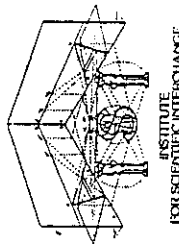


SPIN POLARIZED QUANTUM SYSTEMS

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KINETICS OF SPIN POLARIZED QUANTUM GASES

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ABSTRACT

The kinetic phenomena in gases of particles with internal degrees of freedom (such as spin-polarized gases) is discussed with the emphasis on new results not included in reviews /1/. The only one longitudinal transport process not discussed before in detail concerns the (spin) pressure diffusion. The expressions for pressure diffusion coefficient are given for different types of dilute gases, and the importance of viscous renormalization is demonstrated.

The transverse kinetic phenomena are discussed on the basis of the generalized Waldmann-Snyder kinetic equation including the effects of quantum identity of (quasi)particles and taking into account non-local effects in the interaction. The necessary conditions are formulated for existence of coherent collective effects in Boltzmann gases of particles with arbitrary internal degrees of freedom, and propagation of different types of collective modes is described. The accurate description of non-local interaction terms permits one to compare the classical kinetic and Fermi liquid approaches to kinetics of dilute gases, and to express the non-local part of the Fermi liquid interaction through the elements of the T-matrix and their derivatives.

1. INTRODUCTION

Probably the most striking macroscopic quantum phenomena in spin-polarized gases occur in kinetic processes /1/. Many of these quantum phenomena can be observed irrespective of a degree of quantum degeneracy of a gas - both in the classic Boltzmann temperature region $T \gg T_0$ and in the completely degenerate case $T \ll T_0$ (T_0 is the degeneracy temperature of a gas). Such phenomena are macroscopic manifestations of quantum identity of (quasi)particles, symmetry properties of wave functions and commutation relations for corresponding operators.

The well pronounced quantum effects may be expected in gases with large

quantum parameter $\Lambda/\lambda_0 > 1$; such gases may be called "quantum gases" (here $\Lambda = \hbar/p$ is the de Broglie wavelength of particles with characteristic

momentum p and dimension a_0). In degenerate gases $T \ll T_0$ the "quantum" condition $a_0/\lambda \ll 1$ is even less strong than a condition of diluteness $1 \gg a_0 N^{1/3} \sim a_0 \rho / \bar{n} \sim a_0 \rho / \bar{n} \sim a_0/\lambda$ (N is the number of particles per unit volume), and the evaluation of quantum corrections in a_0/λ coincides with the expansion in density $a_0 N^{1/3}$ leading to a practical impossibility of exact calculations beyond the first order. In Boltzmann gases the wavelength $\lambda = \hbar/p = \hbar/(2mT)^{1/2}$ is determined only by the temperature T resulting in the difference of characteristic parameters for diluteness $a_0^3 N$ and quantum effects $a_0/\lambda = a_0(2mT)^{1/2}/\hbar$. Thus in Boltzmann gases there always exists a possibility for exact calculations of quantum corrections in the frames of the lowest approximation in density and on the basis of two-particle interaction.

Kinetic phenomena in dilute spin-polarized gases may be separated into two groups - longitudinal and transverse effects. Such a separation is exact if one neglects weak dipole or spin-lattice interactions coupling spin and orbital variables. The first group contains all kinetic processes without changes in the direction of magnetization vector - usual transport such as viscosity and thermal conductivity, and diffusion of spin polarization without rotations (usual spin diffusion, spin thermal and pressure diffusion). Such processes are described by the dynamics of diagonal (in spin) components of single-particle density matrix.

The transverse processes are determined by the dynamics of the off-diagonal components of the density matrix and describe the rotation of magnetization without changes in its absolute value, i.e. the dynamics of the appearing small transverse components of magnetization. Certainly, such processes include the propagation of spin waves.

Since these two types of phenomena are quite different, it is convenient to study longitudinal and transverse effects separately by independent approaches.

2. LONGITUDINAL EFFECTS. SPIN PRESSURE DIFFUSION

Diagonal in spin components of the distribution function responsible for longitudinal kinetic effects correspond to the pure spin states of particles, i.e. to the particles with definite values of spin projection. If from formal point of view one considers the particles in different pure quantum (spin) states as

different sorts of particles, then the kinetic equations for diagonal (in spin) components of distribution function for identical particles with spin S turn out to be just the same as the set of $2S+1$ kinetic equations for the gas mixture of $2S+1$ types of different particles. Such analogy may be useful for all sorts of longitudinal transport phenomena in polarized gases.

The utilization of this formal analogy of kinetic equation for diagonal components of distribution function of a dilute gas with a set of kinetic equations for some multicomponent gas is not new. Such an analogy (the so-called Wang Chang and Uhlenbeck approach) is often used to describe transport phenomena in gases of particles with a complicated internal energy spectrum (e.g. for gases of polyatomic molecules). This approach makes it possible to apply for polyatomic gases the well-developed schemes of transport calculations in multicomponent mixtures of monoatomic gases such as Chapman-Enskog or Grad expansions for Boltzmann gases and Brooker-Sykes technique for degenerate ones (see e.g. reviews [1,2] and references therein). Note, that in spin-polarized gases this approach leads to accurate results only in the exchange approximation when coupling of spin and orbital variables is negligible.

In our case of spin-polarized gas of particles with spin S the longitudinal transport is described by the same set of kinetic coefficients as $(2S+1)$ -component gas mixture. Therefore the macroscopic quantum effects in longitudinal transport will appear only as a result of proper symmetrization of wave functions for identical particles in calculations of scattering probabilities entering the corresponding collision integrals.

The most pronounced effects of this sort are so-called magnetokinetic effects: the nearly infinite increase of the particles mean free path (accompanied with corresponding increases of viscosity and thermal conductivity coefficients) at polarization of ultraquantum gases of fermions $a_0/\lambda \ll 1$ (for details and references see reviews [1]).

Certainly, the simplest and the most realistic case is represented by a gas of spin-1/2 fermions for which the longitudinal transport is analogous to the situation in a binary mixture of gases of particles with equal masses. Then the spin diffusion is governed by an ordinary diffusion equation $\partial^2/\partial x^2$ for binary mixtures while the role of the mixture concentration is played by the spin polarization vector of the gas.

Therefore all longitudinal transport coefficients may be evaluated from the already known formal results for binary mixtures of dilute gases. For example, one may use for polarized spin-1/2 Boltzmann gas just the formulae of the classic kinetic theory expressing the transport coefficients of binary gases through the well-known integrals Ω . The only remaining problem is to express the scattering amplitudes entering Ω -integrals via parameters of two-particle potential, and to exploit the proper symmetrization procedure. Such calculations have already been performed in most of the interesting cases for all longitudinal transport coefficients (see reviews /1/ and references therein).

Probably the single exception is the spin pressure diffusion. Usually the pressure diffusion ratio is determined on the basis of irreversible thermodynamics /3,4/ providing in our case

$$\rho = (P/\rho^2) \left\{ \frac{\partial}{\partial \mu} (\mu_+ \mu_+ + \mu_- \mu_-) / \partial \beta \right\}^{-1} P, T \quad (2.1)$$

where $\rho = m_+ N_+ + m_- N_-$ is the density of the gas with pressure P ; m_+, m_- , μ_+ , and μ_- are the (effective) masses, chemical potentials and densities of (quasi)particles with up and down spins, $N_+ + N_- = N$ (normally in dilute exchange spin-polarized gases $m_+ = m_-$). The calculations based on Eq.(2.1) have already been reported /1/. Note, that the spin pressure diffusion ratio (2.1) is equal to zero in Boltzmann gases reflecting the well-known fact of absence of usual pressure diffusion in binary mixtures of ideal Boltzmann gases of particles with equal masses.

Unfortunately, the standard equation (2.1) is not valid practically in all real situations of viscous flows in liquid or gaseous systems /5/. This results from almost unavoidable motion of a liquid/gas in presence of pressure gradients. Since the second spatial derivatives of mass velocity u of a liquid/gas are of the same order as gradients of pressure (according to Navier-Stokes equation, $\Delta u \sim \nabla P / \eta$, η is the viscosity), the second derivatives of u must be included into the expression for the diffusion (spin) current along with ∇P . Of course, it does not mean that the second derivatives of u are usually anomalously large and have the same order as ∇T or $\nabla \alpha$ ($\alpha = (N_+ - N_-)/(N_+ + N_-)$ is the degree of spin polarization); on the contrary, it means that the pressure gradients in stationary situations are usually small. The second spatial derivatives of gas velocity u being included into the expression for (spin) diffusion currents lead to important "viscous" renormalization of pressure diffusion and to considerable difference from Eq.(2.1) /5/.

Recently we made an attempt to generalize results of Ref./5/. It was shown /6/ that with the viscous renormalization the value of k_{sp} in low-density binary mixtures (or spin-polarized gases of spin-1/2 particles) is equal to

$$k_{sp} = P \left\{ (1/m_+) (\partial \mu_+ / \partial P)_{\alpha, T} - (1/m_-) (\partial \mu_- / \partial P)_{\alpha, T} - (\eta_+ / m_+ N_+ - \eta_- / m_- N_-) / \eta \right\} / \left\{ (1/m_+) (\partial \mu_+ / \partial \alpha)_{P, T} - (1/m_-) (\partial \mu_- / \partial \alpha)_{P, T} \right\} \quad (2.2)$$

at all degrees of quantum degeneracy of a gas. Here η_{\pm} are the "partial" viscosities of (spin) components of the gas. The difference between Eqs. (2.1) and (2.2) is due to the term with viscosity in the first brackets.

In dilute Boltzmann $T \gg T_0$ gases Eq.(2.2) reduces to

$$k_{sp} = 2(N_+ N_- / \eta N) (\eta_+ / N_- - \eta_- / N_+) \quad (2.3)$$

while in the degenerate case, $T \ll T_0$,

$$k_{sp} = (3/10) \eta \left\{ \eta_+ (1 + \alpha)^{5/3} - \eta_- (1 - \alpha)^{5/3} \right\} / \left\{ (1 + \alpha)^{5/3} + (1 - \alpha)^{5/3} \right\} \quad (2.4)$$

Generally, the partial viscosities η_{\pm} in Eq.(2.3) for the mixture of classic gases may be expressed via the two-body interaction potential or scattering amplitude using Chapman-Enskog or Grad expansions for binary mixtures (such formulae may be easily obtained from the detailed results of Ref./7/). Below I will give the expression for η_{\pm} / η only in the ultraquantum limit, $a_0 / \Lambda \ll 1$,

$$\eta_+ / \eta = 1 - \eta_- / \eta = (2N_+^2 + N_+ N_- / 2) / (2N_+^2 + 2N_-^2 + N_+ N_-) \quad (2.5)$$

As it was stated in the Introduction, the calculation of quantum corrections in a_0 / Λ is quite sensible and is not beyond accuracy.

The partial viscosities for binary (or polarized) degenerate gases (2.4) may be evaluated on the basis of exact solution of kinetic equation /1,8,9/. In the ultraquantum limit $a_0 / \Lambda \ll 1$ it results in

$$\eta_+ = \frac{(6/24) (2N)^{5/3} h^5}{3 a^2 m^2 T^2} \frac{C(\Lambda)}{d^3 (1 + d^3)^{5/3} (5 - 3d^2)},$$

$$\eta_- = \frac{(6/48) (2N)^{5/3} h^5}{3 a^2 m^2 T^2} \frac{d^5 C(\Lambda)}{(1 + d^3)^{5/3}}, \quad (2.6)$$

$$c(\lambda) = ((1 - \lambda)/4) \sum_{\nu=0}^{\infty} (4\nu + 3)^{\nu} [(\nu + 1)(2\nu + 1) [(\nu + 1)(2\nu + 1) - \lambda]]^{\nu}$$

$$\eta \neq \eta = 1 - \eta \quad \eta = 2 c(\lambda_{\downarrow}) / [2 c(\lambda_{\uparrow}) + (5d^8 - 3d^{10}) c(\lambda_{\downarrow})]$$

$$\lambda_{\downarrow} = 1 - 2d^2 + 6d^4/5, \quad \lambda_{\uparrow} = 1/5, \quad d = (N_{\downarrow}/N_{\uparrow})^{1/3}$$

where a is the two-particle s -wave scattering length. Note, that in accordance with the Introducing one has to calculate quantum corrections in d/λ to Eq.(2.6) simultaneously with the density corrections in $nN^{1/3}$ to Eqs.(2.2), (2.4), (2.6). At present it is possible to obtain only the first order correction.

Some analogous results may be obtained phenomenologically also in the case of dense spin-polarized Fermi liquids /6/.

Dealing with the problems of flows of dilute spin-polarized gases one must take into account also the slip effects near boundary walls. Such boundary slip results in a considerable spin diffusion in the Knudsen boundary layers at the walls. This leads to the following boundary condition on the mass velocity component u_i parallel to the wall ($z=0$):

$$u_i(z=0) = \left\{ \alpha_{11} \partial u_i / \partial z + \alpha_{12} \nabla_i \mu + \alpha_{13} \nabla_i T \right\}_{z=0} \quad (2.7)$$

(instead of usual hydrodynamic condition $u_i(z=0) = 0$), and to the following expressions for the boundary (spin) diffusion current $j_i^{(0)}$ and heat flow $q_i^{(0)}$:

$$j_i^{(0)} = \left\{ \alpha_{21} \partial u_i / \partial z + \alpha_{22} \nabla_i \mu + \alpha_{23} \nabla_i T \right\}_{z=0} \quad (2.8)$$

$$q_i^{(0)} = \left\{ \alpha_{31} \partial u_i / \partial z + \alpha_{32} \nabla_i \mu + \alpha_{33} \nabla_i T \right\}_{z=0}$$

where $\mu = \mu_{\downarrow}/m_{\downarrow} - \mu_{\uparrow}/m_{\uparrow}$, and the slip coefficients α_{ik} are proportional to the particles mean free path l_i ,

$$\alpha_{12} \sim 1/c,$$

c is the sound velocity. Nondiagonal elements of the matrix α_{ik} are related to each other by Onsager symmetry conditions. For example,

$$\alpha_{21} = \alpha_{12} - N D_s (\partial \mu / \partial \alpha)^{-1}_{P,T} (\gamma_{\downarrow} N_{\downarrow} m_{\downarrow} - \gamma_{\uparrow} N_{\uparrow} m_{\uparrow}),$$

where D_s is the usual (longitudinal) spin diffusion coefficient. Unfortunately, the exact calculations of slip coefficients have not been performed yet for spin-polarized gases even in the ultracold limit.

The above results may be applied to spin-polarized gases ${}^3\text{He} \uparrow$, $\text{H} \uparrow$, $\text{D} \uparrow$

(provided that hydrogen gases contain only two hyperfine components) and for other types of polarized or binary gases and Fermi liquids similar to dense normal ${}^3\text{He} \uparrow$. One of the most interesting objects is a system of ${}^3\text{He}$ quasiparticles in dilute liquid ${}^3\text{He} \uparrow$ - ${}^4\text{He}$ mixtures: this system may be studied at arbitrary degrees of quantum degeneracy and spin polarization, and in wide range of values of the quantum parameter a_0/λ .

3. TRANSVERSE KINETICS, SPIN WAVES AND RELATED PHENOMENA

The off-diagonal elements of the single-particle density matrix responsible for transverse dynamics are purely quantum objects. Thus the corresponding macroscopic transverse phenomena are basically of the quantum origin.

Usually quantum effects related to off-diagonal elements of density matrix and to commutation rules for operators of internal variables of particles are studied for classical Boltzmann gases on the basis of Waldmann-Snyder kinetic equation (see, e.g., review /2/ and references therein). In our case (e.g., for internal spin variables) the standard Waldmann-Snyder equation cannot be applied - it does not take into account the quantum identity of (quasi)particles thus overlooking some important collective phenomena of exchange origin. Probably this was the reason why the spin waves propagation in polarized Boltzmann quantum gases was regarded for a rather long time as a unique phenomenon (and with considerable suspicion).

Below we will see how the generalization of Waldmann-Snyder equation in case of identical particles with internal structure leads to the existence of different types of collective modes analogous to spin waves in spin-polarized Boltzmann gases. We will be able to derive the spectra of these modes, conditions of their propagation, and the influence of non-local and gradient terms on the spectra. This will also allow us to study the limits of applicability for the Fermi liquid approaches to the molecular field effects in dilute Boltzmann gases. These results may also help us to understand some of the differences between transverse and longitudinal relaxation times introduced for polarized gases several years ago /10/.

The quantum identity of particles with an arbitrary spectrum of internal degrees of freedom (certainly including the spin states) does not cause any complicated peculiarities in deriving the kinetic equation. One must only provide a more careful symmetrization for operators especially including the procedure

of decoupling of two-particle density matrices before the collision at $t \rightarrow -\infty$. As a result, the kinetic equation for the single-particle Wigner distribution functions $\hat{f}(\mathbf{r}, \mathbf{p})$ (which are still operators in the space of internal states of particles) takes the form

$$\begin{aligned} \partial_t f_{\beta}^{\alpha}(\mathbf{r}, \mathbf{p}) + (i/\hbar) [\hat{H}, \hat{f}]_{\beta}^{\alpha} = & i(2\pi\hbar)^{-3} \int dp_1 dq^l dq^m dr^l dr^m \exp(i/\hbar) \\ & (q^l r^l + q^m r^m) \left\{ -i_{\beta\gamma}^{\alpha M} f_{\gamma}^{\beta}(\mathbf{g} + \mathbf{q}, \mathbf{g} - \mathbf{q}) f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}', \mathbf{p} - \mathbf{q}) f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}'', \mathbf{p}_1) \right. \\ & \left. + i_{\beta\gamma}^{\alpha M} (-\mathbf{g} - \mathbf{q}', \mathbf{g} - \mathbf{q}') f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}', \mathbf{p}_1) f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}'', \mathbf{p} - \mathbf{q}') + f_{\beta}^{\alpha}(\mathbf{r} + \mathbf{r}', \mathbf{p}) f_{\beta}^{\alpha}(\mathbf{r} + \mathbf{r}'', \mathbf{p}) \right. \\ & \left. + i_{\beta\gamma}^{\alpha M} (-\mathbf{g} - \mathbf{q}', \mathbf{g} - \mathbf{q}') f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}', \mathbf{p}_1) f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}'', \mathbf{p}_1) \right. \\ & \left. + i_{\beta\gamma}^{\alpha M} (\mathbf{g} + \mathbf{q}', \mathbf{g} + \mathbf{q}') f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}', \mathbf{p}) f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}'', \mathbf{p}_1) \right. \\ & \left. + i_{\beta\gamma}^{\alpha M} (\mathbf{g} + \mathbf{q}', \mathbf{g} + \mathbf{q}') f_{\beta}^{\gamma}(\mathbf{r} + \mathbf{r}', \mathbf{p}_1) \right\} + \dots \end{aligned} \quad (3.1)$$

where \hat{H} is the single particle Hamiltonian, \hat{t} is the scattering T-matrix, and $\mathbf{g} = \mathbf{p}_1 - \mathbf{p}$. This equation must replace the Waldmann-Snyder equation in the case of identical particles. Above we had written explicitly only terms linear in \hat{t} for the molecular field responsible for possible coherent collective effects. There are also lengthy terms quadratic in \hat{t} (denoted in Eq.(3.1) by ...) which are not interesting in this context.

Eq.(3.1) is rather cumbersome, and for the sake of clarity it is better to introduce some simplifying assumptions. As a rule, the problems of transport or long-wave collective modes involve only slightly inhomogeneous states. Then the r.h.s. of Eq.(3.1) may be simplified by the gradient expansion using

$$\begin{aligned} \int dq dr \exp(iqr/\hbar) F(\mathbf{r} + \mathbf{r}', \mathbf{q}) G(\mathbf{q}) = & (2\pi\hbar)^3 \left\{ F(\mathbf{r}, \mathbf{q}) G(\mathbf{q}) + \right. \\ & i\hbar (\partial/\partial \mathbf{q}) \left[G(\mathbf{q}) (\partial/\partial \mathbf{r}) F(\mathbf{r}, \mathbf{q}) - (\hbar^2/2) (\partial^2/\partial \mathbf{q}^2) \partial \mathbf{q} \right] \\ & \left. + \dots \right\} \quad \mathbf{q} \rightarrow 0 \end{aligned} \quad (3.2)$$

As a second simplification it may be sensible to assume that the interaction potential does not depend on internal states of colliding particles, and therefore has an "exchange" form in respect to internal states:

$$t_{\beta\gamma}^{\alpha M}(\mathbf{p}', \mathbf{p}) = t(\mathbf{p}', \mathbf{p}) \delta_{\beta}^{\alpha} \delta_{\gamma}^M \quad (3.3)$$

Then the first term in the gradient expansion (3.2) for "exchange" interaction (3.3) leads to the kinetic equation

$$\partial_t f_{\beta}^{\alpha} + (i/\hbar) [\hat{H}, \hat{f}]_{\beta}^{\alpha} = (i/\pi\hbar)^3 \int dp_1 \hat{t} [f(\mathbf{p}), f(\mathbf{p}_1)]_{\beta}^{\alpha} + \dots \quad (3.4)$$

where $2\hat{t} = t^+(-\mathbf{g}, \mathbf{g}) + t(\mathbf{g}, -\mathbf{g})$, and we have also used the optic theorem in order to eliminate some of the linear in \hat{t} terms which in reality may be expressed via quadratic terms.

If the only internal states are the states of nuclear spins, and α, β are the corresponding spin indices, then Eq.(3.4) is equivalent to Lhuillier-Lal e kinetic equation $\nabla \cdot \mathbf{J}$. In all the cases, when the single particle internal energy levels are equidistant, the results (including the spectrum of collective oscillations) will be similar to those for spin-polarized gases without other internal degrees of freedom (see below).

The analogy with spin-polarized systems gives one a chance to understand the situation with collective modes in Boltzmann gases with other internal degrees of freedom. If the single-particle Schrodinger equation has Z solutions (eigenstates) including degenerate ones, then Wigner distribution functions and all other single-particle operators for a dilute gas are matrices $Z \times Z$. As the basis \hat{L}_i one may choose the unitary matrix \hat{E}_i , $(Z - 1)$ diagonal matrices $\hat{L}_i^{(0)} = \hat{L}_i^{(0)}$ ($0 < i < Z$), and some convenient set of $Z^2 - Z$ independent non-diagonal matrices \hat{L}_i , constructed using the operators $\hat{I}_{x,y,z}$ of the angular moment operator with $l = (Z - 1)/2$. The corresponding system of matrices provides one with a complete set of basis matrices $Z \times Z$ with commutation relations of the form

$$[\hat{L}_i, \hat{L}_j] = a_{ijk} \hat{L}_k, \quad a_{ijk} = -a_{jik} \quad (3.5)$$

The single particle Hamiltonian and the equilibrium distribution function $\hat{f}^{(0)}$ may be diagonalized,

$$\begin{aligned} \hat{H} = & h_1 \hat{L}_1^{(0)} + (p^2/2m) \hat{E}, \quad \hat{f}^{(0)} = (2\pi\hbar)^{-3} n_1^{(0)}(\mathbf{p}) \hat{L}_1^{(0)} + n_0^{(0)}(\mathbf{p}) \hat{E}, \\ \hat{f} = & (2\pi\hbar)^{-3} n_i(\mathbf{p}) \hat{L}_i. \end{aligned} \quad (3.6)$$

and the linearized kinetic equation (3.4) takes the form

$$\begin{aligned} & [(\omega - kv)\delta_{ik} - \Omega_{ik}^{(0)} - \Omega_{ik}^{(i)}] \hat{v}_{ik} + \beta_{ik} \hat{v}_{ik} \int \hat{v}_i(\mathbf{p}_1) d\mathbf{p}_1 = \\ & = -i \int \hat{v}_i(\mathbf{p}) \hat{v}_{ik}; \quad \hat{v}_i = n_i(\mathbf{p}) - n_i^{(0)}(\mathbf{p}), \end{aligned} \quad (3.7)$$

where the matrix $\Omega_{ik}^{(0)} = h_{i,jk}$ determines the splitting of single-particle internal energy levels, and the matrix $\Omega_{ik}^{(i)}$,

$$\Omega_{ik}^{(i)} = a_{ijk} \int \hat{v}_j^{(0)}(\mathbf{p}_1) d\mathbf{p}_1, \quad (3.8)$$

describe the additional shifts of single-particle states due to molecular field produced by interaction, $\beta_{ik} = a_{ijk} n_j^{(0)}(\mathbf{p})$, and $\hat{\Gamma}$ being quadratic in \hat{v} is a matrix in the indices (i,k) and an integral operator in momenta space (the $\hat{\Gamma}$ -terms in Eq.(3.7) are the only ones responsible for relaxation). The equations (3.7) are equivalent to

$$[(\omega - kv)\hat{\Gamma} - \hat{\Omega}^{(0)} + \hat{\Omega}^{(i)}] \hat{v} + \beta \int d\mathbf{p}_1 \hat{v}(\mathbf{p}_1) = -i \hat{\Gamma} \hat{v} \quad (3.9)$$

where $\hat{\Gamma} = \delta_{ik}$. In most of the cases the exact solution of Eqs.(3.9) is impossible due to the complicated structure of integral operators involved.

If $k=0$ the eigenfrequencies of Eq.(3.9) $\omega = \omega_0^{(s)}$ are determined only by the single particle energy splitting h_i :

$$\text{Det} \|\omega_0^{(s)} \hat{\Gamma} - \hat{\Omega}^{(0)}\| = 0 \quad (3.10)$$

while in the long-wave limit $k \neq 0$ the small changes $\omega_1^{(s)}$ of frequencies $\omega_0^{(s)}$ are quadratic in k and are given by the equation

$$\begin{aligned} & \text{Det} \|(\omega_0^{(s)} \hat{\Gamma} - \hat{\Omega}^{(0)} + \omega_1^{(s)} \hat{\Gamma}) - k^2(T/m)(\omega_0^{(s)} \hat{\Gamma} - \hat{\Omega}^{(0)}) + \\ & \hat{\Omega}^{(i)} - i\hat{\mathcal{C}}^{-1}\| = 0; \end{aligned} \quad (3.11)$$

$$\hat{\Omega}^{(i)} = (a_{ijk} / \hbar m T) \int d\mathbf{p} d\mathbf{p}_1 \mathbf{p} [(\mathbf{p}/3)t^{(0)} - (\mathbf{p}_1/3)t^{(1)}] n_j^{(0)}(\mathbf{p}_1) n_k^{(0)}(\mathbf{p}) / \int d\mathbf{p} n_k^{(0)}$$

where $t^{(0)}$, $t^{(1)}$ are the angular harmonics of the function \hat{v} , and we used the following natural parametrization of the relaxation operator (cf. Ref./10/):

$$\int d\mathbf{p} \hat{\Gamma}(\vec{v}) = 0; \quad \int d\mathbf{p} (\mathbf{p}/m) \hat{\Gamma}(\vec{v}) = -\hat{\mathcal{C}}^{-1} \int d\mathbf{p} (\mathbf{p}/m) \vec{v}$$

Simple estimates show that the order of magnitude of ω_1 is

$$|\omega_1| \sim k^2(T/m) / |\Omega^{(i)} - i/\mathcal{C}|$$

The macroscopic equation of motion responsible for the spectrum (3.11) has the form somewhat analogous to the Leggett equation. Purely dissipative diffusion regime corresponds to the case $|\Omega^{(i)}|/\mathcal{C} \ll 1$, i.e. to the case $\tau \Gamma^{1/2} \gg m^{3/2} \gg 1$. In the opposite case $|\Omega^{(i)}|/\mathcal{C} \gg 1$ it is possible to observe the propagation of the weakly damped collective modes.

The situation becomes transparent when the single-particle internal energy levels are equidistant. In this case the single particle Hamiltonian and equilibrium density matrix (3.6) can always be written as

$$\hat{H} = (\mathbf{p}^2/2m)\hat{E} + h_0\hat{I}_z, \quad \hat{\rho}^{(0)} = (2\pi\hbar)^{-3} n_0(\mathbf{p})(\hat{E} + h_0\hat{I}_z/T) \quad (3.12)$$

If one is interested in small perturbations of the distribution function of the form $\delta\hat{f} = w(\mathbf{p})(\hat{I}_x + i\hat{I}_y)$, then the corresponding equation of motion decouples completely from the rest of the system (3.7). Such an equation becomes a scalar one, and the dispersion relation (3.11) coincides exactly with the spectrum of spin waves in spin-polarized quantum gases/1/:

$$\omega_1 = k^2(T/m) / (\Omega^{(i)} - i/\mathcal{C}), \quad \hbar\omega_0 = h_0 \quad (3.13)$$

(the equidistant splitting of levels plays the same role as an external magnetic field for spin-polarized gases). Certainly, such a simple situation always takes place when one is dealing with a gas of two-level particles.

Generally speaking, the conditions of appearance of coherent mean field effects (either in a form of propagation of low damped collective modes or in a form of small oscillations accompanying diffusion motion) may be formulated in a following way for a classic Boltzmann gas of particles with arbitrary spectrum of internal degrees of freedom:

1. The structure of internal energy levels must provide the existence of solutions of Eq.(3.10) with small (better with zero) imaginary parts
2. The diagonal equilibrium density matrix $\hat{\rho}^{(0)}$ (3.6) should not be reduced to the unitary matrix \hat{E} so that the commutator in linearized Eq.(3.4) will be non-zero at least for some types of perturbations. This means that the equilibrium populations of internal energy levels have to differ from each other, i.e. one must have some "polarization" of nonperturbed state thus providing non-zero internal frequency $\Omega^{(i)}$ proportional to "polarization".
3. The most promising are particles (molecules) with spectra containing degenerate or nearly degenerate levels. The equations corresponding to transitions between these levels may be separated from other Eqs.(3.9) since there are comparatively large gaps for other transitions. If the levels in

question are slightly separated due to some internal or external field, then the equilibrium "polarization" may be already ensured by a temperature factor. If the levels are completely degenerate, and in total equilibrium their occupations are equal, then the "polarization" of nonperturbed distribution must be ensured by some external source (e.g. by some sort of pumping). For "exchange" interaction (in a sense (3.3)) such an artificial "polarization" is conserved, and the collective effects can be observed

4. The analysis is most simple if one is dealing with the system of equidistant levels close to each other. In this case there always exists a branch in the spectrum of collective modes very similar to the spin waves in spin-polarized gases. The examples of such situations are provided by particles with angular momentum (such as ortho-hydrogen molecules) or by all types of two-level particles. Then if the interaction of particles is practically independent of corresponding quantum numbers (which is very often true), the macroscopic equations of motion for the off-diagonal elements of the distribution are the same as in the case of the spin waves.

5. The damping of coherent collective modes in Boltzmann gases of particles with internal structure is small only at low temperatures and/or for rather weak interaction.

* * *

The above results for collective phenomena were obtained exploiting the analogy with spin waves in spin-polarized gases [1,7].

Nevertheless, Eq.(3.1) enables one to obtain some additional data which may be important for systematic study of spin dynamics in spin-polarized gases. The previous studies of polarized gases were based on the assumption of the locality of the interaction (only the first term in the expansion (3.2) was used thus limiting the description to Eq(3.4) and not using all the information contained in Eq.(3.1)). In general situation such a limitation is not always justified. There is a special reason for concern: since we are interested in the terms proportional to k^2 in the spectrum of collective modes and macroscopic equation of motion, we must also consider the gradient terms in Eq.(3.1) proportional to $\partial f(r,p)/\partial r$ and $\partial^2 f(r,p)/\partial r_1 \partial r_2$.

It may be shown that the gradient terms lead to the renormalization of the spectrum (3.11):

$$\text{Det} \| (\omega_1^{(s)} + \omega_0^{(s)}) \hat{\Gamma} - \hat{\Omega}^{(0)} - (T/m)k^2 [(\hat{1} + \hat{A}_1)(\omega_0^{(s)}) \hat{\Gamma} - \hat{\Omega}^{(0)} + \hat{\Omega}^{(1)} - i\hat{C}^{-1}y^{-1}(\hat{1} + m\hat{A}_2T) + m\hat{A}_3T] \| = 0, \quad (3.14)$$

where the matrices $\hat{A}_{1,2}$ appear due to the first spatial gradients, and \hat{A}_3 - due to the second. The explicit expressions relating the matrices \hat{A} to the T -matrix are very cumbersome, and will not be given here. I would only like to discuss the results qualitatively and very briefly.

The non-locality of Eq.(3.1) gives rise to several different types of additional terms two of which are rather interesting. First, there appear terms of the zero-sound type which were naturally introduced in the frames of the Fermi-liquid-like approach to polarized gases [1] and were absent in purely classic description [1].

The second type of non-local terms is represented by off-shell contributions (such as e.g. $\partial_t(g + q, -g + q)/\partial q$ at $q \rightarrow 0$). The presence of such terms in non-local transverse dynamics clearly demonstrates the inevitable difference $1/0$ between transverse and longitudinal relaxation times.

Fortunately, the effects of non-local terms are small for dilute quantum Boltzmann gases. Their contribution to the spectrum may be characterized by the dimensionless parameter $\hbar \hat{\Omega}^{(1)}/T$. As a result, if one wants to include non-local corrections, one also has to derive the kinetic equation more carefully including the first density correction to the Boltzmann equation and using more accurate schemes of description of two-particle correlation functions.

The above non-local corrections become more important for degenerate systems especially with the increase of density and polarization. For dense highly polarized Fermi liquids the non-local contributions to the spectrum are of the same order as the usual local ones [8].

The study of non-local gradient terms gives one a chance to compare two approaches used to describe transverse dynamics of polarized gases: the classical kinetic approach developed by Lhuillier and Laloë [7] and the unified approach to degenerate and nondegenerate gases¹⁾ in the spirit of Landau theory of Fermi liquids. The latter approach was used beginning from the first papers on spin waves in polarized gases [11], and in the main order in density gave the same results for the spectrum as the kinetic approach. This coincidence is not

¹⁾The intermediate region had been studied in Ref.10

amazing since for dilute gases the concept of "statistical quasiparticles" /12/ works quite well even in the Boltzmann region: in dilute Boltzmann gases all the interaction effects are quadratic in the distribution function, $\hat{F}(\mathbf{p}, \mathbf{p}') \hat{f}(\mathbf{p}) \hat{f}(\mathbf{p}')$, and the only remaining problem is to express the "interaction function" $\hat{F}(\mathbf{p}, \mathbf{p}')$ (the analog of the Fermi liquid function) through the T-matrix.

However, there always remained some difference in the form of kinetic equations for these two approaches though the difference in the results was negligible. The kinetic equation in Fermi liquid approach contained /1,10,12/ some zero- sound terms,

$$kv \int \hat{F} \delta f dp,$$

which were absent in classical kinetic approach /7/. The above analysis of non-local effects show that such contributions do appear in classical kinetic approach if one takes into account the gradient non-local terms. However, the same order gradient contributions give rise to some additional terms absent in usual Fermi liquid theory. This is due to the fact (see /8/) that the usual Landau theory is basically the local one. It is quite sufficient for thermodynamic calculations. But we are interested in the effects of the order of k^2 and encounter several commutators of internal variables (these commutators provide for "polarized" systems the non-zero contributions to the kinetic equation even without additional spatial derivatives). For these reasons the exact calculations of the k^2 -spectrum of collective modes within the Fermi liquid approach must involve the effects of non-local Fermi liquid interaction in addition to the usual

Fermi liquid function $F(\mathbf{p}, \mathbf{p}')$:

$$\delta \hat{H} = \int \hat{F} \delta f dp' + \int \hat{F}_1 \mathbf{V} \delta f dp' + \int \hat{F}_{ij} (\partial^2 / \partial r_i \partial r_j) \delta \hat{n} dp'$$

In the case of Boltzmann gases the above non-local kinetic equation permits one to express the phenomenologically introduced functions \hat{F}_1 , \hat{F}_{ij} through the elements of the T-matrix and their derivatives including the off-shell terms. Such a procedure leads to the more complete results than the semiphenomenological approach /13/.

The same analysis for the degenerate systems especially for the dense ones seems to be much more complicated.

The detailed results will be published elsewhere.

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