

CHAPTER 1

SPIN-POLARIZED ^3He - ^4He SOLUTIONS

BY

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Contents

1. Introduction	3
1.1. Spin-polarized quantum systems	3
1.2. ^3He - ^4He II solutions: nearly an ideal Fermi gas	5
1.3. Interaction of quasiparticles: a choice of a model. Quantum gases	8
2. Macroscopic description of polarized solutions	12
2.1. Equations of spin dynamics	12
2.2. Fermi liquid description of degenerate solutions	17
3. Dilute spin-polarized solutions. Thermodynamics	23
4. Transport phenomena and magnetokinetic effects	31
4.1. Viscosity, thermal conductivity and absorption of sound	31
4.2. Spin diffusion, spin thermal and pressure diffusion. Spin second viscosity	39
5. Transverse spin dynamics and spin waves	41
5.1. Theoretical results	41
5.2. Experimental data on the spin dynamics	49
6. ^3He superfluidity in $^3\text{He}\downarrow$ - ^4He solutions	53
6.1. Temperature of the superfluid transition	53
6.2. BCS phase of a polarized solution	55
6.3. Inhomogeneous phases: superfluid liquid crystals	59
7. Other spin-polarized ^3He Fermi systems. Concluding remarks	65
7.1. Solid, liquid and gaseous $^3\text{He}\downarrow$	65
7.2. Unusual phases of $^3\text{He}\downarrow$	68
7.3. Concluding remarks	70
References	71

1. Introduction

1.1. SPIN-POLARIZED QUANTUM SYSTEMS

The last decade brought into being a new branch of physics, the physics of spin-polarized quantum systems, which combined the achievements in the areas of (ultra)low temperatures and high magnetic fields. The systematic investigation of the influence of spin polarization on the properties of various quantum systems (quantum liquids, quantum solids and quantum gases) has revealed an unexpectedly large number of new and unanticipated quantum phenomena, interesting from both the theoretical and experimental points of view. Certainly, macroscopic quantum effects are more pronounced in systems of light particles - atoms of hydrogen and helium isotopes.

The choice of the special term "spin-polarized quantum systems" has a twofold purpose. First, such a term emphasizes that the majority of phenomena in question are due to the alignment of spins irrespective of the means to achieve such an alignment. Second, some of the polarized systems can be in long-lived metastable states, whose very existence is called forth just by the spin polarization of the particles. For example, in the case of the isotopes of spin-polarized atomic hydrogen the strong spin polarization prevents the recombination of atoms into molecules, resulting in the large lifetimes of the atomic gases $\text{H}\downarrow$ and $\text{D}\downarrow$. For $^3\text{He}\downarrow$ systems the possible metastability is connected with the fact that the polarization of some helium systems may be caused not by direct magnetization in high magnetic fields (the so-called "brute force technique") but by some alternative indirect method. This is so because the interaction of helium atoms is mostly exchange and conserves the total magnetic moment (spin) of the particles, thus leading to the possibility to create polarized states with a total magnetic moment that is not determined by the values of the temperature and the external magnetic field but by the prehistory of the system. This possibility is especially important for such helium systems for which the direct magnetization in practically accessible magnetic fields up to 100 kOe does not result in an appreciable degree of spin polarization. Information on the methods to create spin-polarized quantum systems and a summary of the results obtained up to 1980 can be found in Spin-Polarized Quantum Systems (1980).

Most interesting low-temperature properties of ${}^3\text{He}$ are due to the existence of the nuclear spin of the ${}^3\text{He}$ particles and are a manifestation of magnetic phenomena in different solid, liquid and gaseous phases of ${}^3\text{He}$. At present, the influence of spin polarization on the various properties of ${}^3\text{He}$ has been investigated most thoroughly in the case of solutions of ${}^3\text{He}$ in superfluid ${}^4\text{He}$. Interest in ${}^3\text{He}$ - ${}^4\text{He}$ II solutions is stimulated by several considerations. First, dilute ${}^3\text{He}$ - ${}^4\text{He}$ II solutions are practically the only example of a (nonelectron) Fermi gas that does not condense at any low temperature. For such systems one can easily obtain a comparatively simple and complete theoretical description by the same methods as for other dilute gases. On the other hand, one can investigate the solutions experimentally in a very wide range of parameters: at temperatures between 0.2 mK and 1 K, ${}^3\text{He}$ concentrations from 10^{-4} or 10^{-5} up to 10^{-1} , and at practically arbitrary degrees of spin polarization. Apart from this, in contrast to the majority of other ${}^3\text{He}$ systems, spin polarization of solutions at low temperatures and low ${}^3\text{He}$ concentration can be achieved relatively easily, by direct magnetization in not very high external magnetic fields. The simplicity of the theory and the possibility of experiments in a wide range of parameters allow a consistent comparison of the theoretical and experimental data on the properties of ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions practically without any model assumptions and adjustable parameters. For this reason the information on the properties of the solutions is more complete as compared with other spin-polarized ${}^3\text{He}\downarrow$ systems.

The considerable amount of available data on ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions can be cast in a relatively general form and extended (at least qualitatively) to other spin-polarized quantum systems. It is evident that the properties of ${}^3\text{He}\downarrow$ or $\text{D}\downarrow$ gases are analogous to the properties of nondegenerate ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions, and the Fermi liquid approach to the degenerate solutions can be generalized to the dense normal ${}^3\text{He}\downarrow$ Fermi liquid. Moreover, many of the results on the spin dynamics of nondegenerate ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions can also be applied to the $\text{H}\downarrow$ Bose gas. Thus, the investigation of ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions provides a convenient method to model quantum phenomena in other spin-polarized quantum systems.

Moreover, ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions are the object of specific cryogenic interest. In all modern cryogenic equipment one has to use some phase of ${}^3\text{He}$ (as a working medium or a heat transferring agent) and high magnetic fields. Therefore, the study of spin-polarized helium systems can be useful for the progress in the ultralow-temperature technique. It is important that superfluid ${}^3\text{He}$ - ${}^4\text{He}$ solutions are the only liquids with a considerable (orbital) entropy at temperatures below 1 mK.

1.2. ${}^3\text{He}$ - ${}^4\text{He}$ II SOLUTIONS: NEARLY AN IDEAL FERMI GAS

A ${}^3\text{He}$ - ${}^4\text{He}$ II solution is a system of impurity ${}^3\text{He}$ fermions dissolved in a superfluid ${}^4\text{He}$ Bose background. At sufficiently low temperatures T the number of Bose excitations of the system (phonons, rotons) is negligible, and the normal component of the superfluid ${}^3\text{He}$ - ${}^4\text{He}$ II solutions is determined solely by the ${}^3\text{He}$ impurity system (the condition $T \leq 0.5$ K is sufficient at not very low ${}^3\text{He}$ concentrations). According to the theory of Landau and Pomeranchuk (1948) an isolated ${}^3\text{He}$ impurity atom in superfluid ${}^4\text{He}$ behaves as a delocalized Fermi quasiparticle with its states classified by the values of momentum p and a continuous energy spectrum $\mathcal{E}(p)$. In an external magnetic field H the energy spectrum of the ${}^3\text{He}$ quasiparticles with (nuclear) spin $1/2$ is described by the spin operator $\hat{\mathcal{E}} = \mathcal{E}_0(p)$. At sufficiently low temperatures and ${}^3\text{He}$ concentrations the characteristic velocities of the quasiparticles are small, and the energy spectrum of the bare quasiparticles \mathcal{E}_0 can be represented as a series of even powers of the small momentum p . Taking into account only the first three terms of this expansion, the energy spectrum \mathcal{E} can be written as

$$\mathcal{E}(p) = \left\{ -\Delta + \frac{p^2}{2M} \left[1 - \gamma(p/p_c) \right]^2 \right\} f - \beta \hat{\sigma} H, \quad (1.1)$$

where $\beta = 0.0778$ mK/kOe is the ${}^3\text{He}$ nuclear magnetic moment, $\hat{\sigma} \equiv \sigma_{ab}$ are the Pauli matrices, $f \equiv \delta_{ab}$ is the unit spin operator, M is the effective mass of the ${}^3\text{He}$ quasiparticles, $p_c = m_{4s_0}$ with m_4 the mass of the ${}^4\text{He}$ atom and s_0 the sound velocity in pure ${}^4\text{He}$ II ($p_c/\hbar \sim 1.5 \text{ \AA}^{-1}$ at zero pressure), and the dimensionless parameter $\gamma \leq 1$. The positive sign of the binding energy $\Delta > 0$ corresponds to the finite solubility of ${}^3\text{He}$ in superfluid ${}^4\text{He}$ discovered by Edwards et al. (1965). The effective mass of the ${}^3\text{He}$ quasiparticles M and the binding energy Δ depend on the pressure (density) of the solution; at the saturated vapor pressure $\Delta \sim 2.8$ K, $M \sim 2.3m_3$ (m_3 is the ${}^3\text{He}$ atomic mass). More precise data on the spectrum of isolated ${}^3\text{He}$ impurity quasiparticles can be found in Esel'son et al. (1973), Baym and Pethick (1978), Greywall (1978) and Bashkin and Meyerovich (1981).

At sufficiently low ${}^3\text{He}$ concentrations one can always neglect the interaction of the ${}^3\text{He}$ quasiparticles, and the system of ${}^3\text{He}$ quasiparticles represents an ideal gas of fermions with the spectrum (1.1). Note, that the contribution of the term with γ in (1.1) often lies beyond the accuracy of the calculations in the ideal gas approximation for thermodynamic functions of solutions: the neglected interaction terms (see below) more often

than not are more important than the deviation of the spectrum from a quadratic one. In the approximation of noninteracting particles with quadratic ($\gamma=0$) spectrum the evaluation of the thermodynamic, hydrodynamic and kinetic characteristics of solutions becomes trivial and must be done using the standard formulae of the ideal gas theory. Thus, the Fermi momentum p_0 of the ${}^3\text{He}$ quasiparticles in a (nonpolarized) solution is determined by the number of ${}^3\text{He}$ atoms per unit volume N_3 : $p_0 = \hbar(3\pi^2 N_3)^{1/3}$, and the degeneracy temperature of the solution (i.e. the Fermi energy of the ${}^3\text{He}$ quasiparticles) T_0 is equal to $T_0 = p_0^2/2M = (\hbar^2/2M)(3\pi^2 N_3)^{2/3}$. Numerically, T_0 is equal to $2.6x^{2/3}$ K at the saturated vapor pressure, where x is the concentration of ${}^3\text{He}$ in the solution. Consequently, at $T \gg T_0$ the thermodynamics of the solution is governed in this approximation by the expressions for the classical (Boltzmann) ideal gas, and at $T \leq T_0$ by the expressions for the degenerate ideal Fermi gas. Since the temperature can easily be varied in experiments between 1 K and 1 mK and the ${}^3\text{He}$ concentration between 10^{-4} and 10^{-1} , the ${}^3\text{He}$ quasiparticle system in the solution turns out to be the only gas accessible to experimental investigation in the degenerate, nondegenerate and intermediate cases.

In a spin-polarized solution the Fermi surface of the quasiparticles consists of two Fermi spheres whose radii p_{\pm} are determined by the numbers of particles with up and down spins per unit volume, N_{\pm} ($N_{+} + N_{-} = N_3$): $p_{\pm} = \hbar(6\pi^2 N_{\pm})^{1/3}$. In a fully polarized solution one of these Fermi spheres vanishes: $N_{-} = p_{-} = 0$, $N_{+} = N_3$, $p_{+} = 2^{1/3} p_0$. Below we shall always suppose that $N_{+} \geq N_{-}$. In the discussed approximation the degeneracy temperatures for particles with spin projections $\pm 1/2$ are $T_{\pm} = p_{\pm}^2/2M$. When $N_{-} \rightarrow 0$, $T_{-} \rightarrow 0$, $T_{+} \rightarrow 2^{2/3} T_0 \sim 4.2x^{2/3}$ K. If the spin polarization is an equilibrium one and is determined by an external magnetic field, then the degree of polarization P of a nondegenerate (Boltzmann) solution at $T \gg T_{+}$ is given by

$$P \equiv (N_{+} - N_{-})/N_3 = \tanh(\beta H/T);$$

in the case of a degenerate solution at $T \leq T_{-}$, P is given by the equations

$$P = \frac{p_{+}^3 - p_{-}^3}{p_{+}^3 + p_{-}^3}, \quad p_{+}^2 - p_{-}^2 = 4\beta H M, \quad p_{+}^3 + p_{-}^3 = 6\pi^2 \hbar^3 N_3. \quad (1.2)$$

The solution of eqs. (1.2), $P(H)$, is plotted in fig. 1. At an arbitrary degree of quantum degeneracy of the solution the dependence of the polarization

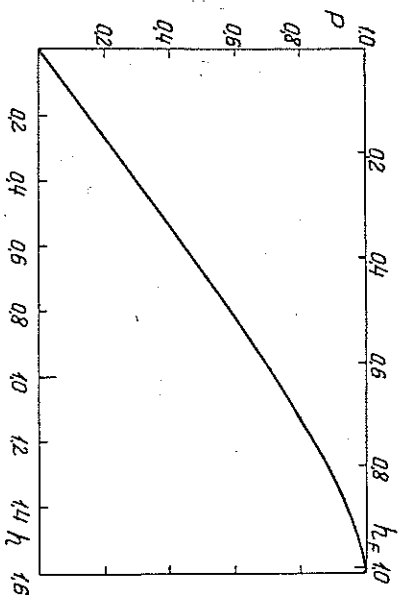


Fig. 1. The dependence of the degree of spin polarization $P = (N_{+} - N_{-})/N_3$ on the magnetic field H for degenerate solutions, $T \leq T_0$, in the ideal gas approximation (1.2), (3.8); $\hbar_F = 2\beta H/(2^{2/3} T_0)$, $h = 2\beta H/T_0$.

on the magnetic field is determined by the well-known Stoner formulae (McDougall and Stoner 1938):

$$P = \frac{g(x_{+}) - g(x_{-})}{g(x_{+}) + g(x_{-})}, \quad x_{\pm} = \frac{\mu \pm \beta H}{T},$$

where the chemical potential μ is given by the equation

$$1 = \frac{3}{4} (T/T_0)^{3/2} \{g(x_{+}) + g(x_{-})\},$$

and the function $g(x)$ has the form of the usual Fermi integral,

$$g(x) = \int_0^{\infty} y^{1/2} dy (1 + \exp(y - x))^{-1}.$$

For a nondegenerate (Boltzmann) solution at $T \gg T_{+}$, most of the thermodynamic functions do not depend on the degree of spin polarization in the ideal gas approximation because the mean velocities of the particles do not depend on the spin projections and are determined only by the temperature of the solution, T . On the contrary, for a degenerate solution the characteristic (Fermi) velocities $v_{\pm} = p_{\pm}/M$ are determined by the spin projections, and the thermodynamic quantities depend considerably on the polarization.

Note that in the case of high polarizations one can find (Meyerovich 1978) a specific region of parameters

$$T_- \ll T \ll T_+, \quad (1.3)$$

for which the spin-up component of the solution is degenerate, and the number of particles with down spins $N_- \ll N_+$ is small enough to make the spin-down component nondegenerate. Such a solution may be called "semidegenerate". In this semidegenerate region the presence and the statistics of spin-down particles in many cases is very important for the spin dynamics and kinetics, despite the very small number of particles with down spins, N_- .

As we shall see below, one can use the ideal gas approximation in order to understand (at least qualitatively) many of the main effects associated with the spin polarization of solutions. This approximation is even more useful for polarized solutions than for nonpolarized ones because it is more convenient to polarize (by an external magnetic field) solutions with low ${}^3\text{He}$ concentrations.

1.3. INTERACTION OF QUASIPARTICLES: A CHOICE OF A MODEL. QUANTUM GASES

The above description of ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions on the basis of the spin-polarized ideal Fermi gas model is effective only for dilute solutions, when the interaction corrections are small due to the smallness of the concentration. At higher concentrations the interaction corrections cannot be ignored. Moreover, there are many interesting effects which cannot be understood without inclusion of the interaction even for very dilute solutions.

Unfortunately, at present there is no consistent accurate microscopic method for a description of the interaction in the ${}^3\text{He}$ impurity quasiparticle gas. The difficulties are associated, first, with the absence of a general evaluation procedure for the interaction effects in comparatively dense systems (in our case, in the ${}^3\text{He}$ quasiparticle system), and, second, with the presence of the ${}^4\text{He}$ superfluid Bose background with soft excitation modes (the sound velocity is small), which leads to an additional complication of the ${}^3\text{He}$ quasiparticle interaction because the interaction processes are to a great extent indirect and are accompanied by emission and absorption of (virtual) phonons, resulting in retardation and nonlocal effects even at not very high velocities/energies of the quasiparticles.

For this reason, the majority of authors, following the well-known suggestion of Bardeen, Baym and Pines (1966, 1967) describe the interac-

tion in a model form [see, e.g., Bardeen et al. (1966, 1967), Radebaugh (1968), Ebner and Edwards (1971), Esel'son et al. (1973), Baym and Pethick (1978), Ghozian and Varoquaux (1979)]: one chooses some relatively realistic model form of the ${}^3\text{He}$ quasiparticle interaction potential which depends on the velocities (momenta) of the interacting particles and contains several arbitrary adjustable parameters. The values of these parameters must be chosen by a best fit of the experimental data (e.g. on the solution thermodynamics) with the results of calculations with a given type of model potential. However, the choice of one or other form of model potential cannot be justified microscopically. Nevertheless, for nearly all model potentials with a sufficient number of adjustable parameters one may expect a comparatively good description of the thermodynamic, hydrodynamic and transport phenomena in concentrated solutions. Considerable differences between all models appear in the calculations of the spectra of high-frequency oscillations and of the ${}^3\text{He}$ superfluid transition temperature in solutions. The latter becomes quite evident if one takes into account that the superfluid transition temperature in a Fermi gas depends exponentially on the interaction (see section 6.1).

Though there is no consistent analysis of polarization effects in solutions on the basis of model interaction potentials, it is quite clear that different types of potentials can provide satisfactory expressions for the thermodynamic, hydrodynamic and transport effects in spin-polarized ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions (maybe one will have to include additional fitting parameters due to the presence of an additional thermodynamic variable — the magnetization vector of the solution).

Nevertheless, one can carry out a fairly general semimicroscopic procedure of description of the ${}^3\text{He}$ quasiparticle interaction in solutions, which is valid at not very high ${}^3\text{He}$ concentrations (Bashkin 1977, Bashkin and Meyerovich 1981). As we shall see below, this description possesses two considerable advantages. First, the interaction is taken into account using just a single microscopic constant, which has a clear physical meaning. Second, such a description, in contrast to the method of model potentials, has a well-defined accuracy. This description of the interaction is based on the simple fact that in a wide range of solution parameters the de Broglie wavelength Λ of the quasiparticles is large compared to the atomic dimension a_0 ,

$$\Lambda \gg a_0. \quad (1.4)$$

The wavelength $\Lambda \sim \hbar/p$, where p is the characteristic momentum of the quasiparticles, which is equal to $p \sim (ME)^{1/2}$, with the characteristic energy of the particles $E \sim \max\{T, T_0\}$. Thus, the condition of a large de Broglie

wavelength (1.4) is equal to the following inequality for the temperature and the Fermi energy of the ${}^3\text{He}$ quasiparticles:

$$T, T_0 \leq \hbar^2/Ma_0^2. \quad (1.5)$$

$\hbar^2/Ma_0^2 \geq 1$ K, $T_0 \sim 2.6x^{2/3}$ K, and the conditions (1.4), (1.5) are easily satisfied at sufficiently low temperatures and ${}^3\text{He}$ concentrations $x \leq 1\%$. Sometimes dilute gases, $Ma_0^2 \ll 1$, are called "quantum gases" if conditions (1.4), (1.5) turn out to be valid. Often the term "quantum gases" is used in the case of a more rigid limitation for the wavelength, $\Lambda \geq N^{-1/3} \gg a_0$. In this case quantum effects are mostly due to the quantum degeneracy of the gas and to the corresponding changes of the distribution function. Below we are interested in the less rigid conditions (1.4), (1.5) (with an unspecified relation of Λ to $N^{-1/3}$), when many of the quantum effects are not related to the degenerate form of the distribution function, but to the (ultra)quantum character of the interaction of long-wavelength particles.

The quantum gas of ${}^3\text{He}$ quasiparticles (1.4), (1.5) forms a gas of slow, $pa_0/\hbar \ll 1$, particles. The scattering amplitudes f_l in channels with orbital moment l decrease sharply for slow particles, $f_l \propto (pa_0/\hbar)^{2l}$ when l increases (if the interaction between particles vanishes rapidly enough at large distances). Therefore, for the quantum gas (1.4), (1.5), the interaction reduces mainly to s-wave scattering with scattering amplitude — a independent of the momenta of the interacting particles. As a result, one can try to describe all interaction effects using just a single microscopic constant — the s-wave scattering length a . By taking into account only the s-wave scattering processes one neglects corrections of the order of $(pa_0/\hbar)^2$. In this case one must restrict all calculations to the first and the second order in the interaction. This also means that the contributions of the terms with γ in the spectrum of the bare quasiparticles (1.1) do not lie beyond the accuracy only when the calculations are performed up to second order in the interaction. Besides, the retardation effects are of the order of $(P/Ms_0)^2$ and are not significant in the s-wave scattering approximation for the interaction (Bashkin and Meyerovich 1981).

The analysis of all available experimental data on the thermodynamics and kinetics of nonpolarized solutions has shown [see e.g. Bashkin and Meyerovich (1981)] that $a \sim -1.5$ Å. As an example, fig. 2 gives the dependence of the ${}^3\text{He}$ chemical potential in degenerate solutions, μ_3 , on the ${}^3\text{He}$ concentration x . The dashed line corresponds to the ${}^3\text{He}$ chemical potential in the ideal gas approximation, and the solid curve takes the interaction into account in the s-wave scattering approximation with $a = -1.5$ Å (the normalizing constant l makes $\mu_3 + l = 0$ at the demixing concentration). One can see that the above approximation is good enough

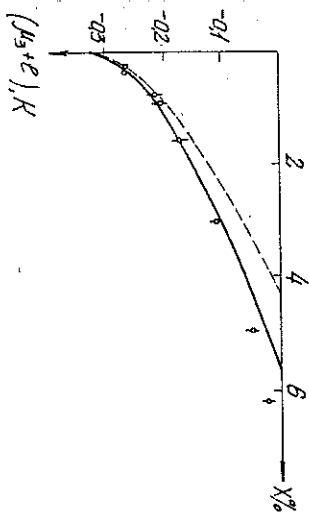


Fig. 2. Concentration dependence of the ${}^3\text{He}$ chemical potential μ_3 for $T \rightarrow 0$, $x = N_3/(N_3 + N_4)$ is the ${}^3\text{He}$ concentration. The dashed curve corresponds to the ideal gas approximation, the solid curve to the s-wave scattering approximation with $a = -1.5$ Å (Bashkin and Meyerovich 1981). The constant l is the latent heat of evaporation of ${}^3\text{He}$, and the condition $\mu_3 + l = 0$ corresponds to the demixing concentration. Circles: experimental results of Seligmann et al. (1969).

for the thermodynamic functions up to concentrations of the order of 3%; an analogous agreement can be seen in the cases of nearly all other thermodynamic and kinetic quantities (Bashkin 1977, Bashkin and Meyerovich 1981). The negative sign of a corresponds to effective attraction between particles in the s-wave channel.

In polarized solutions the quasiparticle interaction can be taken into account using just the same constant a . This is due to the fact that in the nonrelativistic approximation the particle interaction and the scattering amplitude are spin independent. Therefore the dependence of the interaction effects on the polarization in the exchange approximation appears mainly because the polarization changes the distribution functions of the quasiparticles. But in solutions with high spin polarizations this is not exactly so. Due to the Pauli principle, the s-wave scattering of spin-1/2 fermions is effective only for collisions of particles with opposite spins; for this reason, in the case of nearly full polarization, when practically all spins are parallel to each other, the s-wave scattering becomes ineffective, and the main interaction mechanism is associated with the p-wave scattering. Since the ratio of the scattering amplitudes for the p-wave and s-wave channels is small for slow particles as far as $(pa_0/\hbar)^2$ is small, full polarization leads to a decrease of the effective interaction by the factor $\hbar^2/Ma_0^2T_0 \sim x^{-2/3}$ in degenerate solutions, $T \leq T_0$ (x is the ${}^3\text{He}$ concentration in the solution), and by the factor \hbar^2/Ma_0^2T in Boltzmann solutions, $T \gg T_0$.

To a great extent the polarization of quantum gases under the conditions (1.4), (1.5) leads to the same quantum effects independently of the degree of quantum degeneracy of the gas. The role of the form of the distribution

functions at different degrees of degeneracy reduces to an insignificant quantitative difference, which can almost always be eliminated if one replaces the Fermi momentum P_0 by the thermal momentum $P_T = (MT)^{1/2}$. Therefore, in this review we shall discuss the polarization effects, where possible, at an arbitrary degree of quantum degeneracy of the ${}^3\text{He}$ quasiparticle system. Certainly, one can obtain the main analytical quantitative results only in the two limiting cases of completely degenerate and Boltzmann systems. As usual, in the intermediate temperature region one has to use numerical calculations. Unfortunately, many experiments are performed in this intermediate region, thus complicating the comparison between theoretical and experimental data.

2. Macroscopic description of polarized solutions

2.1. EQUATIONS OF SPIN DYNAMICS

One can obtain a macroscopic description of many of the magnetic phenomena in ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions by taking advantage of the fact that the ${}^3\text{He}$ quasiparticle interaction is mainly of the exchange type and conserves the spin, and that the spin-nonconserving nuclear magnetic dipole interaction is extremely weak. In the exchange approximation the equation of motion for the magnetic moment (per unit volume) M takes the simple form of the conservation law of the magnetic moment,

$$\frac{\partial}{\partial t} M + \frac{\partial}{\partial x_k} J_k + \frac{2\beta}{\hbar} [H \times M] = 0, \quad (2.1)$$

and the problem reduces to the evaluation of the spin current J_k . In hydrodynamics the spin current is given by an expansion in the spatial gradients of thermodynamic variables, which are small, and can be represented in the form

$$J_k = M v_k^{(n)} + \hat{D} \frac{\partial}{\partial x_k} \frac{M}{N_3} + A_T \frac{\partial}{\partial x_k} T + A_P \frac{\partial}{\partial x_k} \Pi + A_4 \frac{\partial}{\partial x_k} N_4, \quad (2.2)$$

where $v^{(n)}$ is the normal velocity, Π is the osmotic pressure (the pressure in the ${}^3\text{He}$ quasiparticle gas), and N_3 and N_4 are the numbers of ${}^3\text{He}$ and ${}^4\text{He}$ particles per unit volume. (Sometimes it is more convenient to use, instead of N_4 , the total pressure in the solution \mathcal{P} as an independent thermodynamic variable. This leads to some redefinition of the coefficients A_P and A_4 in eq. (2.2).) Below we are not interested in the effects caused by

changes of the superfluid Bose background density; therefore in this section we shall omit the last term in eq. (2.2) for the sake of simplicity. In a system with a single preferred direction in spin space (the direction of the equilibrium magnetization $e = M/M$), the spin vectors A in eq. (2.2) are always directed along e and can be expressed as

$$A_T = -\beta N_3 D_{ST} e, \quad A_P = -\beta N_3 D_{SP} e, \quad (2.3)$$

while the tensor $\hat{D} = D_{\alpha\beta}$ in the most general form has three independent components proportional to $e_\alpha e_\beta$, the unit tensor $\delta_{\alpha\beta}$ and to $\epsilon_{\alpha\beta\gamma} e_\gamma$ ($\epsilon_{\alpha\beta\gamma}$ is the completely antisymmetric tensor):

$$D_{\alpha\beta} = -\beta N_3 (D_1 \delta_{\alpha\beta} + D_2 e_\alpha e_\beta + D_3 \epsilon_{\alpha\beta\gamma} e_\gamma).$$

Below it will be more useful to introduce new notations K_{ST} , K_{SP} , D_{\parallel} , D_{\perp} and $\Omega_{\text{int}} \tau_{\perp}$ instead of the coefficients D_{ST} , D_{SP} , D_1 , D_2 and D_3 :

$$\begin{aligned} D_{ST} &= D_{\parallel} K_{ST}/T, & D_{SP} &= D_{\parallel} K_{SP}/\Pi, & D_{\parallel} &= D_1 + D_2, \\ D_{\perp} &= D_1 (1 + \Omega_{\text{int}}^2 \tau_{\perp}^2), & \Omega_{\text{int}} \tau_{\perp} &= D_3/D_1, \end{aligned} \quad (2.4)$$

where τ_{\perp} is the diffusion (exchange) relaxation time. The reason for such definitions of the coefficients in the spin current will soon become clear. In the notations (2.4) the spin dynamics equation (2.1) takes the following form (Meyerovich 1983, 1985):

$$\begin{aligned} N_3 \left(\frac{\partial}{\partial t} \mathbf{P} + v_k^{(n)} \frac{\partial}{\partial x_k} \mathbf{P} \right) + \Omega_0 [e \times \mathbf{P}] \\ - \frac{\partial}{\partial x_k} \left\{ \frac{D_{\perp} N_3}{1 + \Omega_{\text{int}}^2 \tau_{\perp}^2} \left(\frac{\partial}{\partial x_k} \mathbf{P} + \Omega_{\text{int}} \tau_{\perp} \left[e \times \frac{\partial}{\partial x_k} \mathbf{P} \right] \right) \right. \\ \left. + \frac{D_{\parallel} (1 + \Omega_{\text{int}}^2 \tau_{\perp}^2) - D_{\perp}}{D_{\perp}} e \left(e \frac{\partial}{\partial x_k} \mathbf{P} \right) \right\} \\ - D_{\parallel} N_3 e \left(\frac{K_{ST}}{T} \frac{\partial}{\partial x_k} T + \frac{K_{SP}}{\Pi} \frac{\partial}{\partial x_k} \Pi \right) = 0, \end{aligned} \quad (2.5)$$

where $\mathbf{P} = M/\beta N_3$ is the polarization vector of the solution, and $\Omega_0 = 2\beta H/\hbar$ is the usual frequency for the precession of the magnetic moment in an external magnetic field H .

We choose as the z -axis the direction of the vector e . Then the linearized equation for the longitudinal component P_z of the polarization,

$$N_3 \left(\frac{\partial}{\partial t} + v_k^{(n)} \frac{\partial}{\partial x_k} \right) P_z - \frac{\partial}{\partial x_k} \left\{ D_{\parallel} N_3 \left(\frac{\partial}{\partial x_k} P_z + \frac{K_{sT}}{T} \frac{\partial}{\partial x_k} T + \frac{K_{sp}}{\Pi} \frac{\partial}{\partial x_k} \Pi \right) \right\} = 0, \quad (2.6)$$

and the linearized equations for the transverse components of the magnetization $M_{\pm} = M_x \pm iM_y$,

$$\frac{\partial}{\partial t} M_{\pm} + i\Omega_0 M_{\pm} - \frac{D_{\perp}}{1 + \Omega_{\text{int}}^2 \tau_{\perp}^2} (1 + i\Omega_{\text{int}} \tau_{\perp}) \frac{\partial^2}{\partial x_k^2} M_{\pm}, \quad (2.7)$$

decouple completely.

The representation of the coefficients in the spin current (2.2) has been chosen in the form of eqs. (2.3), (2.4) in order to emphasize the analogy of the longitudinal equation of motion for P_z , eq. (2.6), with the standard hydrodynamic diffusion equation for binary mixtures [see, e.g., Landau and Lifshitz (1978)]

$$N \left(\frac{\partial C}{\partial t} + v \nabla C \right) - \nabla \left(DN \left(\nabla C + \frac{K_T}{T} \nabla T + \frac{K_P}{\Pi} \nabla \Pi \right) \right) = 0, \quad (2.8)$$

where C is the concentration of the mixture with density N , v is the hydrodynamic velocity, D is the diffusion coefficient, and K_T and K_P are the thermal diffusion and pressure diffusion ratios. In our case (2.6) the longitudinal component of the polarization, $P_z = M_z/\beta N_3$, behaves like the concentration of a dissolved component, D_{\parallel} stands for the usual longitudinal spin diffusion coefficient, and $D_{sT} \equiv D_{\parallel} K_{sT}/T$ and $D_{sp} \equiv D_{\parallel} K_{sp}/\Pi$ can naturally be called the spin thermodiffusion and spin pressure diffusion coefficients (K_{sT} and K_{sp} are the spin thermal diffusion and spin pressure diffusion ratios). The analogy between the longitudinal equation of spin dynamics (2.6) and the diffusion equation for binary mixtures (2.8) is not accidental and corresponds to the conservation of the total spin projection in exchange processes. In the exchange approximation a spin-polarized system of identical spin-1/2 particles behaves (for processes that do not change the direction of the magnetization, $M_{x,y} \equiv 0$) as a binary mixture of particles with spin projection $+1/2$ (the density N_+) and particles with spin projection $-1/2$ (the density N_-), while all exchange interactions

conserve the number of particles in each component. Using the analogy with the diffusion theory, one can easily write the equation of heat transfer in polarized solutions:

$$\frac{\partial T}{\partial t} + \frac{K_{sT}}{C_p} \left(\frac{\partial \mu_p}{\partial P_z} \right)_{\Pi, T} \frac{\partial P_z}{\partial t} = \chi \Delta T,$$

where χ is the thermal diffusivity, C_p is the heat capacity, and μ_p is the chemical potential corresponding to the polarization P . The similarity of longitudinal spin dynamics to diffusion in binary mixtures helps not only to derive macroscopic equations of motion, but also to evaluate the transport coefficients.

Equation (2.7) for the transverse components of the magnetic moment M_{\pm} determines the spin wave spectrum in polarized solutions:

$$\omega = \Omega_0 + \frac{D_{\perp} \Omega_{\text{int}} \tau_{\perp}}{1 + \Omega_{\text{int}}^2 \tau_{\perp}^2} \left(1 - \frac{i}{\Omega_{\text{int}} \tau_{\perp}} \right) k^2. \quad (2.9)$$

The coefficient $D_{\perp} \propto \tau_{\perp}$ determines the transverse spin diffusion in the limit $\Omega_{\text{int}} \tau_{\perp} \rightarrow 0$. The damping of spin waves is small only when $\Omega_{\text{int}} \tau_{\perp} \gg 1$. The possibility of large values of the parameter $\Omega_{\text{int}} \tau_{\perp}$ does not contradict the hydrodynamic condition used in the small-gradient expansion in eqs. (2.1), (2.2): the hydrodynamic condition $\omega - \Omega_0 \equiv \delta\omega \ll 1/\tau$ means only that the frequency $\delta\omega \ll \Omega_{\text{int}}$ when $\Omega_{\text{int}} \tau_{\perp} \gg 1$. It is clear that the appearance of a high "internal" frequency Ω_{int} is absolutely necessary for the existence of weakly damped spin waves. We shall see below that the frequency Ω_{int} determines the precession frequency of the magnetic moment in a molecular field. In general, the possibility of propagation of spin waves with a quadratic spectrum $\omega \propto k^2$ is not surprising for an exchange system with a nonzero total magnetic moment; the spin dynamics equation (2.7) is to a great extent analogous to the Landau-Lifshitz equation for ferromagnets.

The above spin dynamics equations were derived in the exchange approximation. In the case of $^3\text{He} \downarrow - ^4\text{He}$ solutions, the extremely weak magnetic dipole interaction of the ^3He nuclear magnetic moments and the ^3He quasiparticle collisions with the walls are the only spin-nonconserving processes. The nuclear magnetic dipole relaxation is characterized by a very long relaxation time $\tau_d \sim \hbar E/\beta^4 N_3^2$, where E is the characteristic energy of the quasiparticles. (In nondegenerate solutions $E \sim T$, in degenerate solutions $E \sim T_0$ and the expression for τ_d contains the additional large factor $(T_0/T)^2 \gg 1$.) Numerically the quantity $\hbar E/\beta^4 N_3^2$ is of the order of $10^4 E/x^2$ s/K (x is the ^3He concentration). Nevertheless, even such slow

dipole processes sometimes lead to considerable consequences. The influence of the dipole processes on the properties of spin-polarized solutions is to some extent analogous to the influence of very slow chemical reactions on the properties of usual binary mixtures.

Magnetic dipole processes lead to violation of the magnetic moment conservation law (2.1), i.e. to violation of the conservation law for the number of particles with a given spin projection:

$$\begin{aligned} \frac{\partial N_{\pm}}{\partial t} + N_{\pm} \operatorname{div} v^{(\omega)} &= -\frac{N_{\pm} - N_{\pm}^{(0)}}{\tau_d}, \\ \frac{\partial N_3}{\partial t} + N_3 \operatorname{div} v^{(\omega)} &= 0, \end{aligned} \quad (2.10)$$

where $N_{\pm}^{(0)}(H, T, N_3)$ are the local equilibrium values of the spin densities N_{\pm} for particles with spin projections $\pm 1/2$ ($N_+ + N_- = N_+^{(0)} + N_-^{(0)} = N_3$); in eq. (2.10) we omitted for the sake of simplicity the diffusion spin currents described above. The dipole processes tending to level off the chemical potentials μ_{\pm} for up and down spins are the only processes that establish the equilibrium values of the spin densities $N_{\pm}^{(0)}$. Therefore, for all oscillation phenomena the value of the parameter $\omega\tau_d$ (ω is the frequency of the oscillations) becomes very important, though many of the quantities are formally independent of $\omega\tau_d$. At low frequencies $\omega\tau_d \ll 1$ the densities N_{\pm} relax to $N_{\pm}^{(0)}$ during the period of the oscillations, and the oscillations are truly hydrodynamic. In the opposite case of high frequencies $\omega\tau_d \gg 1$ (this case corresponds to usual experimental conditions) there is no such relaxation, and the oscillations take place at a fixed number of particles in each of the spin components. For example, the oscillating increments of the spin densities δN_{\pm} are given in the (second) sound wave, according to eq. (2.10), by the relations

$$\delta N_{\pm} = -\frac{N_{\pm}^{(0)}}{N_3} \delta N_3 + \frac{1}{\omega\tau_d} (\delta N_{\pm} - \delta N_{\pm}^{(0)}), \quad (2.11)$$

which can be reduced to

$$\frac{\partial N_{\pm}}{\partial N_3} = \frac{1}{1 + i/\omega\tau_d} \left(\frac{N_{\pm}^{(0)}}{N_3} + \frac{i}{\omega\tau_d} \frac{\partial N_{\pm}^{(0)}}{\partial N_3} \right).$$

As a result, the second-sound velocity $s_2^2 = (1/M) \partial \Pi / \partial N_3$ (the second sound in ${}^3\text{He}$ - ${}^4\text{He}$ solutions at low temperatures is practically the sound in

the ${}^3\text{He}$ impurity quasiparticle gas; Π is the osmotic pressure, i.e. the pressure in the ${}^3\text{He}$ quasiparticle gas) depends on the frequency [cf. Landau and Lifshitz (1978)]:

$$s_2^2 = \frac{1}{M} \left(\frac{\partial \Pi}{\partial N_+} \frac{\partial N_+}{\partial N_3} + \frac{\partial \Pi}{\partial N_-} \frac{\partial N_-}{\partial N_3} \right) = \frac{i\omega\tau_d s_2^{(\infty)^2} + s_2^{(0)^2}}{1 + i\omega\tau_d}, \quad (2.12)$$

where $s_2^{(0)}$ is the second sound velocity in the limit $\omega\tau_d \rightarrow 0$ when $\partial N_{\pm} / \partial N_3 = \partial N_{\pm}^{(0)} / \partial N_3$, and $s_2^{(\infty)}$ is the sound velocity in the high-frequency limit $\omega\tau_d \gg 1$ when $\partial N_{\pm} / \partial N_3 = N_{\pm}^{(0)} / N_3$. This frequency dispersion of the oscillations leads to damping of the sound waves:

$$k = \left[\frac{1 + \omega^2\tau_d^2 p^2 + i(p^2 - 1)\omega\tau_d}{1 + \omega^2\tau_d^2 p^4} \right]^{1/2} \frac{\omega}{s_2^{(0)}}, \quad p = \frac{s_2^{(\infty)}}{s_2^{(0)}} \quad (2.13)$$

(k is the wave vector), and gives rise to the second (bulk) viscosity (Meyerovich 1982)

$$\xi = \xi_0 / (1 - i\omega\tau_d), \quad \xi_0 = MN_3\tau_d (s_2^{(\infty)^2} - s_2^{(0)^2}). \quad (2.14)$$

Of course, when the polarization goes to zero, $N_{\pm}/N_3, N_{\pm}^{(0)}/N_3 \rightarrow 1/2$, then the frequency dispersion and the spin second viscosity vanish.

The spin second viscosity dominates over the usual dissipation mechanisms of sound absorption only in the low-frequency case $\omega^2\tau_d \ll 1$ (τ is the usual exchange relaxation time determined by the ${}^3\text{He}$ quasiparticle collisions, $\tau \ll \tau_d$). Note, that the expression for the sound velocity $s_2^2 = (1/M) \partial \Pi / \partial N_3$ is exact only for the quadratic dispersion law of the bare quasiparticles, eq. (1.1) with $\gamma = 0$. The above results for the second-sound frequency dispersion and the spin second viscosity in spin-polarized solutions are analogous to the well-known Mandelstam-Leontovich effect in hydrodynamics (Landau and Lifshitz 1978).

2.2. FERMI LIQUID DESCRIPTION OF DEGENERATE SOLUTIONS

For a degenerate ${}^3\text{He}$ quasiparticle system in superfluid He II one has an additional macroscopic description in the framework of the Landau theory of Fermi liquids [see, e.g., Khalatnikov (1971)]. This description includes interactions and can be used for degenerate spin-polarized ${}^3\text{He}$ - ${}^4\text{He}$ solutions.

The Fermi liquid function of an arbitrary exchange polarized system of

spin-1/2 particles in the most general form can always be linearized in the spin operators of the interacting particles:

$$f(\mathbf{p}, \hat{\sigma}; \mathbf{p}', \hat{\sigma}') = \psi(\mathbf{p}, \mathbf{p}') f f' + \xi(\mathbf{p}, \mathbf{p}') \hat{\sigma} \hat{\sigma}' \\ + e \{ \varphi(\mathbf{p}, \mathbf{p}') \hat{\sigma} f' + \varphi(\mathbf{p}', \mathbf{p}) \hat{\sigma}' f \} \\ + \xi(\mathbf{p}, \mathbf{p}') (\hat{\sigma} e)(\hat{\sigma}' e), \quad (2.15)$$

where $\hat{\sigma}$ are the Pauli matrices, $f \equiv \delta_{\alpha\beta}$ is the unit operator, and $e = M/M$ is the unit vector in the direction of the spin polarization. The energy of the quasiparticles in the Fermi liquid is a linear function of the spin operators:

$$\hat{\epsilon} = \epsilon_0(\mathbf{p}) f - B(\mathbf{p}) \hat{\sigma} e + \delta \hat{\epsilon}, \\ \delta \hat{\epsilon} = -\beta \hat{\sigma} \delta \mathbf{H} + \text{Sp}_{\sigma'} \int f(\mathbf{p}, \hat{\sigma}; \mathbf{p}', \hat{\sigma}') \delta \hat{n}_{\sigma'}(\mathbf{p}') dT', \quad (2.16)$$

where $dT = d^3p/(2\pi\hbar)^3$, and $\delta \hat{n}$ is the deviation of the single-particle density matrix (the distribution function) for the quasiparticles of the Fermi liquid from its equilibrium value $\hat{n}^{(0)}$.

$$\hat{n}^{(0)} = \frac{1}{2} n(\mathbf{p}) \hat{f} + \frac{1}{2} m(\mathbf{p}) \hat{\sigma}. \quad (2.17)$$

The functions $n(\mathbf{p}) = \text{Sp}_{\sigma} \hat{n}^{(0)}$ and $m(\mathbf{p}) = \text{Sp}_{\sigma} \hat{\sigma} \hat{n}^{(0)}$ determine the equilibrium distributions of the particle density and the magnetic moment of the solution. The diagonal components n_{\pm} of the spin density matrix determine the equilibrium distribution functions for quasiparticles with spin projections $\pm 1/2$ on the axis e and are given by the usual Fermi distribution functions,

$$n_{\pm} = \frac{1}{2} \left(1 - \tanh \frac{\epsilon_{\pm} - \mu_{\pm}}{2T} \right), \quad n = n_+ + n_-, \quad m = e(n_+ - n_-), \quad (2.18)$$

where $\epsilon_{\pm} = \epsilon_0 \mp B$ are the diagonal components of the quasiparticle energy, and the chemical potentials μ_{\pm} for up and down spins are equal to each other in the case of equilibrium polarization determined by an external magnetic field. According to Abrikosov and Dzyaloshinsky (1958), the function $B(\mathbf{p})$ in the energy (2.16) can be expressed in the component ξ of

the Fermi liquid function (2.15) using the transformation rule for the energy of Fermi liquid excitations at infinitesimal rotations:

$$B(\mathbf{p}) = \beta \mathbf{H} - \int \xi(\mathbf{p}, \mathbf{p}') [n_+(\mathbf{p}') - n_-(\mathbf{p}')] dT'. \quad (2.19)$$

Most of the final results must include the harmonics in the expansion of the Fermi liquid function in Legendre polynomials on the Fermi surfaces of radii $P_{\pm} = \hbar(6\pi^2 N_{\pm})^{1/3}$. Let us denote the corresponding harmonics as

$$\Psi_{ab}^{(n)} = (n + \frac{1}{2}) \frac{P_a m_b}{2\pi^2 \hbar^3} \int \psi(\mathbf{p}_a, \mathbf{p}'_b) P_n(\cos \chi) d \cos \chi, \quad (2.20)$$

and similarly $Z_{ab}^{(n)}$, $\Phi_{ab}^{(n)}$ and $\Xi_{ab}^{(n)}$ for the functions ξ , φ and ξ . Here the indices (a, b) take the values (+) or (-), P_n are Legendre polynomials, χ is the angle between the vectors \mathbf{p}_a and \mathbf{p}_b , and m_{\pm} denote the effective masses of quasiparticles on the corresponding Fermi surfaces. Many results can be expressed through the following four linear combinations of Fermi liquid harmonics:

$$A_{\pm}^{(n)} = \Psi_{\pm\pm}^{(n)} + Z_{\pm\pm}^{(n)} + 2\Phi_{\pm\pm}^{(n)} + \Xi_{\pm\pm}^{(n)} + 1, \\ B_{\pm}^{(n)} = \Psi_{\pm\pm}^{(n)} - Z_{\pm\pm}^{(n)} \pm \Phi_{\pm\pm}^{(n)} \mp \Xi_{\pm\pm}^{(n)}. \quad (2.21)$$

If we are interested in phenomena without changes of the direction of magnetization, then in the exchange approximation a polarized Fermi liquid of spin-1/2 particles is equal to a binary system of components corresponding to systems of particles with spin projections $\pm 1/2$. For such phenomena all spin operators remain diagonal, and all calculations must be carried out exactly as for other two-component Fermi liquids [see, e.g., Bashkin and Meyerovich (1981), Oliva and Ashcroft (1981), Meyerovich (1983), Bedell (1985)], while the Fermi liquid function enters all final results only in the combinations A , B [eq. (2.21)]. For example, the effective masses m_{\pm} of Fermi liquid quasiparticles are equal to

$$\frac{m_{\pm}}{M_{\pm}} = \frac{A_{\pm}^{(0)} A_{\pm}^{(0)} - B_{\pm}^{(0)} B_{\pm}^{(0)}}{A_{\pm}^{(0)} - B_{\pm}^{(0)} P_{\pm}/P_{\pm}}, \quad (2.22)$$

where the effective masses of the bare quasiparticles $M_{\pm}^{-1} = (\partial \epsilon_{\pm} / \partial p_{\pm}) / p_{\pm}$ are determined with the help of the spectrum (1.1):

$$M_{\pm} = M / (1 - 2\gamma p_{\pm}^2 / p_{\pm}^2).$$

A density change of the spin-up or spin-down component δN_{\pm} causes

changes of the chemical potentials $\delta\mu_{\pm}$ and the osmotic pressure $\delta\Pi$ of ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions:

$$\begin{aligned}\delta\mu_{\pm} &= \frac{2\pi^2\hbar^3}{m_{\pm}p_{\pm}} A_{\pm}^{(0)} \delta N_{\pm} + \frac{2\pi^2\hbar^3}{m_{\mp}p_{\mp}} B_{\pm}^{(0)} \delta N_{\mp}, \\ \delta\Pi &= 2\pi^2\hbar^3 \left\{ \frac{\delta N_{+}}{m_{+}p_{+}} (N_{+}A_{+}^{(0)} + N_{-}B_{-}^{(0)}) + \frac{\delta N_{-}}{m_{-}p_{-}} (N_{-}A_{-}^{(0)} + N_{+}B_{+}^{(0)}) \right\}.\end{aligned}\quad (2.23)$$

These expressions allow one to determine the second-sound velocities (2.12):

$$\begin{aligned}s_2^{(0)2} &= \frac{(N_3/M)(A_{+}^{(0)}A_{-}^{(0)} - B_{+}^{(0)}B_{-}^{(0)})}{p_{-}(A_{+}^{(0)} - B_{-}^{(0)}) + p_{+}(A_{-}^{(0)} - B_{+}^{(0)})}, \quad v_{\pm} = \frac{p_{\pm}m_{\pm}}{2\pi^2\hbar^3}, \\ s_2^{(\infty)2} &= \frac{1}{MN_3} \left\{ \frac{1}{p_{+}} N_{+}^2 A_{+}^{(0)} + \frac{1}{p_{-}} N_{-}^2 A_{-}^{(0)} + N_{+}N_{-} \left(\frac{1}{p_{+}} B_{-}^{(0)} + \frac{1}{p_{-}} B_{+}^{(0)} \right) \right\}.\end{aligned}\quad (2.24)$$

(Equation (2.12) is valid only for the quadratic spectrum (1.1) with $\gamma = 0$; if $\gamma \neq 0$ the sound velocity s_2^2 differs from $(1/M) \partial\Pi/\partial N_3$ by the factor $\{(M_{+}/M)(\partial N_{+}/\partial N_3) + (M_{-}/M)(\partial N_{-}/\partial N_3)\}^{-1}$ with which eq. (2.12) and, consequently, eq. (2.24) should then be multiplied.

The transport coefficients of the spin-polarized Fermi liquid can be expressed by phenomenological scattering probabilities for quasiparticle pair collisions using the method developed by Abrikosov and Khalatnikov (1957) for nonpolarized Fermi liquids. For polarized systems one has to use the generalization of this method to the case of multicomponent Fermi liquids. The corresponding, rather cumbersome, formulae can be found, for example, in Meyerovich (1983).

In spin-polarized or multicomponent Fermi liquids one may be confronted with an additional peculiarity pointed out in some detail by Troian and Mermn (1985). The quasiparticle energy $\hat{\epsilon}$ appearing in the local equilibrium distribution function $\hat{n}^{(0)}(\hat{\epsilon})$ is determined by the nonequilibrium distribution function (2.15)–(2.18):

$$\begin{aligned}\epsilon_{\pm} &= \epsilon_0 \mp B + \int dT' \{ \delta n_{\pm}(\psi + \xi \pm \varphi \pm \varphi' + \xi) \\ &+ \delta n_{\mp}(\psi - \xi \pm \varphi \mp \varphi' - \xi) \}.\end{aligned}\quad (2.25)$$

For a nonpolarized system, i.e. for a single-component Fermi liquid, this fact is not very critical because this rescaling of the energy takes place in both (left- and right-hand) sides of the kinetic equation, and the corresponding rescaling factors usually cancel. For a spin-polarized Fermi liquid these rescaling functions (2.25) form the matrix A_{\pm} , B_{\pm} [eq. (2.21)] with more complicated rules of cancellation, and may enter the final expressions. As a result one has to be somewhat more careful with the kinetic equation for a spin-polarized or a multicomponent Fermi liquid.

The transverse spin dynamics (processes accompanied by a change in the direction of the magnetic moment) is more complicated for a spin-polarized Fermi liquid than the longitudinal dynamics. These processes correspond to the dynamics of the nondiagonal elements of the single-particle density matrix \hat{n} . The nondiagonal elements of the density matrix do not represent the well-defined Fermi liquid quasiparticles with definite spin projections and with long lifetimes, remaining near the Fermi surfaces. This can lead to a large damping of the nondiagonal elements (to the absence of the large factor $(T_0/T)^2 \gg 1$ in the lifetime) and to the appearance of the forbidden integration deep into the Fermi spheres [integrals like in eq. (2.19)].

Besides, in the case of transverse spin dynamics, e.g. in the case of spin waves, there are also some additional problems caused by the appearance of gradient (nonlocal) terms of the following type [see, e.g. Meyerovich (1983)]:

$$\delta\hat{\epsilon} = \text{Sp}_{\sigma} \int f^{(0)} \frac{\partial^2}{\partial x_k^2} \delta\hat{n}_{\sigma}(\mathbf{p}') dT',$$

These terms make a considerable contribution to the spectrum of spin waves. Up to now there is no consistent procedure to overcome these difficulties for a dense spin-polarized Fermi liquid. Some additional comments on the justification of the Landau theory for polarized Fermi systems can be found in a recent paper by Quader and Bedell (1985). Certainly, all these difficulties disappear for weakly polarized Fermi liquids when the relative shift of the Fermi spheres is small, $(p_{+} - p_{-}) \ll p_0$.

A detailed description of the transverse spin dynamics of weakly spin-polarized dense Fermi liquids has been given by Leggett (1970). On the basis of a direct integration of the kinetic equation Leggett has derived the macroscopic equation of the spin dynamics of weakly polarized Fermi liquids (an equation like eqs. (2.5), (2.7) without spin thermal diffusion and pressure diffusion terms) and has shown that the characteristic internal frequency Ω_{int} can be expressed through the first two harmonics of the

Fermi liquid function,

$$\Omega_{\text{int}} = \frac{(Z_0 - Z_1/3)\Omega_0}{(1 + Z_1/3)(1 + Z_0)}. \quad (2.26)$$

(For weakly polarized Fermi liquids the functions φ , ξ in the f -function (2.15) are equal to zero, and the quantities $Z_{0,1}$, being the first two harmonics of the function ξ , are equal to their values in the absence of polarization). Apart from this, in the case of weak polarization $\tau_{\parallel} = \tau_{\perp}$, $D_{\parallel} = D_{\perp} = v_0^2(1 + Z_0)/3$, and the spectrum (2.9) of the spin waves has the form (Leggett and Rice 1968a,b, Leggett 1970)

$$\omega = \Omega_0 + \frac{v_0^2 k^2}{3\Omega_0} \frac{(1 + Z_0)^2(1 + Z_1/3)}{(Z_0 - Z_1/3)(1 + 1/\Omega_{\text{int}}\tau_{\perp}^2)} \left(1 + \frac{1}{\Omega_{\text{int}}\tau_{\perp}}\right). \quad (2.27)$$

Often the spin waves (2.27) in spin-polarized Fermi liquids are called Stijn waves (the existence of spin waves with a quadratic spectrum in weakly magnetized Fermi liquids was predicted by Silin (1957); see also Abrikosov and Dzyaloshinsky (1958), Platzman and Wolf (1973)). The frequency Ω_{int} (2.26) is the frequency of the precession of the magnetic moment in the molecular field originating from the Fermi liquid interaction. If the spin polarization is caused not by an external magnetic field, then instead of the frequency $\Omega_0 = 2\beta H/\hbar$ in eq. (2.26) one must substitute $(2\pi\hbar^2/M\rho_0)(N_+ - N_-)/(1 + Z_0)$. Often the notation μM is used [following Leggett and Rice (1968a,b), Leggett (1970)] for the above main parameter of the transverse spin dynamics $\Omega_{\text{int}}\tau_{\perp}$; for many cases our notation $\Omega_{\text{int}}\tau_{\perp}$ seems to be more convenient.

The transverse spin dynamics of $^3\text{He}\downarrow - ^4\text{He}$ solutions has been studied by Owers-Bradley et al. (1984a) at ^3He concentrations of 5% (pressures 0, 10 and 25 bar) and 9.5% (pressure 10 bar) in the temperature range from 25 down to 0.3 mK. The spin polarization was caused by an external magnetic field of 280 Oe ($\Omega_0 = 925$ kHz). The experiments were performed with the continuous NMR technique. It was found out that the NMR line practically never has a complicated structure. The NMR linewidth and the position of the maximum were temperature dependent. The linewidth increased with the temperature decreasing from 25 mK, reached a maximum at $\Omega_{\text{int}}\tau = 1$ and then decreased. At a ^3He concentration of 5% the maximum linewidth was reached at temperatures $T = 2.15 \pm 0.1$ mK (zero pressure), 1.90 ± 0.1 mK (10 bar) and 1.75 ± 0.1 mK (25 bar). At zero pressure $\tau T^2 = 2.8 \times 10^{-11}$ s K² (Owers-Bradley et al. 1984a, Corruccini et al. 1972), and the condition $\Omega_{\text{int}}\tau = 1$ leads to the following value of the

parameter $\lambda = \Omega_{\text{int}}/\Omega_0$ (2.26): $\lambda = -0.024 \pm 0.002$ (the negative sign of λ is determined by the sign of the NMR frequency shift with respect to Ω_0). This suggests for $Z_0 = 0.08 \pm 0.03$ that the value of the unknown Landau parameter Z_1 is $Z_1 = 0.33 \pm 0.1$. Owers-Bradley et al. (1984a) treated their experimental data in a somewhat different manner. The position of the absorption peak (the NMR frequency shift) and the linewidth were calculated with the help of eq. (2.7) assuming that spin waves with only a single value of the wave vector k are excited. Comparison with the experimental values of the frequency shift and the linewidth resulted in the values $k \sim 1$ mm⁻¹ and $\lambda = -0.028 \pm 0.003$, which leads to the Landau parameter $Z_1 = 0.34 \pm 0.1$, though it is not quite clear why the system would pick out a single oscillation mode: the experimental cell used by Owers-Bradley et al. (1984a) had a rather complicated geometry and did not represent a good resonator.

The equilibrium time for the magnetization — the nuclear magnetic dipole relaxation time — in nonpolarized (and weakly polarized) Fermi liquids is of the order of (Vollhardt and Wölfle 1981a,b) $\tau_d^{-1} \sim (\beta^2 N_s)^2 (T/T_0)^{-2} (v/\hbar T_0)/(1 + Z_0)^3$ at low temperatures (v is a constant of the order of 10–100). The time τ_d reduces in the case of strongly nonequilibrium polarization due to the absence of the factor $(T_0/T)^2$. The depolarization times have not been studied in detail for Fermi liquids with high degrees of spin polarization and for the relaxation processes of quasiparticle collisions with the walls.

3. Dilute spin-polarized solutions. Thermodynamics

In section 1.3 we suggested a method for the description of the quasiparticle interaction in dilute quantum solutions [eqs. (1.4), (1.5)]. Applying this method one has to determine only a single interaction parameter — the s -wave scattering length a . Below we shall restrict ourselves for the sake of simplicity to the first (linear in a) order in the interaction. This corresponds to the first order of perturbation theory and to zero order in the concentration/temperature expansion. Some results up to second order can be found in Bashkin and Meyerovich (1981).

In the first order in the interaction for the s -wave scattering with the amplitude — a independent of the momenta, the interaction correction to the (free) energy of the ^3He quasiparticle gas is given by the quadratic form of the single-particle distribution functions $\hat{n}_\sigma(\mathbf{p})$ of the ideal noninteracting gas,

$$F_1 = \frac{\pi a \hbar^2}{M} \sum_{\mathbf{p}, \mathbf{p}'} \text{Sp}_\sigma \text{Sp}_{\sigma'} \hat{n}_\sigma(\mathbf{p}) \hat{n}_{\sigma'}(\mathbf{p}') [I I' - \sigma \sigma']. \quad (3.1)$$

Direct integration of eq. (3.1) with the distribution functions (2.17), (2.18) results in the following expression for F_1 at an arbitrary degree of quantum degeneracy of the ^3He quasiparticle gas:

$$F_1 = \frac{4\pi a\hbar^2}{M} N_+ N_- \quad (3.2)$$

The fact that the interaction correction F_1 (3.2) is proportional to the product $N_+ N_-$ of the densities N_+ and N_- of the quasiparticles with different spin projections, indicates that, due to the Pauli principle, only the collisions of quasiparticles with antiparallel spins are effective in the s -wave scattering of spin-1/2 fermions. Note, that the use of the quadratic term with γ in the spectrum (1.1) often lies beyond the accuracy of the calculations in the first (lowest) order in the s -wave interaction. As a result, the free energy of the degenerate solution is equal to

$$F = F_4^{(0)} - N_3 \Delta + \frac{3}{10M} (p_+^2 N_+ + p_-^2 N_-) - \beta H (N_+ - N_-) + \frac{4\pi a\hbar^2}{M} N_+ N_- \quad (3.3)$$

and for the nondegenerate (Boltzmann) solution

$$F = F_4^{(0)} - N_3 \Delta - N_+ T \ln \left[\frac{e}{N_+} \left(\frac{MT}{2\pi\hbar^2} \right)^{3/2} \right] - N_- T \ln \left[\frac{e}{N_-} \left(\frac{MT}{2\pi\hbar^2} \right)^{3/2} \right] + \frac{4\pi a\hbar^2}{M} N_+ N_- \quad (3.4)$$

where $F_4^{(0)}$ is the energy of pure He II for $T \rightarrow 0$.

The energy of the single-particle excitations of the interacting quantum gas (the single-particle energy (1.1) with interaction corrections) is equal to

$$\hat{\epsilon} = \hat{\epsilon}_0 + \delta\hat{\epsilon},$$

$$\epsilon_0 = \left(-\Delta + \frac{p^2}{2M} \right) f - \beta\delta H + \frac{2\pi a\hbar^2}{M} N_3 \left(f - \frac{N_+ - N_-}{N_3} \delta e \right), \quad (3.5)$$

$$\delta\hat{\epsilon} = \text{Sp}_\sigma \int d\Gamma' \delta\hat{h}_\sigma(p') (i\Gamma' - \delta\delta') \frac{2\pi a\hbar^2}{M},$$

at an arbitrary degree of quantum degeneracy. The term $\delta\hat{\epsilon}$ in eq. (3.5)

represents the change of the single-particle energy with the change of the ^3He quasiparticle distribution functions, i.e. the molecular field. The existence of this term demands an explanation. A degenerate solution is a low-density Fermi liquid of ^3He quasiparticles, and $\delta\hat{\epsilon}$ (3.5) is the usual Fermi liquid term (2.16) with the f -function (Bashkin and Meyerovich 1981)

$$f(p, \delta; p', \delta') = \frac{2\pi a\hbar^2}{M} (i\Gamma' - \delta\delta') \quad (3.6)$$

In this case the presence of the term $\delta\hat{\epsilon}$ (3.5) is beyond doubt. For nondegenerate systems the introduction of a molecular field like $\delta\hat{\epsilon}$ (3.5) is usually beyond the accuracy, but in quantum gases (1.4), (1.5) the molecular field $\delta\hat{\epsilon}$ (3.5) can be taken into account even for nondegenerate dilute Boltzmann gases. The reason is that the molecular field is linear in the interaction (linear in a), while all incoherent and nonlocal collision corrections are quadratic in a (the scattering probabilities in the collision integral are quadratic in the scattering amplitude). Therefore, all collision corrections, forbidding the application of the molecular-field approach in most cases, turn out to be small for quantum gases (1.4), (1.5) in comparison with $\delta\hat{\epsilon}$ (3.5).

All thermodynamic characteristics of polarized quantum solutions (1.4), (1.5) can easily be derived using the above expression for the free energy (3.2)-(3.4) and the single-particle energy (3.5) in the s -wave interaction limit. Apart from this, for degenerate solutions the expression for the Fermi liquid function (3.6) permits one to use directly the equations of the preceding section (note that only the harmonics $Y^{(0)}$ and $Z^{(0)}$ (2.20), (2.21) differ from zero for the f -function (3.6)). For example, the equilibrium spin polarization of degenerate solutions in an external magnetic field is determined at $T=0$ by the minimization of the energy (3.3) with respect to N_\pm with $N_+ + N_- = N_3 = \text{const.}$:

$$P = \frac{N_+ - N_-}{N_3} = \frac{N_+^{(0)} - N_-^{(0)}}{N_3} \left[1 + 4 \left(\frac{6}{\pi} \right)^{1/3} \frac{N_+^{(0)1/3} N_-^{(0)1/3}}{N_+^{(0)1/3} + N_-^{(0)1/3}} \right] \quad (3.7)$$

where $N_\pm^{(0)}$ being the densities of up and down spins in the ideal noninteracting gas, are determined by the following equations:

$$\left(\frac{N_+^{(0)}}{N_3} \right)^{2/3} - \left(\frac{N_-^{(0)}}{N_3} \right)^{2/3} = h_F,$$

$$\frac{N_+^{(0)} + N_-^{(0)}}{N_3} = 1, \quad h_F = 2\beta H / 2^{2/3} T_0. \quad (3.8)$$

$(2^2/3)T_0$ is the Fermi energy of the fully polarized gas, and the condition $h_F = 1$ gives the value of the field necessary for full polarization of the gas at $T = 0$.) Figure 1 displays the dependence of the polarization $P(h)$, eq. (3.8), on the magnetic field H for an ideal gas of ^3He quasiparticles at $T = 0$. Analogous curves for finite temperatures $T \neq 0$ can be found in Mullin and Miyake (1983) and Greywall and Paalonen (1981, 1982), and for a weakly interacting gas in the s -wave scattering limit (3.7) in Owers-Bradley et al. (1984b).

Usually the influence of spin polarization on the thermodynamic characteristics of a solution is insignificant and reduces to a change of the high-order terms of the thermodynamic functions. However, two quantities are very sensitive to spin polarization. Strong polarization dependences can be observed for the osmotic pressure (the so-called magneto-osmotic effect) and for the second-sound velocity (Bashkin and Meyerovich 1978, 1981). The osmotic pressure is given as the pressure difference between two parts of a cell separated by a membrane with a superleak permeable for the ^4He superfluid component only, while the concentrations of the ^3He impurity (normal) component $N_{\pm}^{(0),(2)}$ are different in the two parts of the cell. The value of the osmotic pressure Π can be easily evaluated for $T \rightarrow 0$ using the condition of equality of the ^4He chemical potentials on the two sides of the membrane,

$$\mu_4(\mathcal{P}, N_+^{(1)}, N_-^{(1)}) = \mu_4(\mathcal{P} - \Pi, N_+^{(2)}, N_-^{(2)}),$$

where \mathcal{P} is the overall pressure in the solution. This equality together with the thermodynamic identity for the pressure at $T = 0$,

$$\mathcal{P} = N_+ d\mu_+ + N_- d\mu_- + N_4 d\mu_4,$$

leads to the following expression for $\delta\Pi$:

$$\begin{aligned} \delta\Pi = & \left\{ N_+ \left(\frac{\partial\mu_+}{\partial N_+} \right)_{N_-} + N_- \left(\frac{\partial\mu_-}{\partial N_+} \right)_{N_+} \right\} \delta N_+ \\ & + \left\{ N_- \left(\frac{\partial\mu_-}{\partial N_-} \right)_{N_+} + N_+ \left(\frac{\partial\mu_+}{\partial N_-} \right)_{N_+} \right\} \delta N_-. \end{aligned} \quad (3.9)$$

In accordance with eq. (3.3) the chemical potentials μ_{\pm} are equal to

$$\mu_{\pm} = \left(\frac{\partial E}{\partial N_{\pm}} \right)_{N_{\mp}} = -\Delta \mp \beta H + \frac{p_{\pm}}{2M} + \frac{4\pi a \hbar^2}{M} N_{\mp}, \quad (3.10)$$

and direct integration of eq. (3.9) results in

$$\Pi(N_+^{(1)}, N_-^{(1)}, N_+^{(2)}, N_-^{(2)}) = \Pi_0(N_+^{(1)}, N_-^{(1)}) - \Pi_0(N_+^{(2)}, N_-^{(2)}), \quad (3.11)$$

$$\Pi_0(N_+, N_-) = \frac{(6\pi^2)^{2/3} \hbar^2}{5M} [N_+^{5/3} + N_-^{5/3}] + \frac{4\pi a \hbar^2}{M} N_+ N_-.$$

Expression (3.11) for Π_0 actually determines the pressure in the ^3He impurity quasiparticle gas. If the spin polarization is determined by a magnetic field, the spin densities N_{\pm} in the square brackets in the lower of eqs. (3.11) can be expressed by the spin densities of a polarized ideal gas $N_{\pm}^{(0)}$ using eq. (3.7). As a result the dependence of the osmotic pressure on the polarization takes the following form in a degenerate solution:

$$\frac{\Pi_0(P)}{\Pi_0(0)} = 2^{2/3} \frac{N_+^{5/3} + N_-^{5/3}}{N_3^{5/3}} \left\{ 1 + \frac{20\pi a N_3^{1/3}}{(6\pi^2)^{2/3}} \left[\frac{N_+ N_- N_3^{-1/3}}{N_+^{5/3} + N_-^{5/3}} - 2^{-4/3} \right] \right\}. \quad (3.12)$$

The polarization dependence of $\Pi_0(P)/\Pi_0(0)$ is shown in fig. 3 in the ideal gas approximation (without interaction corrections). Some numerical results for $\Pi_0(P)$ including the interaction terms at nonzero temperatures can be found in Owers-Bradley et al. (1984b).

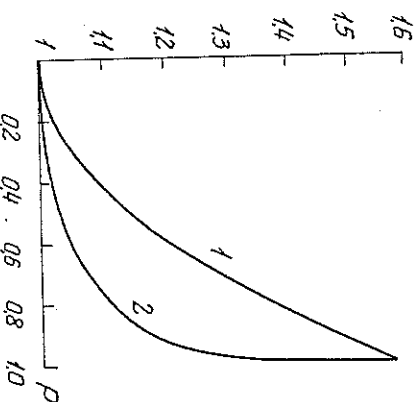


Fig. 3. Polarization dependence of the relative change of the osmotic pressure $\Pi(P)/\Pi(0)$, eq. (3.12) (curve 1), and of the second-sound velocities $s_3^{(0)2}(P)/s_3^2(0)$, eq. (3.14) (curve 1) and $s_3^{(0)2}(P)/s_3^2(0)$, eq. (3.13) (curve 2) in dilute degenerate solutions.

The pressure in a dilute gas is determined by the characteristic momentum of the particles. In a degenerate gas the particle momenta p_{\pm} depend on the polarization, leading to a considerable polarization dependence of the pressure (3.12). In a nondegenerate gas the momentum of the particles is determined by the temperature and practically does not depend on the polarization. Therefore, the magneto-osmotic effect, being strong in a degenerate solution [eq. (3.12)], decreases with decreasing degree of degeneracy, and in the limiting case of Boltzmann solutions the osmotic pressure depends on the spin polarization due to the presence of the small interaction corrections exclusively.

A successful observation of the magneto-osmotic effect has been made by Gully and Schmiedeshoff (1983). In this experiment the osmotic pressure has been measured for a solution with a ^3He concentration of 1.6×10^{-4} ($T_0 \sim 8$ mK) in the temperature range between 50 and 5 mK. The magnetic field of 90 kOe ($2\beta H \sim 14$ mK) made it possible to achieve a spin polarization of the order of 70% at the lowest temperatures. Unfortunately, the detailed results of Gully and Schmiedeshoff (1983) are not available to the present author; it has only been reported that the results of the measurements are in good agreement with a simple gas model.

The other significant effect in thermodynamics is the dependence of the second-sound velocity in a solution (the sound velocity in the ^3He impurity quasiparticle system), $s_2^2 = (1/M) \partial \Pi_0 / \partial N_3$, on the polarization (Bashkin and Meyerovich 1978, 1981, Meyerovich 1982, Greywall and Paalanen 1981, 1982). The expressions (2.24) lead for the second-sound velocity in degenerate solutions with the f -function (3.6) to the following value in the low-frequency (hydrodynamic) limit:

$$s_2^2(\omega \rightarrow 0) = \frac{(6\pi^2)^{2/3} \hbar^2}{3M^2} \frac{N_3^{1/3}}{N_+^{1/3} + N_-^{1/3}} \times \left(1 + 4 \left(\frac{6}{\pi} \right)^{1/3} \frac{a N_+^{1/3} N_-^{1/3}}{N_+^{1/3} + N_-^{1/3}} \right), \quad (3.13)$$

and in the high-frequency limit $\omega \tau_d \gg 1$ to

$$s_2^2(\omega \tau_d \rightarrow \infty) = \frac{(6\pi^2)^{2/3} \hbar^2}{3M^2} \frac{N_+^{5/3} + N_-^{5/3}}{N_3} \times \left(1 + 4 \left(\frac{6}{\pi} \right)^{1/3} \frac{N_+ N_-}{N_+^{5/3} + N_-^{5/3}} \right). \quad (3.14)$$

The relative changes of the second-sound velocities (3.13), (3.14),

$$\frac{s_2^2(P; \omega \rightarrow 0)}{s_2^2(P=0)} = \frac{2^{2/3} N_3^{1/3}}{N_+^{1/3} + N_-^{1/3}} \times \left\{ 1 + 4 \left(\frac{6}{\pi} \right)^{1/3} a N_3^{1/3} \left(\frac{N_+^{1/3} N_-^{1/3} N_3^{-1/3}}{N_+^{1/3} + N_-^{1/3}} - 2^{-4/3} \right) \right\},$$

$$\frac{s_2^2(P; \omega \rightarrow \infty)}{s_2^2(P=0)} = \frac{2^{2/3} (N_+^{5/3} + N_-^{5/3})}{N_3^{5/3}} \times \left\{ 1 + 4 \left(\frac{6}{\pi} \right)^{1/3} a N_3^{1/3} \left(\frac{N_+ N_- N_3^{-1/3}}{N_+^{5/3} + N_-^{5/3}} - 2^{-4/3} \right) \right\},$$

are shown in fig. 3 in the ideal gas approximation (without the last terms in curly brackets).

Actually the second-sound velocity is given by the characteristic velocities of the ^3He quasiparticles. The Fermi velocities v_{\pm} in a degenerate solution depend essentially on the polarization even in the ideal gas approximation, in contrast to a nondegenerate solution, where the characteristic velocities are nearly spin independent and differ from the thermal velocity $v_T = (T/m)^{1/2}$ only to the extent of small interaction effects. For this reason the polarization dependence of the second-sound velocity weakens with decreasing degree of degeneracy of the solutions.

Experimentally the dependence of the second-sound velocity on the degree of polarization of the spin system has been measured by Greywall and Paalanen (1981, 1982) for solutions with three ^3He concentrations: $x_1 = 10^{-2}$ ($T_0 \sim 120$ mK), $x_2 = 3 \times 10^{-3}$ ($T_0 \sim 54$ mK) and $x_3 = 10^{-3}$ ($T_0 \sim 25$ mK), at temperatures above 10 mK in a magnetic field of 93 kOe ($2\beta H \sim 14.5$ mK). The frequency of the sound oscillations was about several kHz. A trivial estimation of the nuclear dipole relaxation time, $\tau_d \sim \hbar T_0^3 / \beta^4 N_3^2 T^2 \sim 10^4$ s K^2 / T^2 shows that $\omega \tau_d \gg 1$, and the experimental results give the value of the second-sound velocity $s_2^{(\infty)}$. The experimental data of Greywall and Paalanen (1981, 1982) are plotted in fig. 4. The dashed lines correspond to the values of $\{s_2^{(\infty)}(H; T) - s_2(0; T)\} / s_2(0; T)$ calculated by Greywall and Paalanen (1981, 1982) in the ideal Fermi gas model at finite temperature. The points A_1, A_2, A_3 denote the values of $\{s_2^{(0)}(H; 0) - s_2(0; 0)\} / s_2(0; 0)$ for the concentrations x_1, x_2, x_3 . It is clear that the agreement between the experimental and theoretical data is very

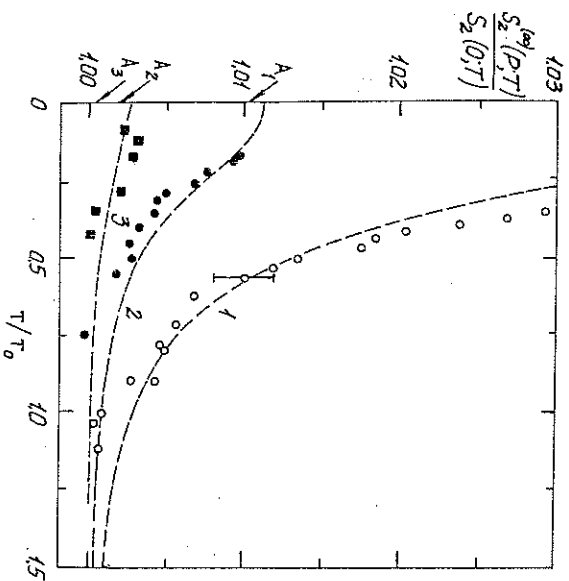


Fig. 4. Temperature dependence of the relative change of the second sound velocity $s_2^{(ex)}(p; T)/s_2(0; T)$. Experimental data and the calculations (in the ideal gas approximation) of Greywall and Paalanen (1981, 1982); \circ (curve 1): ^3He concentration $x_1 = 10^{-3}$ ($T_0 \sim 26$ mK); \bullet (curve 2): $x_2 = 3 \times 10^{-3}$ ($T_0 \sim 54$ mK), \blacksquare (curve 3): $x_3 = 10^{-2}$ ($T_0 \sim 120$ mK). The points A_1, A_2, A_3 correspond to the values of $s_2^{(ex)}(p; T = 0)/s_2(0; 0)$ in a field of $H = 93$ kOe for the concentrations x_1, x_2, x_3 .

good. The results of Greywall and Paalanen (1981, 1982) confirm the main theoretical predictions (Bashkin and Meyerovich 1978, 1981): an increase of the second-sound velocity with increasing polarization and a decrease of the polarization dependence with decreasing degree of the quantum degeneracy. The accuracy of the experimental data is high enough to show the difference between $s_2^{(0)}$ and $s_2^{(ex)}$. Unfortunately, even for the highest ^3He concentrations (1%) the interaction corrections are too small and cannot be extracted from the experimental data.

It was mentioned in section 1.3 that for nearly full polarization the ^3He quasiparticle interaction is associated with p-wave scattering because the s-wave scattering is ineffective for spin-1/2 fermions with equal spin projections. In this approximation the Fermi liquid function of fully spin-polarized $^3\text{He} \downarrow - ^4\text{He}$ solutions is equal to (Bashkin and Meyerovich 1977, 1981)

$$f(\theta) = \frac{24\pi b p_F^2}{M} \sin^2(\theta/2),$$

where $p_F = 2^{1/2} p_0$ is the radius of the Fermi sphere for the fully polarized gas. The value and the sign of the constant $b \sim a^3$ are unknown at present. The effective mass of the Fermi liquid excitation is

$$m_+ = M \left\{ 1 + 2\gamma (p_F/p_0)^2 - 12\pi b N_3 \right\},$$

where it has been taken into account that the effective mass of the bare ^3He quasiparticles (1.1) is equal to

$$M_+ = M / (1 - 2^{5/3} \gamma p_0^2 / p_c^2).$$

The total energy of the fully polarized degenerate solution at $T = 0$, including the quasiparticle interaction, is

$$E = E_4^{(0)} - N_3 \Delta + \frac{3}{10M} p_F^2 N_3 \left\{ 1 - \frac{2}{3} \gamma (p_F/p_0)^2 + \pi b N_3 \right\} - \beta H N_3,$$

and the single-particle energy of the Fermi liquid excitations has the form

$$\epsilon(p) = -\Delta + \frac{p^2}{2M} \left\{ 1 - \gamma (p/p_0)^2 + 12\pi b N_3 \right\} + \frac{36\pi}{10M} p_F^2 b N_3 - \beta H + \delta \epsilon(p).$$

The limiting value of the osmotic pressure is equal to

$$\Pi_0(p = 1) = \frac{2}{3} N_3 \frac{p_F^2}{2M} \left\{ 1 - \frac{10}{3} \gamma (p_F/p_0)^2 + 30\pi b N_3 \right\}$$

in first order in the interaction for a fully polarized solution [for details see, e.g., Bashkin and Meyerovich (1981)].

4. Transport phenomena and magnetokinetic effects

4.1. VISCOSITY, THERMAL CONDUCTIVITY AND ABSORPTION OF SOUND

A gigantic growth of transport coefficients such as the viscosity and the thermal conductivity on polarization of the spin system seems to be one of the most striking and peculiar polarization effects in a quantum gas of fermions. The corresponding phenomena have been called magnetokinetic effects.

Magnetokinetic effects were first predicted for degenerate Fermi gases (Bashkin and Meyerovich 1977, 1978), but it soon became clear (Meyero-

vich 1978) that such effects can be observed at an arbitrary degree of quantum degeneracy of a Fermi gas. The explanation of these effects is quite simple. As was stated above more than once, in a quantum gas (1.4), (1.5) (e.g. in the ${}^3\text{He}\downarrow-{}^4\text{He}$ solutions) the particle collisions reduce mainly to s-wave scattering. Due to the Pauli principle, only collisions of particles with antiparallel spins are effective in s-wave processes for identical spin-1/2 particles. Since the particle mean free paths are inversely proportional to the density of scattering centres, the mean free paths l_{\pm} of particles with spin projections $\pm 1/2$ are of the order of $l_{\pm} \sim 1/N_{\pm} a^2$. It is evident that at full spin polarization $N_{-} \rightarrow 0$, and the mean free path l_{+} of the particles with spin-up increases infinitely, $l_{+} \rightarrow \infty$. This leads to a corresponding increase of the viscosity and thermal conductivity. At full spin polarization the s-wave processes are ineffective, and p-wave scattering becomes the main interaction channel, with a cross-section of the order of $\sigma_p \sim a^2 (pa/\hbar)^4$, where p is the characteristic momentum of the particles. As a result, the ratios of the viscosity coefficient η and the thermal conductivity coefficient κ of a fully polarized gas to their values in a nonpolarized system is inversely proportional to the ratio of p-wave and s-wave cross-sections:

$$\frac{\eta(P=1)}{\eta(P=0)} \sim \frac{\kappa(1)}{\kappa(0)} \sim \frac{g_s}{g_p} \sim \left(\frac{pa}{\hbar}\right)^4$$

$$\sim \begin{cases} \left\{ \left(\frac{\hbar^2/Ma^2}{T} \right)^2, & T \gg T_0, \\ \left\{ \left(\frac{\hbar^2/Ma^2}{T_0} \right)^2 \sim x^{-4/3}, & T \leq T_0, \end{cases} \quad (4.1)$$

where x is the atomic density of the gas (in our case x is the ${}^3\text{He}$ concentration in the ${}^3\text{He}\downarrow-{}^4\text{He}$ solution). For quantum gases (1.4), (1.5) such an increase of the transport coefficients may be very large, and one can observe the transition to the Knudsen regime.

The dependence of the transport coefficients on the spin polarization is determined by a solution of the kinetic equation. In our case the kinetic equation coincides formally with the system of two kinetic equations for a binary mixture of gases with densities N_{+} and N_{-} . These two equations for the distribution functions n_{\pm} of up and down spins are coupled through the collision integrals. Such a coupling must result in the existence of four different longitudinal (exchange) relaxation times. This coupled system of two kinetic equations has to be solved in the same manner as in the case of an arbitrary binary gas mixture.

It turned out that these two kinetic equations always decouple in the case of dilute quantum gases with s-wave interaction, and there are only two

independent longitudinal relaxation times τ_{\pm} . As a result, the viscosity and thermal conductivity coefficients η and κ have the following form for dilute degenerate ${}^3\text{He}\downarrow-{}^4\text{He}$ solutions (Bashkin and Meyerovich 1978, 1981):

$$\frac{\eta(P)}{\eta(0)} = \frac{(p_{+}/p_0)^5 (2 + 5d^8 - 3d^{10})}{2d^3 (5 - 3d^2)}, \quad (4.2)$$

$$\frac{\kappa(P)}{\kappa(0)} = \frac{(p_{+}/p_0)^3 (4 + 3d^4 + d^6)}{2d (3 + d^2)}, \quad d = p/p_{+},$$

where $\eta(0)$ and $\kappa(0)$ are the viscosity and thermal conductivity of nonpolarized solutions. The polarization dependences of the relative changes of η and κ (4.2) are shown in figs. 5, 6. The expressions (4.2) have been obtained on the basis of an approximate method developed by Abrikosov and Khalatnikov (1957) for the kinetic equation in a degenerate nonpolarized Fermi system. It is well known that the exact solutions of the kinetic equation (Brooker and Sykes 1968) differ from the results of Abrikosov and Khalatnikov (1957) by considerable correction factors. In

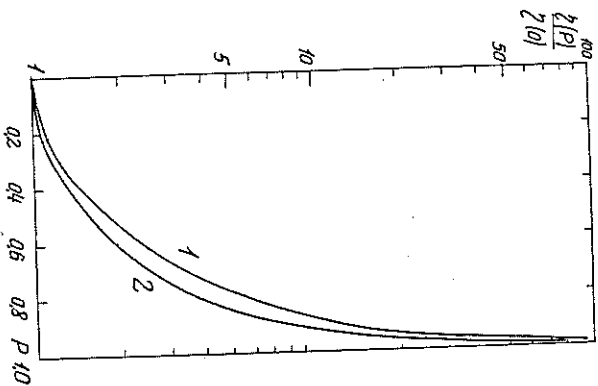


Fig. 5. Polarization dependence of the relative change of the viscosity, $\eta(P)/\eta(0)$: 1: degenerate solutions, eq. (4.2); 2: nondegenerate solutions, eq. (4.5).

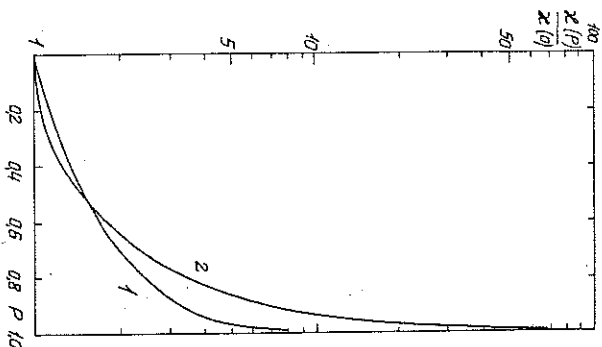


Fig. 6. Polarization dependence of the relative change of the thermal conductivity, $\kappa(P)/\kappa(0)$: 1: degenerate solutions, eq. (4.2); 2: nongenerate solutions, eq. (4.5).

the case of nonpolarized dilute ^3He - ^4He solutions (Bashkin 1977, Bashkin and Meyerovich 1981) these factors are equal to 0.81 and 0.52,

$$\eta(0) = \frac{1}{24\pi^3} \frac{p_0^5}{(MaT)^2} \cdot 0.81, \quad \kappa(0) = \frac{1}{16\pi} \frac{p_0^5}{(Ma)^2 T} \cdot 0.52, \quad (4.3)$$

and in the case of fully polarized solutions in the p-wave scattering approximation (Bashkin and Meyerovich 1977, 1981) the Brooker and Sykes factors are equal to 0.79 and 0.55,

$$\eta(1) = \frac{7}{\pi} \left(\frac{2^{1/3} \hbar^2}{24\pi bMT} \right)^2 P_0 \cdot 0.79, \quad \kappa(1) = \frac{35\pi}{2^{1/3} \cdot 6T} \left(\frac{\hbar^2}{24\pi bM} \right)^2 \cdot \frac{0.55}{P_0}.$$

Nevertheless, the corresponding corrections to the relative changes of the viscosity and thermal conductivity (4.2) turned out to be very small and do not exceed 8% at an arbitrary degree of spin polarization [Mullin and Miyake (1983); the exact formulae of these authors are rather cumbersome]. The difference in $\eta(0)$ and $\kappa(0)$ between Mullin and Miyake (1983) and

Bashkin (1977) seems to be caused by the different definitions of the scattering length a .

The expressions (4.2) for the transport coefficients of a spin-polarized degenerate gas are inadequate at very high degrees of polarization, $N_- \rightarrow 0$. When $N_- \rightarrow 0$, the degeneracy temperature for down spins $T_- \rightarrow 0$, and the corresponding subsystem becomes nondegenerate. In this case one has to treat a $^3\text{He}\downarrow$ - ^4He solution as a "semidegenerate" system [eq. (1.3)]. If $N_- \rightarrow 0$, the viscosity η behaves as (Meyerovich 1978)

$$\frac{\eta}{\eta(0)} = \frac{28^3}{5} \left(\frac{T_0}{T} \right)^{3/2} V(z), \quad z = (2^{2/3} T_0 - 2BH)/T, \quad (4.4)$$

with the function

$$V(z) = \frac{\pi^2}{4} \left\{ \int_0^\infty \frac{t^{3/2} dt}{1 + \exp(t-z)} \frac{\exp(t-z)}{\exp(t-z) - 1} \ln \frac{1 + \exp(t)}{1 + \exp(z)} \right\}^{-1},$$

plotted in fig. 7. (In eq. (4.4) the spin polarization is supposed to be caused by an external magnetic field.) The dimensionless parameter z in eq. (4.4) characterizes the degeneracy of the distribution function n_- . When $z \gg 1$, the system of down spins is degenerate ($z = T_-/T$, $V(z) = z^{-3/2}$), and eq. (4.4) coincides with eq. (4.2) in the limit $p_-/p_+ \rightarrow 0$. In the opposite case $z \rightarrow -\infty$ the function n_- becomes the Boltzmann distribution function and the viscosity η (4.4) depends exponentially on the temperature and the magnetic field: $V(z) \approx 0.70 \exp(-z)$. Thus, at high polarizations the growth

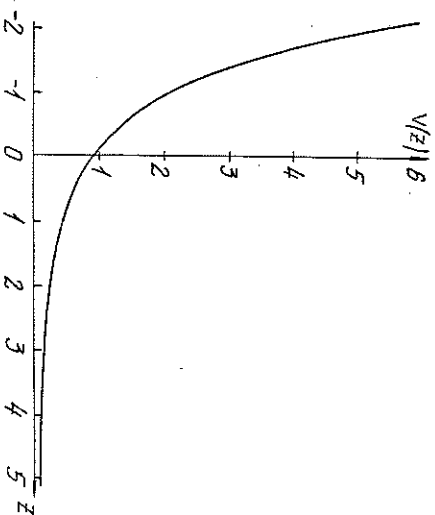


Fig. 7. The function $V(z)$, eq. (4.4).

of the kinetic coefficients becomes somewhat slower in comparison with eq. (4.2), and the limiting values $\eta(1)$ and $\kappa(1)$ are reached through the semidegenerate region (4.4).

For polarized nondegenerate Boltzmann solutions the kinetic equation coincides with that for a binary mixture of classical ideal gases with densities N_+ and N_- . Hence, one can obtain the values of the transport coefficients practically without calculations, using well-known results of the Chapman-Enskog theory for transport phenomena in classical binary mixtures. In our case there are some additional simplifications. First, in the s-wave scattering limit the scattering amplitudes and cross-sections do not depend on the particle momenta. Thus, we are interested in the results of the Chapman-Enskog theory in the simplest hard-sphere approximation. Second, in considering the s-wave $^3\text{He}-^4\text{He}$ collisions one has to take into account only the interaction of particles with opposite spin projections, i.e. the collisions of particles from different components of the classical binary mixture. Therefore, in the expressions of the Chapman-Enskog theory [see, e.g., Hirschfelder et al. (1954)] one has to put formally the scattering cross-sections of particles with atoms of the same component equal to zero, $W_{11} = W_{22} = 0$, and to take into account only the collisions of atoms of different components with the scattering probability $W_{12} = W_{21} = \text{const.}$, independent of the momenta.

Due to these simplifications the cumbersome formulae (Hirschfelder et al. 1954) for the transport coefficients of classical binary mixtures become much more transparent, and after some algebra one can obtain the following expression for the transport coefficients of spin-polarized nondegenerate $^3\text{He}\downarrow-^4\text{He}$ solutions:

$$\begin{aligned} \frac{\eta(P)}{\eta(0)} &= \frac{N_+^2 + \frac{1}{3}(N_+ - N_-)^2}{4N_+N_-}, & \eta(0) &= \frac{5(\pi MT)^{1/2}}{84\pi a^2}, \\ \frac{\kappa(P)}{\kappa(0)} &= \frac{59(N_+^2 + N_-^2) + 54N_+N_-}{172N_+N_-}, & & \\ \kappa(0) &= \frac{75(\pi T/M)^{1/2}}{324\pi a^2}. & & \end{aligned} \quad (4.5)$$

The relative changes of the viscosity and thermal conductivity (4.5) with polarization can be found in figs. 5, 6. The above method of calculation was developed in Meyerovich (1982) for the transport phenomena in nondegenerate $^3\text{He}\downarrow-^4\text{He}$ solutions. Lhuillier and Laloë (1982) analyzed the kinetic equation for the general case of a spin-polarized Boltzmann gas and reproduced the Chapman-Enskog procedure for longitudinal transport

phenomena. Of course, in the low-temperature quantum region (1.4), (1.5) the results of Lhuillier and Laloë (1982) coincide with eq. (4.5) (there are some insignificant numerical differences between the results of Meyerovich (1982) and Lhuillier and Laloë (1982) in the values of the transport coefficient at zero polarization). Certainly, the results of the Chapman-Enskog theory for binary mixtures make it very easy to generalize eq. (4.5) in order to take into account the real energy dependences of the scattering amplitudes and to obtain results valid at higher temperatures.

The growth of the kinetic coefficients (4.2), (4.5) can be limited by p-wave scattering processes, by ^3He quasiparticle collisions with the walls, and by the collisions of ^3He quasiparticles with some other excitations of $^3\text{He}\downarrow-^4\text{He}$ solutions (for example, with phonons). The last scattering mechanism turned out (Meyerovich 1978) to be ineffective and can influence the magnetokinetic effects only in nondegenerate $^3\text{He}-^4\text{He}$ solutions with very low ^3He concentrations,

$$x \leq (\theta/Ms_0^2)^{5/2} (T/\theta)^{11/2},$$

where θ and s_0 are the Debye temperature and the first-sound velocity in pure He II.

The above transport phenomena are stationary processes and cannot demonstrate the existence of two different longitudinal (exchange) relaxation times τ_{\pm} . The presence of different relaxation times can be exposed in dynamic processes with finite frequencies, for example, in sound propagation. The absorption of first sound was investigated in Meyerovich (1982) for degenerate spin-polarized $^3\text{He}\downarrow-^4\text{He}$ solutions. The first-sound waves in dilute $^3\text{He}-^4\text{He}$ solutions are mainly oscillations of superfluid He II. Though ^3He quasiparticles also participate in these oscillations, their influence on the sound velocity is small because of the low ^3He concentration. However, the attenuation of first sound at low temperatures is predominantly due to the ^3He impurity component. Though the polarization of solutions is unimportant for the sound velocity, it must cause a tremendous change of the sound absorption because of magnetokinetic effects. It is clear that in spin-polarized solutions there are two absorption maxima at frequencies $\omega\tau_{\pm} = 1$ instead of a single maximum $\omega\tau = 1$ in a nonpolarized solution.

In a dilute degenerate spin-polarized solution the longitudinal relaxation times τ_{\pm} are given by

$$\tau_{\pm} = \frac{1}{4\pi M} \left(\frac{1}{aT} \right)^2 \frac{5}{5 - 3(p_{\mp}/p_{\pm})^2} \left(\frac{p_{\pm}}{p_{\mp}} \right)^3. \quad (4.6)$$

Both relaxation times τ_{\pm} are related to the corresponding "dynamic" viscosities,

$$\eta_{\pm} = \tau_{\pm} N_{\pm} P_{\pm}^2 / 5M, \quad (4.7)$$

while the "static" viscosity (4.2) is equal to $\eta = \eta_{+} + \eta_{-}$. In the first-order approximation in the ${}^3\text{He}$ concentration the absorption coefficient for first sound takes the form

$$\gamma = \frac{\omega^2}{2m_4 N_4 s_0^3} \left(1 - \frac{m_3}{M} + \alpha \frac{m_4}{M} \right)^2 \left(\frac{4\eta_{+}/3}{1 + \omega^2 \tau_{+}^2} + \frac{4\eta_{-}/3}{1 + \omega^2 \tau_{-}^2} \right), \quad (4.8)$$

where $\alpha = (N_4/m_4 s_0^2) \partial(-\Delta)/\partial N_4$ (at the saturated vapor pressure presumably $\alpha \sim 0.3$). In the truly hydrodynamic regime $\omega \tau_{\pm} \ll 1$ the sound absorption coefficient in degenerate solutions is given, as in an arbitrarily degenerate Fermi system, by the value of the static viscosity $\eta = \eta_{+} + \eta_{-}$.

As the polarization grows, the sound absorption maximum (4.8) at the frequency $\omega = 1/\tau_{+}$ increases in value and moves to vanishingly low frequencies, while the second maximum at the frequency $\omega = 1/\tau_{-}$ remains at the same position and decreases to zero. Note, that, as a result of the magnetokinetic effect, the relaxation time τ_{+} (4.6) increases practically without limit with increasing polarization, and at high degrees of polarization only quasistatic oscillations are truly hydrodynamic: at high degrees of polarization $0 \leq (1-P) \leq 1$ the condition $\omega \tau_{+} \leq 1$ corresponds to the numerical inequality $\omega[s^{-1}] \leq 5 \times (10x^{1/3})^4 T^2 / (1-P)[\text{mK}^{-2}]$, where x is the ${}^3\text{He}$ concentration.

Up to now there are no calculations of the magnetokinetic effects in the intermediate (between Boltzmann and degenerate) temperature region.

As far as the author knows, there have been no direct measurements of the polarization effects on the transport phenomena in ${}^3\text{He} \downarrow - {}^4\text{He}$ solutions, though several groups had planned corresponding experiments. The only experimental result is associated with a weak increase of the second-sound absorption coefficient in a magnetic field observed by Greywall and Paalanen (1981, 1982). But such an increase could not definitely be interpreted as a sign of a magnetokinetic effect: the experimental cell of Greywall and Paalanen (1981, 1982) possessed a very complicated geometry, and the results on sound absorption could not be subjected to a quantitative analysis.

4.2. SPIN DIFFUSION, SPIN THERMAL AND PRESSURE DIFFUSION. SPIN SECOND VISCOSITY

The longitudinal spin diffusion coefficient D_{\parallel} (2.3)-(2.6) has to be calculated using the diagonal (in spin space) kinetic equation for the single-particle density matrix. In a polarized gas of spin-1/2 particles (${}^3\text{He} \downarrow - {}^4\text{He}$ solutions) this coefficient is essentially analogous to the usual particle diffusion coefficient in a binary gas mixture. For degenerate solutions calculations on the basis of Hone's scheme (Hone 1961) lead to the following polarization dependence of the coefficient D_{\parallel} (Meyerovich 1982):

$$\frac{D_{\parallel}(P)}{D(0)} = \left(\frac{P_{+}}{P_0} \right)^5 \frac{N_3 N_2^{2/3}}{N_3^{5/3} + N_2^{5/3}}, \quad D(0) = \frac{1}{8\pi} \left(\frac{\hbar}{M} \right)^3 \left(\frac{P_0}{dT} \right)^2 \quad (4.9)$$

(the small correction to eq. (4.9) similar to the Brooker-Sykes correction factor has been calculated by Mullin and Miyake (1983)), and in nondegenerate solutions the coefficient D_{\parallel} does not depend on the polarization (Meyerovich 1982, Lhuillier and Laloe 1982):

$$D_{\parallel} = \frac{3}{8} \frac{(\pi T/M)^{1/2}}{4\pi a^2 N_3}, \quad (4.10)$$

reflecting the fact that in a classical ideal binary gas mixture the diffusion coefficient does not depend on the concentration of the mixture.

The spin thermal diffusion ratio for nondegenerate solutions is equal to

$$K_{sT} = \frac{5}{43} \frac{N_{+} - N_{-}}{N_3}. \quad (4.11)$$

The results obtained by Meyerovich (1982) and Mullin and Miyake (1983) for the spin thermodiffusion coefficient in a degenerate spin-polarized solution are slightly different. The spin pressure diffusion ratio, calculated with the help of the usual thermodynamic relations (Landau and Lifshitz 1978), is equal to (Meyerovich 1982)

$$K_{sP} \equiv -\Pi_0 \left[(\partial N_{\pm}/N_3) / \partial \Pi_0 \right]_{\mu_{+} = \mu_{-}} \\ = \pm \frac{3}{5} \frac{N_1^{1/3} N_1^{1/3}}{N_3^2} \left(N_2^{5/3} + N_2^{5/3} \right) \left(N_2^{2/3} - N_2^{2/3} \right) \quad (4.12)$$

in degenerate solutions, and is equal to zero in the nondegenerate case

because in binary mixtures of ideal Boltzmann gases the pressure diffusion coefficient differs from zero only when the masses of the particles are different in both gases.

However, this approach to the pressure diffusion on the basis of the thermodynamics of irreversible processes is not always correct. It is well known (Zhdanov et al. 1962) that the pressure diffusion coefficient is often renormalized in the presence of viscous flows in gases. In our case of spin pressure diffusion in polarized solutions (4.12) this renormalization for degenerate solutions, resulting in small corrections in T/T_0 , is insignificant. On the contrary, in the Boltzmann temperature region the thermodynamic pressure diffusion ratio differs from zero only due to small interaction terms, and the viscous renormalization becomes dominant, excluding very high polarizations when the viscosity is very large. In this region the spin pressure diffusion coefficient is given by the expressions of Zhdanov et al. (1962) rather than by the thermodynamic expressions for binary mixtures of gases of particles with equal masses (Landau and Lifshitz 1978, Hirschfelder et al. 1954).

The spin diffusion coefficients (4.9), (4.10) do not depend on the spin polarization in the same way as the viscosity and thermal conductivity coefficients. In degenerate solutions the coefficient D_{\parallel} not only does not increase with the polarization, but even tends to zero. This is due to the fact that the diffusion in binary mixtures is determined using the condition of absence of total mass flow. This condition results in the tendency (4.9) $D_{\parallel} \rightarrow 0$ for $N_- \rightarrow 0$ because the Fermi velocity for down spins v_- decreases when N_- goes to zero: $v_- \propto N_-^{1/3} \rightarrow 0$. For Boltzmann solutions the characteristic velocity does not depend on the spin polarization and is determined solely by the temperature, $v_T = (T/M)^{1/2}$, leading to the absence of a polarization dependence for D_{\parallel} (4.10).

Let us list also the values of the longitudinal diffusion (exchange) relaxation times. For degenerate solutions the corresponding relaxation time is connected with the longitudinal diffusion coefficient as

$$D_{\parallel} = \frac{\tau_{\parallel}}{3} \frac{N_3 v_+^2 v_-^2}{N_+ v_+^2 + N_- v_-^2}. \quad (4.13)$$

In accordance with eq. (4.9) this expression means that τ_{\parallel} does not go to zero when $N_- \rightarrow 0$, but has a finite limit:

$$\tau_{\parallel}(N_- \rightarrow 0) \rightarrow \frac{3}{4\pi} \frac{h}{T} \left(\frac{h^2}{Ma^2} / T \right).$$

For Boltzmann solutions

$$D_{\parallel}/\tau_{\parallel} = v_T^2, \quad (4.14)$$

where $v_T = (T/M)^{1/2}$ is the thermal velocity of the ^3He quasiparticles. For semidegenerate solutions (1.3) the longitudinal relaxation time

$$\frac{D_{\parallel}}{\tau_{\parallel}} = \frac{N_3 v_+^2 v_T^2}{3N_+ v_+^2 + N_- v_T^2} \sim v_T^2/3. \quad (4.15)$$

The calculation of the spin second viscosity (2.14) for dilute solutions is trivial. For degenerate spin-polarized solutions (Meyeroovich 1982)

$$\zeta_0 = \frac{2^{2/3} p_0^2}{3M^2} M \tau_d \frac{N_+^{1/3} N_-^{1/3} (N_+^{2/3} - N_-^{2/3})}{N_+^{1/3} (N_+^{1/3} + N_-^{1/3})}. \quad (4.16)$$

In polarized solutions the ratio of the first and second viscosities is determined by the parameter $\omega^2 \tau_+ \tau_d$, where τ_+ is the viscous relaxation time (see section 4.1). A rough estimate shows that at $T \sim 1$ mK the second viscosity dominates in dissipative phenomena only for quasistationary processes. In the Boltzmann temperature region the bulk viscosity is equal to zero in the ideal gas approximation when in eq. (2.11) $\partial N_{\pm}^{(0)}/\partial N_{\pm} \equiv N_{\pm}^{(0)}/N_{\pm}$. In this nondegenerate case the spin bulk viscosity is always small and differs from zero due to subtle interaction effects only. Certainly, this also concerns the (second) sound frequency dispersion.

5. Transverse spin dynamics and spin waves

5.1. THEORETICAL RESULTS

Other interesting peculiarities of spin-polarized ^3He \downarrow - ^4He solutions are the transverse spin dynamics and the possibility of spin wave propagation. Basically, long-wavelength spin oscillations with a quadratic dispersion law (of the ferromagnetic type),

$$\omega = \Omega_0 + (A - iB)k^2, \quad (5.1)$$

can propagate through an arbitrary exchange system with an intrinsic or induced magnetic moment. Here $\Omega_0 = 2BH/\hbar$ is the usual precession frequency of the magnetic moment in an external magnetic field, k is the

wave vector of the oscillations, and the real coefficients A and B are determined by exchange interaction processes and give the velocity and attenuation of the spin waves; the condition of low damping has the form $A \gg B$. The coefficient A is related to the existence of a molecular field, and the coefficient B to the relaxation processes (the incoherent interaction). The fact that the spin waves (5.1) may be low-damped in polarized degenerate solutions is not surprising: a degenerate solution is a low-density Fermi liquid in which the spin polarization leads to the existence of Slinn waves (see section 2.2). In this case the low damping of spin waves is associated with a strong molecular field (a Fermi liquid interaction) and with a long characteristic relaxation time proportional to $(T_0/T)^2 \gg 1$. The surprising fact is the possibility of spin wave propagation with low damping in nondegenerate (Boltzmann) solutions: usually the incoherent interaction in classical Boltzmann gases is more effective than the coherent interaction, and $B \gg A$. Maybe the only exception is a quantum gas (1.4), (1.5). In quantum gases the large wavelength of the quasiparticles (compared with the interaction radius) makes the coherent interaction effects strong enough to provide spin wave propagation even in the Boltzmann temperature region. Note that for Boltzmann spin-polarized quantum gases the existence of spin waves is the most striking consequence of the presence of a strong molecular field, and for pure quantum gases the spin waves (5.1) represent the only possible type of nonsound collective modes. To some extent, the possibility of spin wave propagation in spin-polarized quantum Boltzmann gases contradicts the usual views on collective phenomena in gases.

The spin dynamics in spin-polarized ${}^3\text{He}$ – ${}^4\text{He}$ solutions can be easily investigated in the dilute quantum gas approximation (1.4), (1.5), irrespective of the degeneracy of the gas. The kinetic equation for the single-particle distribution function (density matrix) $\hat{n}_a(\mathbf{p})$ has the usual quasiclassical gas form

$$\frac{\partial \hat{n}}{\partial t} - \frac{\partial \hat{n}}{\partial \mathbf{p}} \frac{\partial \hat{\epsilon}}{\partial \mathbf{r}} + \frac{\partial \hat{n}}{\partial \mathbf{r}} \frac{\partial \hat{\epsilon}}{\partial \mathbf{p}} + \frac{i}{\hbar} [\hat{\epsilon}, \hat{n}] = \hat{L}(\hat{n}), \quad (5.2)$$

where $\hat{L}(\hat{n})$ is the collision integral, and $[\hat{\epsilon}, \hat{n}]$ is the spin commutator of the spin matrices of the single-particle energy $\hat{\epsilon}$ and the distribution function \hat{n} . The kinetic equation can be linearized in the small deviation $\delta \hat{n}$ of the distribution function from its equilibrium value, $\hat{n} = \hat{n}_0 + \delta \hat{n}$. In the quantum gas approximation (1.4), (1.5) for the nearly ideal spin-polarized quantum gas of ${}^3\text{He}$ quasiparticles the equilibrium distribution function is diagonal in spin space,

$$\hat{n}_0 = \frac{1}{2}(n_+ + n_-)\hat{I} + \frac{1}{2}(n_+ - n_-)\hat{\sigma}_z, \quad (5.3)$$

when the z -axis is chosen along the direction of the magnetic moment $e = M/M$ ($\hat{\sigma}$ are the Pauli matrices, $\hat{I} = \delta_{\alpha\beta}$ is the unit spin operator, and n_{\pm} are the (scalar) gas distribution functions for the particles with up and down spins). The functions n_{\pm} are given by

$$n_{\pm} = \{1 + \exp[(\epsilon_{\pm} - \mu_{\pm})/T]\}^{-1}. \quad (5.4)$$

If the spin polarization is the equilibrium one and is determined by an external magnetic field, then the chemical potentials μ_{\pm} for up and down spins are equal to each other, $\mu_+ = \mu_- = \mu_3(N_3, H)$, where

$$\mu_3 = (\partial F / \partial N_3)_{N_{\pm}, H} = \mu_3^{(0)} + \partial F_1 / \partial N_3,$$

$\mu_3^{(0)}$ is the chemical potential of the particles of the ideal gas with the energy spectrum (1.1) in an external field, and F_1 is the interaction correction (3.2) to the free energy F of the gas.

The single-particle energy of the interacting particles is given by eq. (3.5), and the diagonal (in spin space) components of the equilibrium energies for up and down spins are given by

$$\epsilon_{\pm} = -\Delta + \frac{p^2}{2M} \mp \beta H + \frac{2\pi a \hbar^2}{M} N_3 \left(1 \mp \frac{N_+ - N_-}{N_3}\right). \quad (5.5)$$

The molecular field in the Hamiltonian (3.5),

$$\delta \hat{\epsilon} = \frac{2\pi a \hbar^2}{M} \int \frac{d^3 p'}{(2\pi \hbar)^3} \{ \hat{f} \text{Sp } \delta \hat{n}(\mathbf{p}') - \hat{\sigma} \text{Sp } \hat{\sigma} \delta \hat{n}(\mathbf{p}') \}, \quad (5.6)$$

coincides formally with the Fermi liquid interaction determined by the f -function (3.6) at an arbitrary degree of quantum degeneracy of the gas.

The inclusion of this term demands special justification for nondegenerate systems (see section 3). Usually in nondegenerate systems this term lies beyond the accuracy of the calculation due to the damping of the single-particle excitations, to nonlocal, quantum and incoherent corrections to the left- and right-hand sides of the kinetic equation (5.2), etc. However, in the case of quantum gases (1.4), (1.5) one may always take the molecular field (5.6) into account. The reason is that the molecular field in gases is always proportional to the forward scattering amplitude (linear in a) and leads to corrections to the particle energy of the order of a/Λ . Nonlocal and relaxation corrections to the single-particle energy result for dilute gases in higher-order terms in this parameter: the damping is always quadratic in

the interaction (quadratic in a), and nonlocal terms contain an additional differentiation $\partial/\partial r^2 \sim \hat{p}^2/\hbar^2 \sim 1/\Lambda^2$, also leading to the additional factor $(a/\Lambda)^2$. Our quantum limit (1.4), (1.5) formally corresponds to the condition $a \rightarrow 0$, and the molecular field, being the leading term in a , can be taken into account. More detailed comments on the validity of the molecular field approach to the spin dynamics of spin-polarized quantum gases have been given by Bashkin (1981, 1984a,b) [see also Meyerovich (1985)].

The possibility of propagating spin oscillations is determined by a strong molecular field (5.6) in quantum gases (1.4), (1.5) and by the spin polarization of gases, which results in a nontrivial role of the spin commutator $[\hat{e}, \hat{n}]$ in the kinetic equation (5.2). The transverse spin dynamics is governed by the equation for the transverse (to the direction of magnetization e) components m_x, m_y of the vector $m = \text{Sp}_0 \hat{\sigma} \delta \hat{n}$. The linearized kinetic equation for the Fourier components of the circular projections of the magnetization vector $m_{\pm} = m_x \pm im_y$ is easily reduced to

$$\begin{aligned} & (\omega - kv - \Omega_0 + \Omega_{\text{int}}) m_{\pm} \\ & - \frac{2\pi a \hbar^2}{M} \left\{ kv \left(\frac{\partial n_{+}}{\partial \epsilon_{+}} + \frac{\partial n_{-}}{\partial \epsilon_{-}} \right) - \frac{2}{\hbar} (n_{+} - n_{-}) \right\} \int m_{\pm} \frac{d^3 p}{(2\pi \hbar)^3} \\ & = \text{Sp}_0 \hat{\sigma}_{\pm} \hat{L}(\hat{n}), \end{aligned} \quad (5.7)$$

at an arbitrary degree of degeneracy of polarized solutions. Here $\Omega_0 = 2\beta H/\hbar$, and the internal frequency Ω_{int} ,

$$\Omega_{\text{int}} = -\frac{4\pi a \hbar}{M} (N_{+} - N_{-}), \quad (5.8)$$

determines the precession frequency of the magnetization vector in the molecular field. The exchange interaction of the ^3He quasiparticles conserves the total spin, and

$$\int \text{Sp}_0 \hat{\sigma} \hat{L}(\hat{n}) dT = 0. \quad (5.9)$$

Collisions lead only to a spin current. The corresponding collision contribution to the spin current can be described using the relaxation time approximation. In a system with a single preferred direction (the vector $e = M/M$) there are two characteristic exchange relaxation times: the

longitudinal and the transverse relaxation times τ_{\parallel} and τ_{\perp} . In this case the collision integral must be parametrized as

$$\int v_i \text{Sp}_0 \hat{\sigma} \hat{L}(\hat{n}) dT = -\frac{1}{\tau_{\perp}} J_i - \left(\frac{1}{\tau_{\parallel}} - \frac{1}{\tau_{\perp}} \right) e_i (eJ_i), \quad (5.10)$$

where J_i is the usual spin current,

$$J_i = \int v_i \text{Sp}_0 \hat{\sigma} \delta \hat{n} dT.$$

It is evident, that for a dilute ($N_3 a^3 \ll 1$) quantum gas (1.4), (1.5) the inequality $N_3 a^3 \ll EMa^2/\hbar^2$ (E is the characteristic energy of the particles) can be rewritten as $\partial n/\partial \epsilon \sim 1/E \ll (Ma^2/\hbar^2)/(Na^3)$, and the first term in the curly brackets in eq. (5.7) is negligible in comparison with the second term. We are interested in the spin wave spectrum in the long wavelength limit $\delta\omega, kv \ll \Omega_{\text{int}}$ ($\delta\omega \equiv \omega - \Omega_0$). In this limit the spectrum of spin oscillations can be easily derived by a simple expansion in $\delta\omega/\Omega_{\text{int}}$ and kv/Ω_{int} at an arbitrary degree of degeneracy (Meyerovich 1985)

$$\omega = \Omega_0 + \frac{1}{3} \frac{k^2}{\Omega_{\text{int}}} \frac{1 - i/\Omega_{\text{int}}\tau_{\perp}}{1 + 1/\Omega_{\text{int}}\tau_{\perp}^2} \frac{N_{+}\langle v^2 \rangle_{+} - N_{-}\langle v^2 \rangle_{-}}{N_{+} - N_{-}}. \quad (5.11)$$

Here and below the mean velocities $\langle v^2 \rangle_{\pm}$ are defined as

$$\langle v^2 \rangle_{\pm} = \frac{1}{N_{\pm}} \int v^2 n_{\pm} \frac{d^3 p}{(2\pi \hbar)^3}.$$

The damping of spin waves is small when $\Omega_{\text{int}}\tau_{\perp} \gg 1$. The relaxation time is of the order of $\tau \sim 1/N_0 a^2$, and, according to eq. (5.8), the condition of small damping is always met for a gas with not very low polarization,

$$P = \frac{N_{+} - N_{-}}{N_3} \gg \frac{a}{\Lambda}.$$

The last condition can be easily met for a quantum gas (1.4), (1.5) since $(a/\Lambda) \ll 1$. In the limiting case of degenerate solutions, $\langle v^2 \rangle_{\pm} = \frac{3}{2} v_{\pm}^2$ ($v_{\pm} = p_{\pm}/M$ are the Fermi velocities of the particles with up and down spins), the spectrum (5.11) takes the form (Bashkin and Meyerovich 1978, 1981)

$$\omega = \Omega_0 + \frac{1}{5} \frac{k^2}{\Omega_{\text{int}}} \frac{1 - i/\Omega_{\text{int}}\tau_{\perp}}{1 + 1/\Omega_{\text{int}}\tau_{\perp}^2} \frac{N_{+}v_{+}^2 - N_{-}v_{-}^2}{N_{+} - N_{-}}. \quad (5.12)$$

In the Boltzmann temperature region $T \gg T_{\pm}$ the values of the mean velocities are equal to each other, $\langle v^2 \rangle_{\pm} = 3v_T^2$ ($v_T = (T/M)^{1/2}$ is the thermal velocity), and the spectrum (5.11) is reduced to (Bashkin and Meyerovich 1979, 1981) *

$$\omega = \Omega_0 + \frac{k^2}{\Omega_{\text{int}}} \frac{1 - i/\Omega_{\text{int}}\tau_{\perp}}{1 + 1/\Omega_{\text{int}}^2\tau_{\perp}^2}. \quad (5.13)$$

In semidegenerate solutions (1.3) with $T_+ \gg T \gg T_-$ the spectrum has the form (Meyerovich 1985)

$$\omega = \Omega_0 + \frac{k^2}{\Omega_{\text{int}}} \frac{1 - i/\Omega_{\text{int}}\tau_{\perp}}{1 + 1/\Omega_{\text{int}}^2\tau_{\perp}^2} \frac{v_+^2 N_+ / 5 - v_-^2 N_-}{N_+ - N_-}, \quad (5.14)$$

and practically coincides with eq. (5.12) because $N_- \ll N_+$, $v_T \ll v_+$.

The macroscopic equation of motion for the transverse components of the magnetization vector $M_{x,y}$,

$$M_{x,y} = \int dT \text{Sp}_0 \hat{\sigma}_{x,y} \hat{h}_\sigma,$$

can be obtained using the procedure developed by Leggett (1970) for a degenerate Fermi system with low polarization. Direct integration of eq. (5.7) yields

$$\begin{aligned} \dot{M}_{\perp} + \nabla_j J_{j\perp} + \Omega_0 [e \times M] &= 0, \\ J_{j\perp} + (\Omega_0 - \Omega_{\text{int}}) [e \times J_j] + \frac{1}{\tau_{\perp}} J_{j\perp} \\ &+ \frac{1}{2} \nabla_j M_{\perp} \frac{N_+ \langle v^2 \rangle_+ - N_- \langle v^2 \rangle_-}{N_+ - N_-} = 0, \\ M_{\perp} &\equiv M - e(eM), \quad J_{j\perp} \equiv J_j - e(eJ_j). \end{aligned} \quad (5.15)$$

* Note that eq. (2.22) of Bashkin and Meyerovich (1979) for the spectrum is invalid; the misprint is corrected in eq. (4.8.5) of Bashkin and Meyerovich (1981).

The solution of eqs. (5.15) coincides with the equation of motion (2.7) at arbitrary degrees of degeneracy and polarization (Meyerovich 1985):

$$\dot{M}_{\perp} + \Omega_0 [e \times M] - \frac{D_{\perp}}{1 + \Omega_{\text{int}}^2\tau_{\perp}^2} \left\{ \frac{\partial^2}{\partial r^2} M_{\perp} + \Omega_{\text{int}}\tau_{\perp} \left[e \times \frac{\partial^2}{\partial r^2} M_{\perp} \right] \right\} = 0, \quad (5.16)$$

where the transverse spin diffusion coefficient D_{\perp} is related to the corresponding relaxation time τ_{\perp} (5.10) by

$$D_{\perp} = \frac{1}{2} \tau_{\perp} \frac{N_+ \langle v^2 \rangle_+ - N_- \langle v^2 \rangle_-}{N_+ - N_-}. \quad (5.17)$$

In the Boltzmann region relation (5.17) takes the form

$$D_{\perp} = v_T^2 \tau_{\perp}, \quad (5.18)$$

in a degenerate system

$$D_{\perp} = \frac{\tau_{\perp}}{5} \frac{v_+^2 N_+ - v_-^2 N_-}{N_+ - N_-}, \quad (5.19)$$

and in semidegenerate solutions

$$D_{\perp} = \tau_{\perp} \frac{v_+^2 N_+ / 5 - v_-^2 N_-}{N_+ - N_-} \sim \tau_{\perp} v_+^2 / 5, \quad (5.20)$$

while the longitudinal diffusion coefficient D_{\parallel} is related to the longitudinal relaxation time τ_{\parallel} (5.10) by (Meyerovich 1985)

$$D_{\parallel} = \frac{\tau_{\parallel} N_3}{N_+ \langle v^{-2} \rangle_- + N_- \langle v^{-2} \rangle_+}. \quad (5.21)$$

Relation (5.21) coincides with eqs. (4.13)-(4.15) in the corresponding limits.

The macroscopic equation of motion (5.16) and the spin wave spectrum (5.11) can be used both in the "hydrodynamic" limit $\delta\omega \tau \ll 1$ and in the "collisionless" limit $\delta\omega \tau \gg 1$. The only limitation on the frequency is $\delta\omega \ll \Omega_{\text{int}}$. This condition and the condition of low damping, $\Omega_{\text{int}}\tau_{\perp} \gg 1$, do not depend on the value of $\delta\omega \tau$. Such a situation is common for oscillations in systems with a large internal frequency caused by interactions. In

our case the role of the "plasma" frequency is played by the precession frequency Ω_{int} (5.8) of the magnetization in the molecular field. Note that the denominator $1 + \Omega_{\text{int}}^2 \tau_{\perp}^2$ in eqs. (5.11), (5.16) can be taken into account only in the low-frequency limit $\delta\omega \tau \ll 1$ when $\delta\omega/\Omega_{\text{int}} \ll 1/\Omega_{\text{int}} \tau$. In the opposite case $\delta\omega \tau_{\perp} \gg 1$ the neglected higher-order terms in $\delta\omega/\Omega_{\text{int}}$ may be more important than small corrections of the order of $1/\Omega_{\text{int}} \tau_{\perp}$.

When $D_{\parallel} = D_{\perp}$, the equations for the spin dynamics become isotropic and formally coincide with the equation of Leggett (1970). Certainly, this is true for low polarization when $D_{\parallel} = D_{\perp}$, $\tau_{\parallel} = \tau_{\perp}$ and $D/\tau = 1/\langle v^{-2} \rangle$. The isotropy is also likely to be observed in a polarized Boltzmann quantum gas when, according to Lhuillier and Laloë (1982), $D_{\parallel} = D_{\perp}$ and [see eqs. (4.14) and (5.18)] $D_{\parallel}/\tau_{\parallel} = D_{\perp}/\tau_{\perp} = v^2$. However, according to Bashkin (1985), there are reasons to suppose that even in polarized Boltzmann quantum gases the spin dynamics may turn out to be anisotropic, and the question seems to be unsettled. Nevertheless, it is clear that the degeneracy of a spin-polarized gas of fermions always leads to a strong anisotropy (Meyerovich 1985). While the degeneracy of a Fermi gas results in proportionality of τ_{\parallel}^{-1} to $(T/T_0)^2$, the expression for τ_{\perp}^{-1} in polarized gases contains a term without such a small factor. The value of τ_{\perp} determines the damping of the nondiagonal elements of the density matrix for single-particle states. In the general case of a degenerate polarized Fermi system these elements do not correspond to well-defined single-particle excitations with long lifetimes remaining near the definite Fermi surface. Mathematically this means that the collision integral for evaluation of τ_{\perp} contains a term not with a δ -function of the total energy of the colliding particles $\delta(\epsilon)$, but with $P(1/\epsilon)$. Thus, in a considerably polarized degenerate Fermi system $\tau_{\perp}^{-1} \gg \tau_{\parallel}^{-1}$ and τ_{\perp}^{-1} is small for a spin-polarized degenerate quantum gas only due to the low density of the gas. Unfortunately, there are no consistent calculations of τ_{\perp} in spin-polarized gases even in the limiting cases of degenerate or Boltzmann systems in the s -wave scattering approximation for slow particles.

For a degenerate spin-polarized dilute gas the spectrum of spin waves (5.12) was derived in Bashkin and Meyerovich (1978), and the equation for the spin dynamics in Meyerovich (1983, 1985). Note that these results differ essentially from the well-known data on the spin dynamics of polarized Fermi liquids (Silin 1957, Abrikosov and Dzyaloshinsky 1958, Leggett and Rice 1968a,b, Leggett 1970, Platzman and Wolf 1973): for a dense Fermi liquid all results were obtained and can be used only for low spin polarizations. For a degenerate polarized dilute Fermi gas there are no difficulties associated with nonlocal effects and with the use of the nondiagonal elements of the single-particle density matrix, and eqs. (5.12), (5.16) are valid at arbitrary degree of spin polarization.

The possibility of spin wave propagation in polarized Boltzmann systems was predicted for the first time, presumably, by Aronov (1977) for the case of an electron gas in semiconductors. However, the model approach used by Aronov (1977) cannot provide a correct description of the spin dynamics in real gases. Spin waves in polarized Boltzmann gases of heavy particles (atoms) were introduced by Bashkin and Meyerovich (1979, 1981) for the example of ${}^3\text{He}$ - ${}^4\text{He}$ solutions, using the above molecular field approach. Later Bashkin (1981, 1984a,b) and Lhuillier and Laloë (1982) generalized these results to other quantum gases (including Boltzmann gases of bosons) and obtained the relations for the spin dynamics beyond the s -wave scattering and the molecular field approximations. The macroscopic isotropic equation for the spin dynamics in polarized Boltzmann gases [an equation like eq. (5.16)] was derived in Lhuillier and Laloë (1982). The anisotropy of the spin dynamics and the intermediate (between Boltzmann and degenerate) temperature region were investigated in Meyerovich (1983, 1985).

5.2. EXPERIMENTAL DATA ON THE SPIN DYNAMICS

The transverse spin dynamics of ${}^3\text{He}$ - ${}^4\text{He}$ solutions have been studied in experiments by Owers-Bradley et al. (1984a) and by Gully and Mullin (1984). The results of the former authors cannot be interpreted on the basis of the above theory of dilute quantum ${}^3\text{He}$ - ${}^4\text{He}$ solutions: in these experiments the ${}^3\text{He}$ concentration was several percent, and the quantum condition (1.4), (1.5) for the ${}^3\text{He}$ quasiparticle gas was not fulfilled. The theory of a dilute quantum gas can be applied for ${}^3\text{He}$ concentrations $x \leq 1\%$. At higher concentrations the expansion parameter pa/h for the interaction is not small. This can be illustrated by the fact that the second term in the expansion in pa/h for, e.g., the Fermi liquid harmonics becomes greater than the first if $x \sim 3\%$. In the experiments of Owers-Bradley et al. (1984a) this fact was confirmed by the opposite sign of the NMR frequency shift in comparison with the conclusion of the dilute-solution model with $a < 0$. As a result, the data of these authors can be interpreted only with the help of the macroscopic theory of spin-polarized Fermi liquids (see section 2.2).

On the contrary, the experimental results of Gully and Mullin (1984) represent a rather good test of the theory of the spin dynamics in spin-polarized quantum gases. In their experiments the ${}^3\text{He}$ concentration in ${}^3\text{He}$ - ${}^4\text{He}$ solutions was equal to $x = 3.7 \times 10^{-4}$ (the degeneracy temperature $T_0 \sim 13$ mK), and the experiments were performed in the temperature range between 15 and 200 mK. The spin polarization was caused by an external magnetic field of 89 kOe ($2\mu_B H/T_0 \sim 1.04$) and exceeded 30% at the

lowest temperatures. Thus, the measurements were conducted at relatively high polarizations and at temperatures from the Boltzmann region down to the degeneracy temperature, and the quantum condition (1.4), (1.5) was met except in the high-temperature region.

The measurements were performed by a usual spin echo technique in a magnetic field gradient of 2.6 kOe/m. The experimental data were treated on the basis of the spin dynamics equation (5.16) (including the field gradient term) by two independent methods. In the first method the dependences of the echo amplitude on the time between pulses and on the tipping angle were measured. These dependences could be described in terms of eq. (5.16), and the condition of best fit determined the values of the adjustable parameters D_{\perp} and $\Omega_{\text{int}}\tau_{\perp}$. The results of the measurements of the phase shift for the spin echo signal (the second method) were consistent with the amplitude measurements. The obtained temperature dependences of the main parameters of the transverse spin dynamics, D_{\perp} and $\Omega_{\text{int}}\tau_{\perp}$, are shown in fig. 8.

The main problem in a quantitative comparison of the data of Gully and Mullin (1984) with theoretical predictions is due to the above mentioned absence of direct calculations for D_{\perp} and τ_{\perp} . Therefore, the theoretical curves in fig. 8 correspond to the temperature dependences of $D_{\parallel}(T)$, eqs. (4.9), (4.10), and of $\Omega_{\text{int}}(T)\tau_{\parallel}(T)$, eqs. (4.13), (4.14), (5.8), for Boltzmann

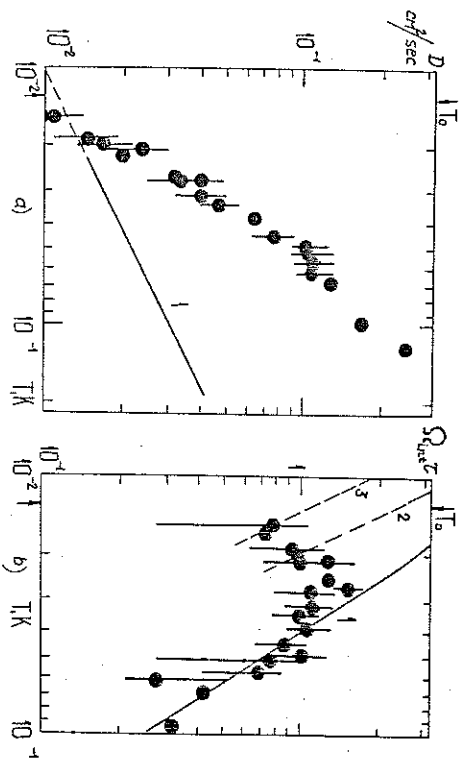


Fig. 8. Temperature dependences of (a) the spin diffusion coefficient D_{\perp} , (b) the parameter $\Omega_{\text{int}}\tau_{\perp}$: \bullet : experimental results of Gully and Mullin (1984) for D_{\perp} and $\Omega_{\text{int}}\tau_{\perp}$. Theoretical curves (calculated with $a = -1.5 \text{ \AA}$): (a) curve 1: $D_{\parallel}(T)$ in nondegenerate solutions, eq. (4.10); (b) curve 1: $\Omega_{\text{int}}\tau_{\parallel}$ in nondegenerate solutions, eqs. (4.10), (4.14), (5.8); curve 2: $\Omega_{\text{int}}\tau_{\parallel}$ in degenerate solutions, eqs. (4.9), (4.13), (5.8); curve 3: $\Omega_{\text{int}}\tau$ calculated with Ω_{int} from eq. (5.8), D_{\parallel} from eq. (4.9) and the ratio D/τ given by eq. (5.19) for D_{\perp} and τ_{\perp} .

(curve 1) and degenerate (curve 2) solutions (there are no calculations in the intermediate region). For the s-wave scattering length we have taken the value of Bashkin and Meyerovich (1981), $a = -1.5 \text{ \AA}$.

The agreement between the experimental data and the simplest quantum gas theory is fairly good. There are three main reasons for the divergence of the experimental and theoretical results. At low (and maybe high) temperatures the divergence may be related to the anisotropy of the spin dynamics and to the lack of calculations for τ_{\perp} . The experimental data seem to confirm that at low temperatures $T \leq T_0$ the quantities τ_{\parallel} , τ_{\perp} ($\tau_{\parallel} \gg \tau_{\perp}$) and their temperature dependences $\tau_{\parallel}(T)$ and $\tau_{\perp}(T)$ are quite different, though in the Boltzmann region these dependences are similar. Figure 8 also shows $\Omega_{\text{int}}\tau$ (curve 3) with τ calculated using the coefficient D_{\parallel} , eq. (4.9), and the ratio D/τ given by eq. (5.19) for D_{\perp} and τ_{\perp} . The difference between curves 2 and 3 clearly demonstrates the difference between the ratios $D_{\parallel}/\tau_{\parallel}$ and D_{\perp}/τ_{\perp} for degenerate solutions.

The other reason for the divergence is important at high temperatures and is related to the fact that conditions (1.4), (1.5) are not very well satisfied: at $T = 100 \text{ mK}$ the expansion parameter of the theory is not small enough, $pa/\hbar = a/\lambda \sim 0.2$. Besides, at high temperatures the ^3He transport is affected by collisions with phonons, and the spin diffusion coefficient and the relaxation time are smaller than the values given by eq. (4.10).

However, there is a method to exclude the shortcomings of the theoretical description of the data for D_{\perp} , τ_{\perp} on the basis of data for D_{\parallel} , τ_{\parallel} . The ratio $\Omega_{\text{int}}\tau_{\perp}/D_{\perp}$ does not contain any unknown parameters associated with the anisotropy of the spin dynamics and is given by (Meyerovich 1985)

$$\frac{\Omega_{\text{int}}\tau_{\perp}}{D_{\perp}} = \frac{12\pi a\hbar}{M} \frac{(N_{+} - N_{-})^2}{N_{+}\langle v^2 \rangle_{+} - N_{-}\langle v^2 \rangle_{-}} \quad (5.22)$$

$$= \begin{cases} -\frac{4\pi a\hbar}{T} (N_{+} - N_{-}), & T \gg T_0, \\ -\frac{20\pi a\hbar (N_{+} - N_{-})^2}{M(N_{+}v_{+}^2 - N_{-}v_{-}^2)}, & T \leq T_0. \end{cases}$$

Figure 9 shows the ratio of the parameters $\Omega_{\text{int}}\tau_{\perp}$ and D_{\perp} measured by Gully and Mullin (1984) and the curve for eq. (5.22) at $T \gg T_0$ with $a = -1.5 \text{ \AA}$. In contrast to fig. 8, the experimental and theoretical curves in fig. 9 are much more similar. Maybe the reason for the remaining difference between the curves is explained by the fact that the real value of a is nearly two times smaller than the value -1.5 \AA (this will result in a very large

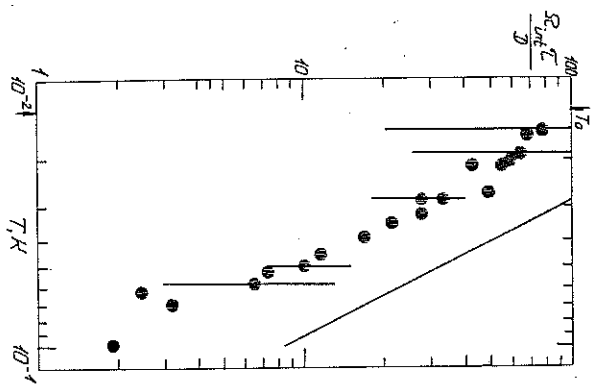


Fig. 9. Temperature dependence of $\Omega_{int} \tau_{\perp} / D_{\perp}$ (s/cm²). ●: experimental results of Gully and Mullin (1984). The theoretical curve corresponds to eq. (5.22) at $T > T_0$ with $\alpha = -1.5 \text{ \AA}$.

decrease of the superfluid transition temperature for ³He in ³He-⁴He solutions). The extrapolation of the curves in fig. 9 shows that these curves reach the limiting value of the ratio (5.22), 2.2×10^3 , for low temperatures $T \leq T_0$ at a temperature of about 4–5 mK.

The decrease of α suggested by fig. 9 provides excellent agreement between the experimental and theoretical data for $\Omega_{int} \tau_{\perp} / D_{\perp}$ if $\alpha \sim -(0.5 - 0.7) \text{ \AA}$, and improves agreement for D_{\perp} in fig. 8, though the difference completely by a mere change of α : the data for D_{\perp} and $\Omega_{int} \tau_{\perp}$ taken separately are sensitive to the anisotropy of the spin dynamics and to the presence of phonons, while the ratio $\Omega_{int} \tau_{\perp} / D_{\perp}$ [eq. (5.22)] is not sensitive to these factors in dilute quantum gases. For these reasons a precise measurement of the ratio $\Omega_{int} \tau_{\perp} / D_{\perp}$ by NMR methods seems to be one of the most reliable ways to determine the main interaction parameter of dilute ³He-⁴He solutions – the s-wave scattering length a .

On the whole, the results of Gully and Mullin (1984) confirm the main conclusion of the theory of the transverse spin dynamics in ³He-⁴He solutions – the existence of spin waves, including the Boltzmann region. The experimental data on the transverse spin dynamics can be interpreted

on the basis of the simple quantum gas theory and this interpretation is considerably more transparent than for other spin-polarized quantum systems (see section 7.1). Some additional experiments seem to be desirable (especially at low temperatures) for a detailed analysis of the anisotropy of the spin dynamics in polarized systems. Calculations of D_{\perp} and τ_{\perp} are also necessary, including the intermediate temperature region. The possibility of an accurate determination of α by the NMR technique is very important for the estimation of the ³He superfluid transition temperature in ³He-⁴He solutions.

6. ³He superfluidity in ³He-⁴He solutions

6.1. TEMPERATURE OF THE SUPERFLUID TRANSITION

After the successful discovery of the superfluid transition in pure liquid ³He several experimental groups made attempts to observe the superfluid transition in the ³He quasiparticle system in ³He-⁴He solutions. Up to now all these attempts have failed [see, e.g., Ahonen et al. (1976), Osheroff and Corruccini (1981), Guénault et al. (1983), Mueller et al. (1983), Owers-Bradley et al. (1984a)], even though a temperature of 0.2 mK was reached. Thus the ³He-⁴He solution remains the only liquid with considerable entropy at temperatures below 1 mK. To a great extent, the interest in the ³He superfluid transition is caused by the unique character of the system – below the transition the solution will represent the only system with two Bose condensates, ³He and ⁴He. Maybe the only analogous system is superfluid star matter, where, along with a neutron condensate, one can find a small fraction of proton condensate.

Up to now the question of a reliable theoretical estimate of the ³He superfluid transition temperature in solutions remains unsettled. The available results range from $T_c \sim 10^{-3}$ – 10^{-4} K for optimistic predictions down to $T_c \sim 10^{-8}$ K for the most unfavourable estimates. Such a difference in predictions for T_c is regrettable but not unexpected. As was mentioned in section 1.3, there is no consistent microscopic theory for the interaction in relatively dense ³He quasiparticle systems dissolved in a dense superfluid Bose background with complicated intrinsic excitation modes. All descriptions of the ³He interaction in solutions are completely or partially based on a model, and the parameters for different models are obtained from fits to thermodynamic and (rarely) kinetic experiments.

The thermodynamic and kinetic results are not very sensitive to the choice of a model if it has a sufficient set of adjustable parameters. On the contrary, the temperature of superfluid pairing for ³He quasiparticles

depends exponentially on the values of the interaction parameters and is very sensitive to the choice of a model. Besides, even if one regards some model as an exact description of the interaction, then there always remain questions as to the accuracy of the calculations for T_c within the framework of this model and of the limits of ${}^3\text{He}$ concentrations for which the results of this model are applicable. As a result, modern theories cannot give reliable values of T_c , nor do they provide an answer to the question of the advisability of further attempts to discover ${}^3\text{He}$ superfluidity in ${}^3\text{He}$ - ${}^4\text{He}$ solutions with the help of modern low-temperature equipment. The answer to the last question depends completely on the optimism of the experimentalists involved. In this context I want to emphasize that without hope of a rapid cardinal progress in obtaining ultralow temperatures it may be useful to pay attention not to the direct observation of the superfluid transition, but to the search of fluctuation effects associated with superfluid fluctuations above the transition.

The above comments on the ${}^3\text{He}$ superfluid transition in ${}^3\text{He}$ - ${}^4\text{He}$ solutions can be illustrated on the example of the description of solutions with the help of the dilute quantum gas theory with s-wave interaction, which is adopted throughout this review. From this point of view, the system of ${}^3\text{He}$ impurity quasiparticles in dilute solutions may represent the sole example of a system for which the BCS theory does not give just a model but an exact description of the superfluid pairing (Bashkin 1978, Bashkin and Meyerovich 1981):

$$T_c = \frac{\gamma}{\pi} \left(\frac{2}{e} \right)^{7/3} T_0 \exp(-1/g), \quad g = \frac{2p_0|a|}{\pi\hbar}, \quad (6.1)$$

where $\ln \gamma = C = 0.5772\dots$ is Euler's constant, p_0 and T_0 stand for the Fermi momentum and temperature of nonpolarized solutions and $a < 0$ is the s-wave scattering length. The index of the exponent in eq. (6.1) is equal to $-1.20/x^{1/3}$ for $a = -1.5 \text{ \AA}$ and at the saturated vapor pressure (x is the ${}^3\text{He}$ concentration in the solution). This means that for $a = -1.5 \text{ \AA}$ the expression (6.1) gives for T_c a value of about 1.4 mK for $x = 0.03$, but if a is two times smaller (as was suggested in the preceding section) the value of T_c for $x = 0.03$ decreases to 0.03 mK . Of course, at lower ${}^3\text{He}$ concentrations the change in T_c with a decrease of a is even more drastic. Apart from the uncertainty in a in eq. (6.1), one has to clear up the question of the limits of applicability of eq. (6.1). The BCS theory has been developed under the assumption that the coupling constant g in the exponent in eq. (6.1) is small, $2p_0|a|/\pi\hbar \ll 1$. When $a = -1.5 \text{ \AA}$ this condition is equivalent to the numerical inequality $x^{1/3}/1.20 \ll 1$. Apparently, this means that the expression (6.1) may be interpreted literally for solutions with ${}^3\text{He}$

concentrations below 10^{-2} - 10^{-3} (for $x = 10^{-3}$ and $a = -1.5 \text{ \AA}$, $g \sim 0.083$). It is absolutely unknown how T_c deviates from eq. (6.1) at higher ${}^3\text{He}$ concentrations. If $a \sim -1.5 \text{ \AA}$, then the experimental data suggest that an increase of the ${}^3\text{He}$ concentration leads to a decrease of T_c as compared with eq. (6.1).

One should also keep in mind two additional circumstances when trying to observe the ${}^3\text{He}$ superfluid transition in solutions. First, one has to take into account the growth of the relaxation times for the temperature and for the magnetic moment of the ${}^3\text{He}$ impurity quasiparticle system. The superfluid transition is determined by the temperature in the quasiparticle system, while all thermometry is based on measurement of phonon temperatures. It is known that at temperatures $T \sim 1 \text{ mK}$ one may be confronted with a considerable difference between the phonon and ${}^3\text{He}$ temperatures. In the experiments one also has to control the spin polarization of about 10^{-3} - 10^{-4} can result in a drastic decrease of the transition temperature T_c . Such a polarization can be caused not only by a weak external magnetic field, but also by the prehistory of the sample, because the (dipole) magnetization relaxation times are very long.

The second circumstance is related to the fact that BCS pairing is possible for not too fast spin-lattice relaxation, $T_c \gtrsim \hbar/\tau^*$. From this point of view, the dipole processes do not lead to complications (the corresponding relaxation time τ_0 is long enough), but the collisions of ${}^3\text{He}$ quasiparticles with the walls with the magnetic coverage or impurities may be important. Simple estimates of the mean free paths of the quasiparticles in $T, T_c \lesssim 0.1 \text{ mK}$ show that this factor may become essential especially in experiments in a restricted geometry, when the solution is cooled in a porous medium in order to achieve effective cooling.

In summary, the situation concerning the ${}^3\text{He}$ superfluidity in ${}^3\text{He}$ - ${}^4\text{He}$ solutions is rather unclear. One can make with certainty only one assertion: since all experimental data suggest attraction in the s-wave channel for ${}^3\text{He}$ quasiparticles in ${}^3\text{He}$ - ${}^4\text{He}$ solutions (Bashkin and Meyerovich 1981) (in contrast to pure liquid ${}^3\text{He}$), ${}^3\text{He}$ superfluidity will be connected, at sufficiently low ${}^3\text{He}$ concentrations, with the usual BCS s-wave pairing of ${}^3\text{He}$ quasiparticles.

In the two following sections we discuss the influence of the spin polarization on such a pairing.

6.2. BCS PHASE OF A POLARIZED SOLUTION

Spin polarization of ${}^3\text{He}$ - ${}^4\text{He}$ solutions has two appreciable effects on the BCS phase of superfluid ${}^3\text{He}$ ↓ in ${}^3\text{He}$ ↓- ${}^4\text{He}$ solutions: a considerable decrease of the ${}^3\text{He}$ superfluid transition temperature and a change of the

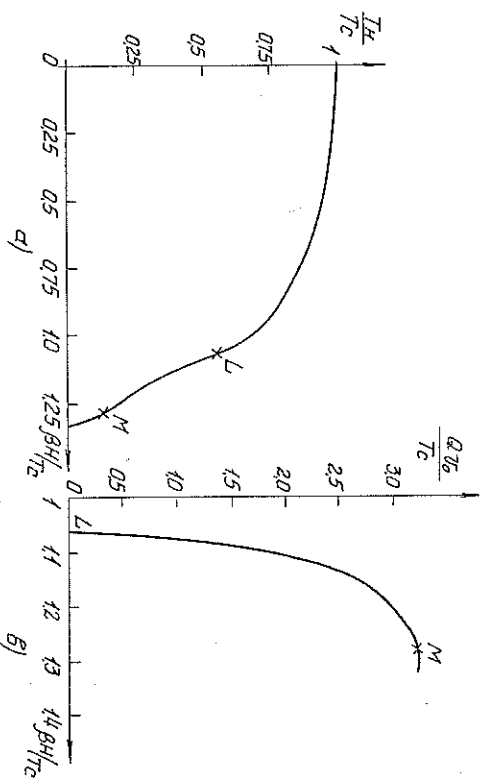


Fig. 10. (a) Temperature of superfluid transition and (b) momentum Q of Cooper pairs as functions of the field. Point L: eq. (6.10), point M: eq. (6.16), the final point: eq. (6.11). T_c is the superfluid transition temperature in the absence of the field.

spectrum of low-frequency hydrodynamic modes. The dependence of the ^3He transition temperature $T_H(H)$ on the magnetic field (spin polarization) is shown in fig. 10. Note that the characteristic scale of polarizations which change the thermodynamics of the transition is quite small and is determined by the parameter $\beta H/T_c$, where T_c is the transition temperature in the absence of a field (polarization). Pairing in the s-wave channel becomes absolutely impossible when $\beta H/T_c \geq 1.33$ (for $T_c \leq 0.1$ mK, this limitation on the magnetic field corresponds to $H \leq 1.6$ kOe).

Spin polarization also results in some peculiarities of the propagation of temperature waves. Temperature waves in ^3He - ^4He solutions with two Bose condensates are the analogues of second sound in pure He II at finite temperatures. In our case the temperature oscillations are accompanied by oscillations of the magnetization, leading to changes of the conditions of wave propagation and to some new experimental opportunities.

In the case of s-wave pairing the ^3He superfluid motion is not connected with the transport of magnetization, and in spin-polarized ^3He - ^4He solutions the dissipationless equations for the three velocity hydrodynamics of systems with two condensates (Khalatnikov 1957, Andreev and Bashkin 1975, Volovik et al. 1975, Bashkin and Meyerovich 1981) must be supplemented by the equation of motion for the longitudinal component of the magnetic moment M_z ,

$$M_z + \text{div}(M_z v^{(n)}) = 0. \quad (6.2)$$

The expression for the differential of the pressure \mathcal{P} has to be rewritten to include the magnetic contribution, which has the following form in a constant magnetic field:

$$\delta\mathcal{P} = (M_z/\chi) \delta M_z, \quad (6.3)$$

where $\chi = M/H$ is the longitudinal susceptibility of the solution. The expressions (6.2), (6.3) are analogous to the corresponding equations for the entropy S of solutions, because the entropy also takes part in the normal motion only:

$$\zeta + S \text{div} v^{(n)} = 0, \quad \delta\mathcal{P} = (TS/c) \delta S, \quad (6.4)$$

where c is the heat capacity of the solution. Comparison of eqs. (6.2)–(6.4) demonstrates that the propagation of the corresponding waves is accompanied by simultaneous coupled spin-temperature-entropy oscillations, and the velocity of wave propagation differs from the velocity of temperature waves in nonpolarized solutions (Andreev and Bashkin 1975, Bashkin and Meyerovich 1981) by the formal substitution of the quantity $TS^2/c + M_z^2/\chi$ for TS^2/c (Meyerovich 1984):

$$s^2 = \left(\frac{TS^2}{c} + \frac{M_z^2}{\chi} \right) \frac{\rho_s}{\rho_3 [\rho_3 - M\rho_s/m_3]}, \quad (6.5)$$

where $\rho_3 = M_3 N_3$, and $\rho_s(T, H)$ stands for the superfluid density in the superfluid gas of ^3He impurity quasiparticles.

As magnetic measurements are much simpler and more precise than calorimetric ones, spin-temperature waves can be investigated by exciting the waves by a change of the longitudinal component of the external magnetic field. In this case one has to add to eq. (6.3) the exciting force $-M_z \delta H$.

The velocity of spin-temperature waves (6.5) is extremely small, while the wave attenuation is rather high, resulting in some additional difficulties for experiments. Near the superfluid transition

$$s^2 \sim \frac{M_z^2}{\chi} \frac{\rho_s}{\rho_3^2} \sim \frac{\beta H}{m_3} \frac{\beta H}{T_0} \frac{\rho_s}{\rho_3} \lesssim \left(\frac{T_c}{T_0} \right)^2 \frac{\rho_s}{\rho_3} v_0^2, \quad (6.6)$$

where v_0 is the Fermi velocity. (In the last inequality we have used the relation $\beta H \leq T_c$, which is valid because in higher fields $\beta H > 1.33T_c$ s-wave pairing is impossible.) The attenuation of waves is governed by the

viscosity, thermal conductivity, spin diffusion and longitudinal spin relaxation. The last two mechanisms can be described by addition of the terms $D\Delta M_z$ and $\delta M_z/\tau^*$ to eq. (6.2). When one uses resonant excitation of spin-temperature waves, then low damping means that

$$D\omega/s^2 \ll 1, \quad \omega\tau^* \gg 1. \quad (6.7)$$

With the help of eq. (4.9) for the spin diffusion coefficient,

$$D \sim \frac{\hbar}{M} x^{-2/3} \left(\frac{T_0}{T} \right)^2$$

(x is the ^3He concentration), and eq. (6.6) for s , the former of inequalities (6.7) can be reduced to

$$\omega \leq \frac{T_c}{\hbar} x^{2/3} \left(\frac{T_c}{T_0} \right)^3 \frac{\rho_s}{\rho_3}, \quad (6.8)$$

and for low T_c low damping means that the oscillations are quasistatic. Condition (6.8) shows that resonant excitation of waves is difficult; at the same time nonresonant excitation considerably lowers the amplitude of the oscillations. The compatibility of conditions (6.7) demands that

$$s^2\tau^*/D \gg 1. \quad (6.9)$$

Whether this inequality holds or not depends on the type of longitudinal spin relaxation. When this is dipole relaxation, then $\tau^* = \tau_d$,

$$1/\tau_d \sim 10^{-15} x^{2/3} T_c^2 / \hbar T_0;$$

then condition (6.9) is equal to

$$(T_c/T_0)^2 (\rho_s/\rho_3) \gg 10^{-15},$$

and can easily be met. Additional difficulties for the observation of spin-temperature waves arises due to the large growth of the mean free path of ^3He quasiparticles and the transition to the Knudsen regime.

The above difficulties for the observation of spin-temperature waves are common not only to magnetic methods of wave generation and observation but also to the usual thermal methods. In this case the role of the spin diffusion and longitudinal relaxation are played by the viscosity, thermal conductivity and the scattering of quasiparticles by the walls, while the

limitation on the frequency of the oscillations (6.8) practically does not change.

6.3. INHOMOGENEOUS PHASES: SUPERFLUID LIQUID CRYSTALS

In not very high magnetic fields the superfluidity of ^3He in solution is caused, as in the absence of a field, by the s -wave pairing of ^3He quasiparticles. The s -pairs are formed from particles with opposite directions of spin projections and momenta. In a magnetic field the Fermi momenta of particles with opposite spin projections p_{\pm} are not equal to each other, and the relative shift of the Fermi spheres prevents the formation of BCS pairs with zero total momentum. On the other hand, the formation of pairs with nonzero total momentum raises the energy of the condensate. The competition of these effects results in a decrease of the temperature of the ^3He superfluid transition with increasing field, and in some range of fields the formation of pairs with zero total momentum becomes unfavourable. In this range pairing with nonzero total momentum is efficient, leading to a spatial inhomogeneity of the appearing superfluid phase.

The dependence of the superfluid transition temperature $T_H(H)/T_c$ on the value of the external field $\beta H/T_c$ (T_c is the transition temperature at zero field/polarization) is plotted in fig. 10a. To the left of point L, determined by

$$\beta H\sqrt{T_c} = 1.06, \quad T_H/T_c = 0.56 \quad (6.10)$$

(in low fields), pairing takes place with zero momentum, and the superfluid phase is of the ordinary BCS type. In higher fields $H > H_L$ pairing with nonzero momentum $Q(H)$ (see fig. 10b) takes place, and the corresponding phase is spatially inhomogeneous. Some properties of such a phase are described below.

In high fields,

$$\beta H \gg 1.33T_c, \quad (6.11)$$

s -wave pairing becomes absolutely impossible. In such fields the superfluid transition is due to p -wave pairing (if the interaction in the p -wave channel corresponds to an effective attraction), and the superfluid phase is somewhat similar to the A_1 -phase of superfluid pure ^3He in very high magnetic fields, $\beta H \gg \Delta$ (Δ is the energy gap).

An inhomogeneous phase with nonzero momentum of the pairs was first predicted for superconductors (Fulde and Ferrell 1964, Larkin and Ovchinnikov 1964). From the thermodynamic point of view the formation of

inhomogeneous phases in ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions and superconductors is very similar. However, there are essential differences between the properties of superconductors and ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions, which are caused by the absence of charges and the presence of two condensates (${}^3\text{He}$ and ${}^4\text{He}$) in ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions. For real superconductors the model of a spatially inhomogeneous phase is rather rough because it does not take into account the spin-orbit interaction, electron diamagnetism, scattering of quasiparticles from impurities, etc. The presence of impurities in superconductors seems to be the most important obstacle since the scattering of pairs with nonzero momentum from any impurities (in contrast to ordinary pairs with zero total momentum) leads to a loss of coherence and to the break-up of the pairs. For these reasons the superconducting inhomogeneous phase was not detected for nearly two decades. Only recently some signs of the existence of this phase have been observed [see, e.g., Buzdin et al. (1984)]. On the contrary, ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions represent a unique exchange and pure system of uncharged fermions, and consistent theoretical and experimental investigation of the inhomogeneous phase of these solutions seems to be more convenient.

The order parameter (the energy gap) in spatially inhomogeneous phases obtains some sort of "crystalline" structure,

$$\Delta(\mathbf{r}) = \sum_m \Delta_m \exp(i\mathbf{Q}_m \mathbf{r}/\hbar), \quad (6.12)$$

and the phase represents a peculiar superfluid liquid crystal. Near the transition point all vectors \mathbf{Q}_m in eq. (6.12) have the same magnitude, $|\mathbf{Q}_m| = Q$, given by the function $Q(H)$ in fig. 10b. The exact type of the expansion (6.12) corresponding to the minimum of the free energy is not yet known. Usually one investigates the phase with a single harmonic (the so-called Fulde-Ferrell phase):

$$\Delta(\mathbf{r}) = \Delta_0 \exp(i\mathbf{Q}\mathbf{r}/\hbar), \quad (6.13)$$

the layered phase,

$$\Delta(\mathbf{r}) = 2\Delta_0 \cos(\mathbf{Q}\mathbf{r}/\hbar), \quad (6.14)$$

or the cubic phase

$$\Delta(\mathbf{r}) = 2\Delta_0 \{ \cos(Qx/\hbar) + \cos(Qy/\hbar) + \cos(Qz/\hbar) \}. \quad (6.15)$$

Most probable are the structures (6.14) or (6.15). The order of the superfluid phase transition depends on the type of phase and on the strength of

the magnetic field even in the mean field approximation. In this approximation the transition is of second order for the Fulde-Ferrell phase and of first order for the cubic phase. For the layered phase (6.14), the superfluid transition is of first order in low fields $H_L < H < H_M$ (see fig. 10), with the point M determined by

$$\beta H_M/T_c = 1.28, \quad T_{H_M}/T_c \sim 0.3, \quad (6.16)$$

and of second order in higher fields (Malaspina and Rice 1971). If one takes the fluctuations into account (Brazovski 1975), then the divergence of the fluctuations near the transition point always makes the transition into an inhomogeneous phase to be of first order. Due to the rotational invariance of the normal phase of ${}^3\text{He}$ in ${}^3\text{He}\downarrow$ - ${}^4\text{He}$ solutions, the spatial orientation of the structure remains arbitrary in the zeroth-order approximation. The orientation of the structure (the orientation of the system of vectors \mathbf{Q}_m) is determined by the walls, the magnetic dipole interaction or the superfluid current (Meyerovich 1981, 1984). The first factor is the most important one. In the case of specular reflection from the walls, the most favourable structure is the structure (6.12) with one of the symmetry planes parallel to the wall.

The thermodynamics of the inhomogeneous phases of solutions is analogous to that of the inhomogeneous phases of superconductors (Larkin and Ovchinnikov 1964, Takada and Izuyama 1969). Thus, in the inhomogeneous phase the quasiparticle velocity in some directions can be close (or equal) to zero, resulting in a slow decrease of the specific heat with temperature and a strong anisotropy of the transport coefficients. In the equilibrium state of the inhomogeneous phases (6.12) the spatial distribution of ${}^3\text{He}$ particles does not depend on the coordinates, the superfluid currents are absent, but one can find a spin density wave. For example, the equilibrium magnetic moment M per unit volume of the layered structure (6.14) is equal to (Bashkin and Meyerovich 1981)

$$M(\mathbf{r}) = M_0 \left\{ 1 - \frac{|\Delta_0|^2}{(\beta H)^2} [F_1 + F_2 \cos(2\mathbf{Q}\mathbf{r}/\hbar)] \right\} \quad (6.17)$$

near the transition point, where $M_0 = \frac{3}{2}N_3(\beta H/T_0)$ is the magnetic moment above the transition, and the functions $F_1(H)$, $F_2(H)$ are plotted in fig. 11.

The most striking consequences of the inhomogeneity can be found in the hydrodynamics of the phases (6.12). The corresponding effects are associated with the anisotropy of the hydrodynamic equations and with the appearance of a new hydrodynamic variable — a translation vector \mathbf{u}

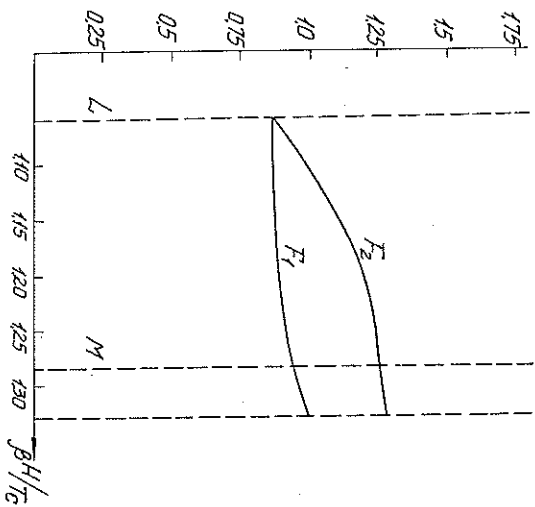


Fig. 11. The functions $F_{1,2}(H)$, eq. (6.17) (Bashkin and Meyerovich 1981).

(Meyerovich 1981, 1984). For example, the direction of ${}^3\text{He}$ mass flow does not coincide with the directions of velocities,

$$J_3 = \frac{1}{2m_3} \sum_m \rho_s^{(m)} \frac{q Q_m}{Q^2} Q_m + m_3 N_3 v^{(n)}, \quad (6.18)$$

$$q \equiv 2m_3 v_3 + 2(M - m_3) v_4 - 2M v^{(n)},$$

where $v^{(n)}$ is the normal velocity, and v_3, v_4 are the superfluid velocities for the ${}^3\text{He}$ and ${}^4\text{He}$ condensates. Near the transition the superfluid densities $\rho_s^{(m)}$ are equal to

$$\rho_s^{(m)} = \frac{m_3^2 N_3}{M} \frac{|\Delta_m|^2}{(\beta H)^2} \varphi^2(H). \quad (6.19)$$

The function $\varphi(H)$ is shown in fig. 12.

The existence of the new dynamic variable \mathbf{u} can be traced to the broken translational symmetry of the phases (6.12) (the translations $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{u}$ change the order parameter $\Delta(\mathbf{r})$). The translations \mathbf{u} may be introduced only for directions for which the transformations $\Delta(\mathbf{r}) \rightarrow \Delta(\mathbf{r} + \mathbf{u})$ are not identical: in the case of the one-dimensional phases (6.13), (6.14) only one

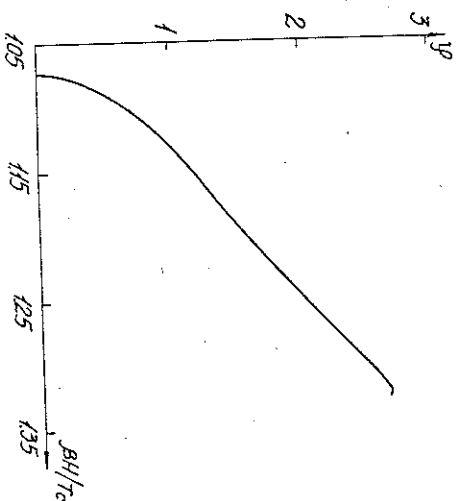


Fig. 12. The function $\varphi(H)$, eq. (6.19) (Meyerovich 1981).

component (parallel to \mathbf{Q}) of the vector \mathbf{u} has sense. Note, that the vector \mathbf{u} is not an entirely new independent variable: the vector \mathbf{u} determines the transformation $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{u}$ and is connected with the normal velocity.

In hydrodynamics we are interested in the linear response of the system to static deformations $\mathbf{u}(\mathbf{r})$ which are slowly varying in space. In this case the vector \mathbf{u} enters the linearized equations of motion via the symmetric deformation tensor $u_{ik} = \frac{1}{2}(\partial u_i / \partial x_k + \partial u_k / \partial x_i)$. Inversion $\mathbf{r} \rightarrow -\mathbf{r}$ changes the direction of mass flows, but does not change the tensor u_{ik} . For this reason, in the case of symmetric phases (6.12), $\Delta(\mathbf{r}) = \Delta(-\mathbf{r})$, the deformation u_{ik} does not lead to mass flows but results in an additional momentum flow:

$$\frac{\partial \Pi_{ik}}{\partial x_k} = \frac{Q^2}{m_3^2} \sum_m \rho_s^{(m)} (Q_m)_i \frac{\partial^2 (Q_m \mathbf{u})}{\partial (Q_m \mathbf{r})^2}. \quad (6.20)$$

The hydrodynamics of inhomogeneous symmetric phases of ${}^3\text{He}$ - ${}^4\text{He}$ solutions with two condensates was studied in Meyerovich (1981). The existence of the new dynamic variable gave rise to a new low-frequency Goldstone mode, and the anisotropy of the hydrodynamic equations caused a considerable anisotropy of the spectra of low-frequency modes. Three types of low-frequency modes can propagate through the usual BCS phase of solutions with two condensates (Bashkin and Meyerovich 1981): density oscillations (an analogue of first sound), oscillations of the ${}^3\text{He}$ concentration (an analogue of second sound in ${}^3\text{He}$ - ${}^4\text{He}$ II solutions above the

transition) and (spin-)temperature waves (see section 6.2). Such oscillations can also propagate in inhomogeneous phases; apart from these, one can find also in the inhomogeneous phase specific shear-wave-like oscillations of the structures.

The elasticity of the structure and the anisotropy of the equations practically have no influence on the oscillations of the density and the ^3He concentration: such oscillations are not very sensitive to the superfluid properties of the ^3He impurity component. In the inhomogeneous phase, as in the absence of a field, the velocity of propagation of the density oscillations is close to the sound velocity in pure He II, and the concentration oscillations propagate with a velocity close to $v_0/\sqrt{3}$ (v_0 is the ^3He Fermi velocity).

The inhomogeneity of the structure affects the slowest oscillations with a velocity exponentially small in the ^3He concentration. The velocity of spin-temperature waves in inhomogeneous phases of solutions is equal to

$$s^2 = s_T^2 \sum_m |\Delta_m|^2 \alpha_m^2 / \sum_m |\Delta_m|^2, \quad (6.21)$$

where $\alpha_m = \mathbf{n}e_m$ determines the angle between the wave vector of the oscillations $\mathbf{n} = \mathbf{k}/k$ and the axes of the "crystal" (6.12) $\mathbf{e}_m = \mathbf{Q}_m/Q$, and the quantity

$$s_T^2 = \frac{\rho_s}{m_3^2 N_3^2} \left(\frac{TS^2}{c} + \frac{M_s^2}{\chi} \right)$$

coincides formally with eq. (6.5) for the velocity of spin-temperature waves in the homogeneous phase. The velocity of elastic waves is of the same order of magnitude as the velocity of spin-temperature waves. The corresponding expression is rather cumbersome:

$$s^2 = \frac{\rho_s Q^2}{2m_3^2 M N_3} \left\{ \sum_m |\Delta_m|^2 \alpha_m^2 (1 - \alpha_m^2) \pm \left(\sum_{m,n} |\Delta_m \Delta_n|^2 \alpha_m^2 \alpha_n^2 \{ 2(r_{mn} - \alpha_m \alpha_n)^2 - (1 - \alpha_m^2)(1 - \alpha_n^2) \} \right)^{1/2} \right\} \times \left(\sum_m |\Delta_m|^2 \right)^{-1}, \quad r_{mn} \equiv \mathbf{e}_m \mathbf{e}_n. \quad (6.22)$$

In the case of the layered structure (6.14) the expressions for the velocities of the spin-temperature and elastic waves, eqs. (6.21), (6.22), reduce to

$$s^2 = s_T^2 \alpha^2, \quad s^2 = \frac{\rho_s Q^2}{m_3^2 M N_3} \alpha^2 (1 - \alpha^2), \quad \alpha = kQ/kQ. \quad (6.23)$$

The Fulde-Ferrell phase (6.13) differs considerably from the symmetric phase discussed above. First, in this phase without inversion symmetry, $\Delta(\mathbf{r}) \neq \Delta(-\mathbf{r})$, a deformation $u_{i,k}(\mathbf{r})$ results not only in a momentum flow, but also in a mass flow (Meyerovich 1981), etc. Second, in this phase a translation $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{u}(\mathbf{r})$ cannot be distinguished from superfluid motion (change of the phase of the order parameter (6.13) $\varphi \rightarrow \varphi + \delta\varphi(\mathbf{r})$), and the broken symmetry of this phase is a relative gauge-translation symmetry (Stern and Liu 1983). As a result, the number of Goldstone modes in the Fulde-Ferrell phase is less than in other inhomogeneous phases (6.12). In this phase the analogue of the spin-temperature and elastic waves is the coupled spin-temperature-shear wave with a velocity given by a combination of the velocities of the spin-temperature and shear elastic waves (6.23). The dynamics of the Fulde-Ferrell phase was studied in detail by Stern and Liu (1983) on the basis of general symmetry considerations.

The problem of structure defects in inhomogeneous phases has not been studied. The question of possible resonant properties of these phases is also unsettled.

7. Other spin-polarized ^3He Fermi systems. Concluding remarks

7.1. SOLID, LIQUID AND GASEOUS $^3\text{He}\downarrow$

Most of the above results for dilute $^3\text{He}\downarrow$ -He II solutions have a very simple and transparent form. Therefore, one faces a natural question: to what extent may these results be generalized to other types of spin-polarized quantum systems, especially to other phases of $^3\text{He}\downarrow$.

The simplicity of the results for $^3\text{He}\downarrow$ - ^4He solutions can be traced to two reasons: to the low ^3He concentration and to the quantum condition of long wavelength for the ^3He quasiparticles. Certainly, these two conditions are valid also for the $^3\text{He}\downarrow$ gas at low temperatures, and the $^3\text{He}\downarrow$ gas may be described by the same formulae as $^3\text{He}\downarrow$ -He II solutions in the Boltzmann temperature range with the trivial substitutions of the ^3He atomic mass m_3 for the effective mass M of the bare ^3He quasiparticles and of the scattering length a_{33} of ^3He atoms in vacuum for the s-wave

scattering length in solutions *a*. This procedure can be applied successfully at temperatures of the ^3He gas below 10^{-2} K.

At higher temperatures/energies for a number of reasons one has to take into account the energy dependence of the scattering amplitudes. Nevertheless, the available ^3He gas even at $T \geq 10^{-2}$ K is very dilute (typical values of the density are about $N_3 \leq 10^{17}$ – 10^{18} cm^{-3}), and the methods of calculation for dilute nondegenerate $^3\text{He}\downarrow$ – ^4He solutions can be applied to a $^3\text{He}\downarrow$ gas. Thus, the kinetic equation for a polarized ^3He gas has the same form as for $^3\text{He}\downarrow$ – He II solutions, eq. (5.2). Therefore, calculations of transport coefficients for a spin-polarized $^3\text{He}\downarrow$ gas can be performed using the results of the Chapman–Enskog theory for classical binary mixtures, though not in the hard-sphere model, but taking into account the exact energy dependencies of the scattering amplitudes and cross-sections (the usual integrals Ω in the classical theory of gases). In this case one can also observe very strong magnetokinetic effects – these effects are based on the smallness of the scattering amplitudes in channels with higher moments compared with the s-wave scattering amplitude, and the presence or absence of an energy dependence of the s-wave scattering amplitude is not essential. Calculations of the transport coefficients in the $^3\text{He}\downarrow$ gas with a realistic scattering amplitude can be found in Lhuillier and Laloté (1982) and Lhuillier (1983). In the $^3\text{He}\downarrow$ gas the magnitude of the magnetokinetic effects may be restricted by the presence of ^4He atoms in the gas. The values of the transport coefficients in a $^3\text{He}\downarrow$ – ^4He gaseous mixture can be obtained using the results of the Chapman–Enskog theory for ternary mixtures of usual ideal gases (Meyerovich 1983).

In $^3\text{He}\downarrow$ gas one can also observe spin waves, and the macroscopic equation of the transverse spin dynamics has the form (5.16). In this equation the frequency of precession of the magnetic moment in the molecular field Ω_{int} is determined by the exact scattering amplitude (Lhuillier and Laloté 1982, Lhuillier 1983, Bashkin 1981, 1984a,b).

Successful experiments with a spin-polarized $^3\text{He}\downarrow$ gas were performed by the Paris group. The gas was polarized using optical pumping (see, e.g., Leduc et al. (1983)); the brute force technique could not be applied because the ratio $\beta H/T$ is very small at temperatures $T \geq 1$ K). This group managed to obtain a $^3\text{He}\downarrow$ gas with a density $N_3 \sim 10^{18}$ cm^{-3} and a polarization $P \leq 70\%$ at temperatures $T \geq 2$ K. The lifetime of the polarized state was more than 60 hours. In this gas spin waves were detected irrespective of the presence of ^4He impurities (Nacher et al. 1984, Tastevin et al. 1985). The experimental results were in very good agreement with the predictions of the theory of spin-polarized quantum gases.

Note that spin polarization may help one to obtain ^3He gas at lower temperatures and higher densities than is possible for usual nonpolarized

^3He . The saturated vapour pressure of ^3He gas falls exponentially with decreasing temperature, and the main factor preventing the investigation of ^3He gas at low temperatures is condensation of the gas. Polarization of the ^3He spin system leads to a considerable increase of the energy of liquid ^3He , resulting in a decrease of the index in the exponent for the ^3He saturated vapour pressure.

If one neglects the effects associated with the excitation of the ^4He superfluid background, then the macroscopic properties of concentrated $^3\text{He}\downarrow$ – He II solutions differ only numerically from those of spin-polarized liquid normal $^3\text{He}\downarrow$. Both these systems are described by the same equations of the Landau theory for spin-polarized Fermi liquids (see section 2.2). Recent experiments on spin waves and the transverse spin dynamics in pure liquid $^3\text{He}\downarrow$ (Masuhara et al. 1984, Einzel et al. 1984) have once again confirmed the validity of the Leggett theory for the spin dynamics of weakly polarized Fermi liquids.

High degrees of spin polarization for normal liquid $^3\text{He}\downarrow$ have been achieved using the elegant method (suggested by Castaing and Nozières (1979)) of rapid melting of spin-polarized ^3He crystals. (The brute force technique, being ineffective for liquid ^3He due to the low ratio $\beta H/T$, is quite useful for solid ^3He at low temperatures.) This method made it possible to obtain liquid $^3\text{He}\downarrow$ with polarizations $P \leq 50\%$ with the depolarization time exceeding tens of minutes. However, the characteristic parameters of polarized liquid ^3He have not been measured yet. The available experimental results concern mainly the polarization shifts of the ^3He melting curve [see, e.g., Bonfait et al. (1984)]. Certainly, polarization of liquid ^3He has to result in lowering of the melting curve, and full polarization results in the disappearance of the Pomeranchuk minimum on the melting curve (Castaing and Nozières 1979, Lhuillier and Laloté 1979).

The properties of spin-polarized solid $^3\text{He}\downarrow$ seem to be quite different from those of liquid $^3\text{He}\downarrow$ – He II solutions. The low-temperature properties of solid ^3He are usually determined by Bose excitations of the system (phonons, magnons), while the properties of ^3He – He II solutions are related mainly to Fermi excitations (^3He quasiparticles). Nevertheless, ^3He crystals can be characterized also by a peculiar system of Fermi excitations corresponding to delocalized vacancies in crystals [see, e.g., the review by Andreev (1982)]. There are some indirect indications that one can find on the magnetic phase diagram of solid ^3He a region of the so-called vacancy ferromagnetic phase (Andreev et al. 1977, Andreev 1982). In this phase all ^3He spins are ferromagnetically ordered, and its energy is determined at $T \rightarrow 0$ by a small fraction of zero-point vacancies. The properties of this phase are very similar to those of dilute fully polarized $^3\text{He}\downarrow$ – He II solutions with the ^3He interaction in the p-wave channel.

Spin polarization leads also to interesting peculiarities of two-dimensional helium systems (^3He - ^4He solutions, helium films, adsorbed ^3He , wall boundary layers, etc.). However, the properties of such phases can hardly be described using an analogy with $^3\text{He}\downarrow$ - ^4He solutions due to the fundamental differences between three- and two-dimensional quantum systems.

An important confirmation of the fact that many quantum effects in spin-polarized quantum gases (1.4), (1.5) do not depend on the degree of degeneracy or the statistics of the gas, but are determined by the ultra-quantum character of the interaction of long-wavelength particles, has been given by the recent discovery of spin waves in a Bose gas (the spin-polarized gas of atomic hydrogen) in the Boltzmann temperature region (Johnson et al. 1984). The theory of spin wave propagation in the $\text{H}\downarrow$ gas practically completely coincides with the above theory of the transverse spin dynamics for a quantum gas of ^3He quasiparticles in the Boltzmann region [see, e.g., Levy and Ruckenstein (1984)].

7.2. UNUSUAL PHASES OF $^3\text{He}\downarrow$

Above we have already mentioned three new unusual phases of $^3\text{He}\downarrow$ whose very existence is due to the polarization of the ^3He spin system: semidegenerate $^3\text{He}\downarrow$ - ^4He solutions, the inhomogeneous phase of $^3\text{He}\downarrow$ - ^4He solutions with two Bose condensates, and the vacancy ferromagnetic phase of solid ^3He . While the existence of the first two phases is nearly beyond doubt, the question of the existence of the third phase may be settled only on the basis of experimental data. Spin polarization may provide the existence of some other unusual phases of $^3\text{He}\downarrow$ in the pressure-temperature-polarization phase diagram of ^3He .

The appearance of some of these new phases is connected with the properties of the $^3\text{He}\downarrow$ / $^3\text{He}\downarrow$ - ^4He phase equilibrium (Meyerovich 1983). It is already known (Castaing and Nozières 1979, Lhuillier and Laloe 1979, Spin-Polarized Quantum Systems 1980, Bashkin and Meyerovich 1981, Castaing et al. 1982) that the polarization of ^3He significantly changes the ^3He phase diagram, and, in particular, alters the mutual solubility of the helium isotopes. Nevertheless, there are two additional possibilities which seem to be interesting from both the theoretical and experimental points of view.

In time intervals less than the dipole depolarization time τ_d the equilibrium between $^3\text{He}\downarrow$ and a $^3\text{He}\downarrow$ - ^4He mixture is determined by two independent conditions, $\mu_1^+ = \mu_2^+$ and $\mu_1^- = \mu_2^-$, where μ_1^\pm and μ_2^\pm are the chemical potentials of ^3He particles with up and down spins in the pure phase and in the mixture. If the energy scales are considerably different in

both phases, polarization of the ^3He spin system leads to quite different polarization dependences $\mu_1^\pm(P_1)$ and $\mu_2^\pm(P_2)$. As a result the equilibrium conditions $\mu_1^\pm(P_1) = \mu_2^\pm(P_2)$ are fulfilled at different degrees of polarization $P_{1,2}$ of both phases, and the equilibrium ^3He concentration in the solution also depends on $P_{1,2}$.

The influence of polarization on the phase equilibrium is evident in the case of weak polarization when the change of energy with polarization is quadratic in $P_{1,2}$. In this case

$$P_{1,2} = \chi_{1,2} P_1 N_1 / (\chi_1 N_1 + \chi_2 N_2), \quad P_1 / P_2 = \chi_1 / \chi_2,$$

where $\chi_{1,2}$ are the susceptibilities per ^3He particle in pure ^3He and in the mixture, $P = (P_1 N_1 + P_2 N_2) / N_3$ is the total degree of spin polarization of the system, and N_1, N_2, N_3 ($N_1 + N_2 = N_3$) are the numbers of ^3He atoms in the pure phase and the mixture. Due to the evident dependence of the ^3He distribution between both phases, N_1/N_2 , on the relation between the numbers N_3 and N_4 of ^3He and ^4He atoms in the system, one can easily vary the polarizations P_1, P_2 by changing N_3/N_4 . The main difficulty in solving the phase equilibrium equations is usually caused by the lack of information on the functions $\mu^\pm(P)$ for dense $^3\text{He}\downarrow$ phases.

At low temperatures ^3He is not soluble in solid ^4He . The demixing temperature of the solid mixture into the pure components is about 0.1 K, and the difference in chemical potential of ^3He atoms within ^4He crystals and in pure nonpolarized (solid or liquid) ^3He is $\delta\mu_{\text{sol}} \leq 0.1$ K. ^3He and in pure nonpolarized (solid or liquid) ^3He is $\delta\mu_{\text{sol}} \leq 0.1$ K. ^3He polarization does not significantly change the ^3He chemical potentials in solid phases, and at low temperatures the solid $^3\text{He}\downarrow$ - ^4He mixture will continue to separate into pure $^3\text{He}\downarrow$ and ^4He crystals. Though the polarization leads to some decrease of the ^3He melting pressure, this decrease is not very large and is of the order of magnitude of the lowering of the ^3He melting curve caused by the Pomeranchuk effect. Thus, the $^3\text{He}\downarrow$ crystals melt at higher pressures than ^4He . As a result we see one of the following situations as the pressure decreases: the system consists either of two phases (solid ^4He -liquid $^3\text{He}\downarrow$) or of three phases (solid ^4He - liquid $^3\text{He}\downarrow$ - concentrated $^3\text{He}\downarrow$ - ^4He solution). In the absence of polarization both situations are possible. Below, it is not very important which of these possibilities is realized in the polarized system. What is important is that the polarization change of the ^3He chemical potential in liquid $^3\text{He}\downarrow$ or in concentrated $^3\text{He}\downarrow$ - ^4He II solutions is of the order of several tenths of a degree and is likely to exceed $\delta\mu_{\text{sol}} \sim 0.1$ K. This means that rapid changes of the pressure in some situations could result in the penetration of ^3He atoms into the ^4He crystal. This unique opportunity may give one a chance to study for the first time impurity quasiparticles in quantum crystals at a

temperature comparable with the bandwidth and the interaction energy of the quasiparticles.

There is an essential difference between the gas of ${}^3\text{He}$ quasiparticles in dilute solid and liquid ${}^3\text{He}$ – ${}^4\text{He}$ solutions. The bandwidth of ${}^3\text{He}$ impurity quasiparticles in ${}^4\text{He}$ quantum crystals is very small, $\Delta \sim 10^{-3}$ – 10^{-4} K, and at not very low temperatures $T \geq \Delta$ the quasiparticle band is filled uniformly. Therefore the characteristic momenta of the impurity quasiparticles are rather high, $p \sim \hbar/a_0$ (a_0 is the atomic dimension) even at low temperatures, leading to the violation of the quantum condition (1.4), (1.5) and to fundamental differences between the properties of ${}^3\text{He}$ impurity gas in solid and liquid ${}^4\text{He}$. At very low temperatures $T \ll \Delta$ the situation changes: all impurity quasiparticles in crystals are distributed near the bottom of their energy band, where their velocities are small and the energy spectrum is quadratic. This makes the properties of ${}^3\text{He}$ impurity gas in ${}^4\text{He}$ crystals similar to those of ${}^3\text{He}$ gas in ${}^3\text{He}$ – ${}^4\text{He}$ II solutions.

The increase of the solubility of ${}^3\text{He}$ in He II as a result of polarization widens the range of possible ${}^3\text{He}$ concentrations and provides means of investigating the gradual transition from an ideal Fermi gas to a dense Fermi liquid. The temperature of the ${}^3\text{He}$ superfluid transition in these concentrated polarized solutions in the case of p-wave pairing may exceed 10^{-5} K. The properties of the corresponding superfluid phase would be somewhat analogous to the A_1 -phase of superfluid ${}^3\text{He}$ and would be very different from those of superfluid ${}^3\text{He}$ in nonpolarized or weakly polarized ${}^3\text{He}$ – ${}^4\text{He}$ II solutions. The main difference with the ${}^3\text{He}$ A_1 -phase is related to the simultaneous presence of two Bose condensates (${}^3\text{He}$ and ${}^4\text{He}$) and to the corresponding drag effects. The possible increase of the ${}^3\text{He}$ solubility in liquid ${}^4\text{He}$ with polarization can even alter the superfluid properties of ${}^4\text{He}$.

7.3. CONCLUDING REMARKS

Spin polarization of ${}^3\text{He}$ – ${}^4\text{He}$ solutions leads to numerous interesting effects in thermodynamics, hydrodynamics, kinetics and spin dynamics. Some of these phenomena have already been investigated, some effects have not been observed yet. The absence of thorough theoretical calculations in the intermediate (between Boltzmann and degenerate) temperature region still impedes the analysis of experimental data. The more complicated theoretical problems are associated with a consistent evaluation of the parameters of the transverse spin dynamics and with the development of reliable approaches to concentrated solutions beyond the gas approximations. These approaches must not only help to obtain thermodynamic and

kinetic results, but also to investigate weak-coupling phenomena – superfluid pairing of quasiparticles. New experiments on the spin dynamics of solutions also seem to be desirable in a wide range of parameters. Magnetic effects and numerous related phenomena still remain unobserved. Without doubt, the main theoretical and experimental interest is attracted by the problem of ${}^3\text{He}$ superfluidity in ${}^3\text{He}$ – ${}^4\text{He}$ solutions including the case of polarized solutions. This question seems to be one of the most fascinating and promising problems of modern low-temperature physics.

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