; ii) imaginary parts of the interaction operators  $F_{\uparrow\downarrow}$  and renormalization factors  $Z_{\uparrow\downarrow}$ ; iii) derivatives of the vertex in off-shell directions.

Generally, the imaginary terms in  $\alpha$  are of the same order as the real ones, and the (spin) mode is strongly damped. We can follow how  $\alpha''$  disappears at low densities and polarizations where it is possible to get analytical results.

At low densities  $N$ , one can expand all the functions in  $N^{1/3}$ . This expansion coincides with the perturbation expansion in the scattering length  $a$ . The dimensionless expansion parameter<sup>3</sup> is  $x = N^{1/3}a$ . In the zeroth and first orders in  $x$  the energies  $\epsilon_{\uparrow\downarrow} = \epsilon'_{\uparrow\downarrow}$  are real. Imaginary terms in energies and  $F_{\uparrow\downarrow}(K_0; \mathbf{p}, \mathbf{p}')$  appear only in the second order in  $x$ . Up to the second order in  $x$ , the functions on both sides of Eqs.(1) do not depend on the arrow indices, and Eqs.(1) collapse into a single semi-standard equation which is similar to<sup>3</sup> but already includes imaginary terms responsible for the zero-temperature relaxation. The doubling of Landau-like Eqs.(1) and other non-trivial effects show up starting from the third

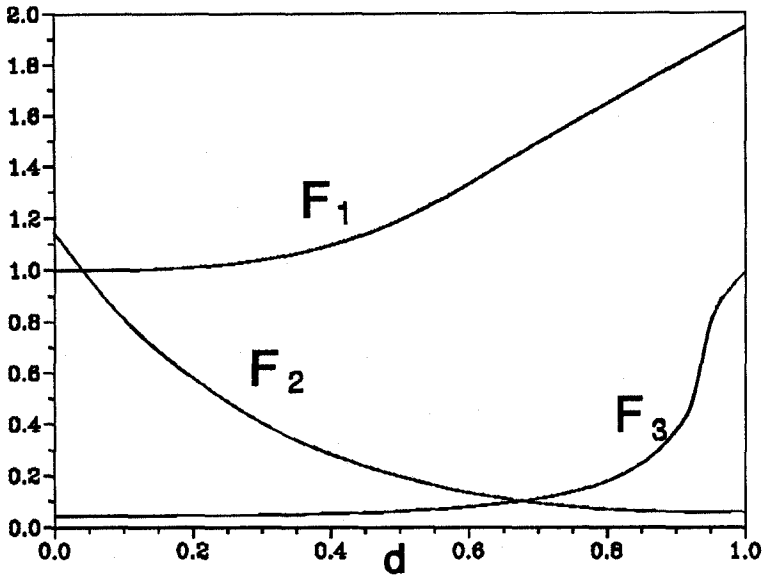


Fig. 1. Functions  $F_1(d)$ ,  $F_2(d)$ , and  $F_3(d)$ .

order in density. Such effects are unaccounted for by previous theories based on some form of a single Landau or Boltzmann kinetic equation<sup>3-5</sup>. The calculation in the main orders yields

$$\alpha' = \frac{5}{6\pi a} \frac{E\uparrow}{n\uparrow} \frac{(1-d^5)}{(1-d^3)^2} - \frac{1}{3m} \frac{1+d}{1-d} + \frac{9}{\pi^3 m} F_2(d), \tag{2}$$

$$\alpha'' = \frac{4}{105m} F_1(d), \quad d = \frac{P\downarrow}{P\uparrow}.$$

(cf.<sup>3,4</sup>). The non-locality of the interaction with the range  $r_0$  also causes the attenuation at  $T = 0$  but in a higher order in density than (2). Here the main terms come from the dependence of the irreducible vertex on the wave vector  $\mathbf{k}$ :

$$\alpha''_{n1} = \frac{8P\uparrow^2 r_0^2}{35m} F_3(d) \tag{3}$$

The attenuation (3) is important if  $r_0 > a$ . This may be the case for  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  mixtures where the  ${}^3\text{He}\text{-}{}^3\text{He}$  interaction contains a phonon-mediated long-range part. A different non-local term originates from off-shell derivatives of the interaction operator  $F_{\uparrow\uparrow}(K; \mathbf{p}, \mathbf{p}')$ , but is of a higher order in  $x$  than (3). Such non-local term is the leading one at  $T \gg T_F$ .

In dense systems, the new effects vanish in the lowest approximation in polarization when Eqs.(1) collapses into the single Leggett-Silin equation. The doubling of the equations and the zero-temperature relaxation show up in the next order in  $H$ . The corresponding term in  $\alpha$  is field-independent with the imaginary part of the same order as the real one. With this accuracy, the equations reduce to two subsets of three linear equations involving the  $\hat{F}$ -functions with momenta  $p_{\uparrow}$ ,  $p_{\downarrow}$ , and  $p_0$ . This result for  $\alpha$  has a simple structure, but is too cumbersome to be given here. In general, in the  $n$ -th order in polarization, Eqs.(1) reduce to two subsets of  $2n+1$  linear equations involving the values of energies and  $F_{\uparrow\uparrow}$  on a set of  $2n+1$  isoenergy surfaces between  $p_{\uparrow}$  and  $p_{\downarrow}$ . This expansion provides a reasonable algorithm for numerical analysis.

In summary, we analyzed microscopic equations for the transverse dynamics in two-level Fermi liquids at  $T = 0$ . The explicit solution of the equations involves the values of the generalized Fermi liquid harmonics on a set of isoenergy surfaces between Fermi spheres for particles on upper and lower levels. The spectrum of the (spin) waves has a finite attenuation, which vanishes only for low densities or polarizations. The polarization and density expansions reveal the sources of zero-temperature attenuation. In cases of very low polarization or densities the theory reduces to the well-known expressions. The best experimental check is provided by spin dynamics in highly polarized  $^3\text{He}\uparrow$  or  $^3\text{He}\uparrow$ - $^4\text{He}$  mixtures.

#### ACKNOWLEDGEMENTS

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