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DEGENERACY EFFECTS IN THE SPIN DYNAMICS OF SPIN-POLARIZED FERMI GASES

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The spin dynamics of spin-polarized Fermi gases with arbitrary degrees of degeneracy is investigated. The spin-wave spectrum, phenomenological diffusion coefficients and the macroscopic equation of motion for the magnetic moment are derived. The degeneracy of the gas leads to a considerable anisotropy of the equations of motion.

1. Recently the theory of the propagation of spin waves in degenerate [1,2] and nondegenerate (Boltzmann) [2–6] spin-polarized quantum gases has been developed. Such spin oscillations were detected during the last several months for nearly all possible polarized quantum gases [7–10]. Moreover, in ref. [10] the results for the intermediate (between Boltzmann and degenerate) temperature region were reported. Below we give some additional comments on the spin dynamics of spin-polarized quantum gases including this intermediate region.

2. The simplest description of the spin dynamics of a dilute ($Na^3 \ll 1$) quantum ($a/\lambda \ll 1$) spin-polarized gas with an arbitrary degree of degeneracy can be given by a Fermi-liquidlike approach [2] (for dilute quantum gases in the main order in the interaction all first-order perturbation schemes and molecular field approaches give the same results). Above N is the number of (^{3}He) particles per unit volume, a is the s-wave scattering length, $\lambda \sim \hbar/(2mE)^{1/2}$ is the characteristic wavelength of particles with energy E and (effective) mass m. In this approximation the energy of single-particle excitations has the form $\hat{\epsilon} = \hat{\epsilon}_0 + \delta\hat{\epsilon}$,

$$\hat{\epsilon}_0 = \frac{p^2}{2m} - \beta\hat{\boldsymbol{\sigma}}\cdot \boldsymbol{H} + \frac{2\pi a\hbar^2}{m} N[1 + (N_+ - N_-)\hat{\boldsymbol{\sigma}}\cdot \boldsymbol{e}/N] ,$$

$$\delta\hat{\epsilon} = \frac{2\pi a\hbar^2}{m}\sum_p (\mathrm{Sp}\,\delta\hat{n} - \hat{\boldsymbol{\sigma}}\,\mathrm{Sp}\,\hat{\boldsymbol{\sigma}}\delta\hat{n}) , \qquad (1)$$

where $\boldsymbol{p}$ is the (quasi-)particle momentum, $\boldsymbol{e}$ is the unit vector along the direction of the equilibrium magnetization $\boldsymbol{M}_0$ (or an external magnetic field $\boldsymbol{H}$) $\boldsymbol{e} = \boldsymbol{M}_0/M_0$, $N_\pm$ correspond to the densities of particles with spin projections $\pm 1/2$ on the axis $e(z)$, $\delta\hat{n}$ is the deviation of the single-particle density matrix $\hat{n}$ (an operator in the spin space) from its equilibrium value $\hat{n}_0$, $\hat{\boldsymbol{\sigma}}$ are Pauli matrices. In the case of ^{3}He↑ gas ($m = m_3$ is the ^{3}He atomic mass, a is the s-wave scattering length of two ^{3}He atoms in vacuum) and dilute solutions of ^{3}He↑ in superfluid ^{4}He (the effective mass of ^{3}He quasiparticles $m = 2.3m_3$ and $a \sim -1.5$ Å at zero pressure) $\beta = 0.08$ mK/kOe is the ^{3}He nuclear magnetic moment. The results for other spin-polarized quantum gases, especially for H↑ bosons in the two lowest hyperfine states, do not differ essentially from the dilute phases of ^{3}He: one only has to change slightly the Zeeman terms and the scattering probability symmetrization [4,5,11].

The linearized kinetic equation for the Fourier components of the circular projections $m_\pm = m_x \pm i m_y$ of the magnetization vector $\boldsymbol{m} = \mathrm{Sp}\,\hat{\boldsymbol{\sigma}}\hat{n}$ is easily reduced to

$$(\omega - \boldsymbol{k}\cdot\boldsymbol{v} - \Omega_0 + \Omega_{\mathrm{int}})m_+ - \frac{2\pi a\hbar^2}{m}\left[\boldsymbol{k}\cdot\boldsymbol{v}\left(\frac{\partial n_+}{\partial\epsilon_+} + \frac{\partial n_-}{\partial\epsilon_-}\right) - \frac{2}{\hbar}(n_+ - n_-)\right] \times \int m_+ \frac{\mathrm{d}^3p}{(2\pi\hbar)^3} = \mathrm{Sp}\,\hat{\sigma}_+\hat{I}(n) , \qquad (2)$$

in accordance with (1) at an arbitrary degree of degeneracy of polarized (Fermi) gases (cf. eq. (4.5.1) of ref. [2] and eq. (2.13) of ref. [3]). Here $I(n)$ is the collisions integral, $\Omega_0 = 2\beta H/\hbar$ is the usual NMR frequency, and the presence of an internal frequency $\Omega_{int} = -(4\pi a\hbar/m)(N_+ - N_-)$ is due to the interaction contribution to the energy (1) and to the spin matrices commutator in the kinetic equation, $n_\pm(\epsilon_\pm)$ are the equilibrium distribution functions for up and down spins:

$$\hat{n}_0 = [n_+ + n_- + \boldsymbol{e}\cdot\boldsymbol{\sigma}(n_+ - n_-)]/2\,,$$

$$n_\pm(\epsilon_\pm) = \{1 - \mathrm{th}[(\mp\beta H + p^2/2m - \mu_\pm)/T]\}/2\,,$$

where the chemical potentials $\mu_\pm$ are equal, $\mu_+ = \mu_-$, if the polarization is in equilibrium and is caused by an external magnetic field. Note, that for dilute quantum gases $Na^3 \ll Ema^2/\hbar$ (this condition is always fulfilled for dilute ($Na^3 \ll 1$) low-temperature ($a/\lambda \ll 1$) gases) the first term in the square brackets in eq. (2) is negligible in comparison with the second one.

We use the following usual τ-representation for the spin-conserving exchange collisions integral:

$$\langle \mathrm{Sp}\, \hat{\boldsymbol{\sigma}}\hat{I}\rangle = 0\,,$$

$$\langle v_i\, \mathrm{Sp}\, \hat{\boldsymbol{\sigma}}\hat{I}\rangle = -\tau_\perp^{-1} J_i - (\tau_\parallel^{-1} - \tau_\perp^{-1})\boldsymbol{e}(\boldsymbol{e}\cdot J_i)\,, \qquad (3)$$

where $\tau_{\perp,\parallel}$ are the transverse and longitudinal relaxation times, $\langle\ \rangle$ stands for the averaging (integration) in the momentum space, and $J_i = \langle v_i\, \mathrm{Sp}\, \hat{\boldsymbol{\sigma}}\delta\hat{n}\rangle$ is the spin current. In the long wavelength limit $\delta\omega$, $kv \ll \Omega_{int}$ ($\delta\omega \equiv \omega - \Omega_0$) the spectrum of spin oscillations can be derived from eqs. (2), (3) by a simple expansion in $\delta\omega/\Omega_{int}$, kv/Ω_{int}:

$$\omega = \Omega_0 + \frac{1}{3}\frac{k^2}{\Omega_{int}}\,\frac{1 - \mathrm{i}/\Omega_{int}\tau_\perp}{1 + 1/\Omega_{int}^2\tau_\perp^2}\,\frac{N_+\langle v^2\rangle_+ - N_-\langle v^2\rangle_-}{N_+ - N_-}\,, \qquad (4)$$

where

$$\langle v^t\rangle_\pm = \frac{1}{N_\pm}\int v^t n_\pm \frac{\mathrm{d}^3p}{(2\pi\hbar)^3}\,,$$

is the average of v^t over the distribution functions $n_\pm$. The spin wave damping is small when $\Omega_{int}\tau_\perp \gg 1$. The last condition is equal ($\tau_\perp \sim 1/Nva^2$) to $(N_+ - N_-)/N \gg a/\lambda$ and is always met for a quantum ($a/\lambda \ll 1$) gas with not very low polarization. The spectrum (4) coincides in the case of a completely degenerate Fermi gas

$$\delta\omega = \frac{k^2}{5\Omega_{int}}\,\frac{1 - \mathrm{i}/\Omega_{int}\tau_\perp}{1 + 1/\Omega_{int}^2\tau_\perp^2}\cdot\frac{N_+v_+^2 - N_-v_-^2}{N_+ - N_-}\,,$$

with the corresponding equations of refs. [1,2], and in the case of nondegenerate Boltzmann gases

$$\delta\omega = \frac{k^2 v_T^2}{\Omega_{int}}\,\frac{1 - \mathrm{i}/\Omega_{int}\tau_\perp}{1 + 1/\Omega_{int}^2\tau_\perp^2}\,,$$

with the results of ref. [2–5] (above $v_T = (T/m)^{1/2}$ is the thermal velocity, $v_\pm = p_\pm/m = (6\pi^2 N_\pm)^{1/3}\hbar/m$ are the Fermi velocities of the particles with up and down spins; note, that eq. (2.22) of ref. [3] for the spectrum is invalid – the misprint is corrected in eq. (4.8.5) of ref. [2]).

3. The macroscopic equation of motion for the transverse components of magnetization $M_{x,y} = \langle \mathrm{Sp}\, \hat{\sigma}_{x,y}\hat{n}\rangle$ can be obtained using the procedure developed by Leggett [11] for degenerate Fermi systems with low polarizations. Integration of eq. (2) yields

$$\dot{M}_\perp + \nabla_i J_{i\perp} + \Omega_0 \boldsymbol{e}\times M = 0\,,$$

$$\dot{J}_{i\perp} + (\Omega_0 - \Omega_{int})\boldsymbol{e}\times J + \tau_\perp^{-1}J_{i\perp} + \tfrac{1}{3}\nabla_i M\,\frac{N_+\langle v^2\rangle_+ - N_-\langle v^2\rangle_-}{N_+ - N_-} = 0\,,$$

$$M_\perp \equiv M - \boldsymbol{e}(\boldsymbol{e}\cdot M)\,. \qquad (5)$$

The equation for the longitudinal component of magnetization $M_z = \boldsymbol{e}\cdot M$ has the form of the usual spin diffusion equations (below we neglect spin thermal and pressure diffusion terms [13,14])

$$(\partial/\partial t)(M_z/N) - D_\parallel N\Delta(M_z/N) = 0\,, \qquad (6)$$

while the phenomenological expression for the longitudinal spin diffusion coefficient can easily be derived with the help of equations from ref. [12]:

$$D_\parallel = \tau_\parallel N/(N_+\langle v^{-2}\rangle_- + N_-\langle v^{-2}\rangle_+)\,. \qquad (7)$$

Combining eqs. (5)–(7) one gets the following equation of motion for the magnetic moment of the spin-polarized quantum gas with an arbitrary degree of quantum degeneracy:

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$$\frac{\partial(M/N)}{\partial t} + \Omega_0 e \times M/N - \frac{D_\perp}{1+\Omega_{\rm int}^2\tau_\perp^2}\{\Delta M/N + [(D_\parallel/D_\perp)(1+\Omega_{\rm int}^2\tau_\perp^2) - 1]e(e\cdot\Delta M/N) + \Omega_{\rm int}\tau_\perp e\times\Delta M/N\} = 0\,,$$

$$D_\perp = \tfrac{1}{3}\tau_\perp \frac{N_+\langle v^2\rangle_+ - N_-\langle v^2\rangle_-}{N_+ - N_-}\,. \qquad (8)$$

This equation is a generalization of the corresponding results of ref. [14] on the case of two different relaxation times.

Eq. (8) and the spectrum (4) can be used both in the "hydrodynamic" $\delta\omega\tau_\perp^2 \ll 1$ and collisionless $\delta\omega\tau_\perp \gg 1$ limits. The only limitation on the frequency $\delta\omega \ll \Omega_{\rm int}$ and the condition of small damping $\Omega_{\rm int}\tau_\perp \gg 1$ do not depend on the value of $\delta\omega\tau_\perp$. Such a situation is common for the oscillations in systems with a large internal frequency caused by interactions (in our case the role of "plasma" or "cyclotron" frequencies is played by $\Omega_{\rm int}$). Note, that the factor $(1+1/\Omega_{\rm int}^2\tau_\perp^2)^{-1}$ in eqs. (4), (8) can be taken into account only in the "hydrodynamic" case when $\delta\omega/\Omega_{\rm int} \ll 1/\Omega_{\rm int}\tau_\perp$; in the opposite case the neglected terms of the order $\delta\omega/\Omega_{\rm int}$ may be more important. Often the notation μM is used (following refs. [6,12]) for our quantity $\Omega_{\rm int}\tau_\perp$ for some cases our notations seem to be more convenient. The values of the relaxation times $\tau_{\parallel,\perp}$ can be calculated using e.g. the general expression [13] for a collisions integral of dilute gases with an additional simplification: in the low-energy s-wave approximation which is valid for dilute quantum $a/\lambda \ll 1$ gases, the scattering probabilities do not depend on momenta.

4. If $D_\parallel = D_\perp$ eq. (8) coincides with the isotropic equations of motion [6,12–14]. Certainly this is true for low polarizations when $D_\parallel = D_\perp$, $\tau_\parallel = \tau_\perp$, and $D/\tau = \langle v^{-2}\rangle_\pm$. Such coincidence also takes place in the case of nondegenerate Boltzmann gases, when according to Lhuillier and Laloë [6] $D_\parallel = D_\perp$, and [see eqs. (7), (8)] $D_\perp/\tau_\perp = D_\parallel/\tau_\parallel = v_T^2$. The degeneracy of polarized gases leads to the violation of the above equalities. In the completely degenerate gas

$$D_\perp = \tfrac{1}{5}\tau_\perp \frac{v_+^2 N_+ - v_-^2 N_-}{N_+ - N_-}\,, \qquad D_\parallel = \tfrac{1}{3}\tau_\parallel \frac{N v_+^2 v_-^2}{N_+ v_+^2 + N_- v_-^2}\,. \qquad (9)$$

Besides, while the degeneracy of a Fermi gas results in a proportionality of $\tau_\parallel^{-1}$ to $(T/T_F)^2$, the expression for $\tau_\perp^{-1}$ contains a term without such a small factor. The value of $\tau_\perp$ determines the damping of the nondiagonal elements of the density matrix. In the general case of a degenerate polarized Fermi system these elements do not correspond to the well-defined single-particle excitations with long life-times remaining near the Fermi surfaces. Thus $\tau_\perp^{-1} \gg \tau_\parallel^{-1}$ and $\tau_\perp^{-1}$ is small for a polarized degenerate quantum gas only due to the low density of the gas.

Most of the experiments on the spin dynamics of polarized gases provide one with data on $\Omega_{\rm int}\tau_\perp$. Using e.g. eq. (8) it is easy to calculate the value of $D_\perp$ starting from these data. In the nondegenerate case $D_\parallel = D_\perp$, and the values of the decrement of the spin oscillation damping lead directly to the temperature dependence of the usual spin diffusion coefficient $D \equiv D_\parallel$. But at lower temperatures $D_\parallel \neq D_\perp$, and one cannot evaluate (at least now) the value of D using the experimental data on $\Omega_{\rm int}\tau_\perp$. This may be the reason for a deviation of the corresponding results of ref. [10] from the expected values of D at low temperatures (at higher temperatures in the Boltzmann region the results [10] for D coincide with the direct calculations of D [6,13,14] with a reasonable scattering amplitude).

Unfortunately the above results cannot be applied directly to the data of ref. [7]: the ^{3}He concentration in the experiments was too high. For ^{3}He concentrations that are greater than $\approx$1% the theory of dilute solutions [3] can be used only qualitatively (e.g. for concentrations of 3% or more the equations of ref. [3] lead to the conditions $|F_1^{a,s}| > |F_0^{a,s}|$ for the harmonics of the Fermi liquid function which are invalid for dilute gases).

5. For a polarized Fermi gas one can find [3,16] a specific region of parameters for which a spin-up component is degenerate $T \ll T_+ = (\hbar^2/2m)(6\pi^2 N)^{2/3}$, and the number of particles with down spins is small enough $N_- \ll N_+$ to make the spin-down component nondegenerate. In this "semidegenerate" region the above relations reduce to

$$\delta\omega = \frac{k^2}{\Omega_{\rm int}}\,\frac{1 - {\rm i}/\Omega_{\rm int}\tau_\perp}{1 + 1/\Omega_{\rm int}^2\tau_\perp^2}\,\frac{v_+^2 N_+/5 - v_T^2 N_-}{N_+ - N_-}\,,$$

$$D_{\parallel} = \tau_{\parallel} N v_+^2 v_T^2/(3N_+ v_+^2 + N_- v_T^2) \sim \tau_{\parallel} v_T^2/3 \,,$$

$$D_{\perp} = \tau_{\perp}(v_+^2 N_+/5 - v_T^2 N_-)/(N_+ - N_-) \sim \tau_{\perp} v_+^2/5 \,.$$

Notice that the limiting value of $D_{\parallel}/\tau_{\parallel}$ in polarized degenerate gases is nonzero [cf. eq. (9)]. These expressions for a semidegenerate gas are valid only if the contribution of spin-down particles exceeds the neglected high order terms in $N_+^{1/3}a$: $1 \gg (T/T_F)^{3/2} \gg N_-/N \gg (Na^3)(\hbar^2/mTa^2)$.

6. The above results were obtained within the framework of the molecular field approach and low-energy s-wave scattering approximation. The latter approximation can be justified in the spin-wave region: the conditions $\Omega_{int}\tau_{\perp} \gg 1$ and $a/\lambda \ll 1$ for a polarized gas practically coincide. But this approximation is not unavoidable – the molecular field approach developed can be easily generalized in order to take into account the exact energy dependence of the scattering amplitudes (see e.g. ref. [11]). More important is the question of the validity of the molecular field approach. Possible limitations may be caused by the nonlocal and quantum corrections to the left and right sides of the kinetic equation, damping of the single-particle excitations, etc. (see refs. [5,14]). The nonlocal terms in the kinetic equation (including the collision integral) seem to be the most dangerous ones. But it is easy to show that in the spin-wave region $\Omega_{int}\tau_{\perp} \gg 1$ all these difficulties disappear, and one can use the molecular field results in a straightforward manner (e.g. the first such results on the spectrum of spin waves in Boltzmann polarized gases [2,3] were confirmed by alternative methods). There are two alternative techniques for the spin dynamics of quantum gases [4,5]. Both methods go beyond the mean field approach and in principle can provide one with precise high order terms. But these formalisms though seemingly more consistent and accurate also do not take into account nonlocal contributions which may be necessary for $\Omega_{int}\tau_{\perp} \lesssim 1$ while in the case $\Omega_{int}\tau_{\perp} \gg 1$ the results of all first order perturbation schemes coincide and give quite accurate results.

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