Spin-Polarized ³He Fermi Systems

A. E. Meyerovich

Institute for Physical Problems, USSR Academy of Sciences, Moscow, USSR

Transport phenomena in spin-polarized ³He systems are studied (normal Fermi liquid ³He↑, ³He↑–He II solutions, gas ³He↑, ³He↑–⁴He gaseous mixtures). The transport coefficients, including the spin diffusion, spin thermodiffusion, and spin bulk viscosity coefficients, are calculated for spin-polarized Fermi liquids and gaseous mixtures. The analogy of the "spin rotation effect" in polarized nondegenerate gases with similar phenomena in degenerate Fermi systems and with collisionless spin oscillations is discussed.

1. INTRODUCTION

Considerable progress has been achieved in the study of spin-polarized quantum systems. Experimental[1-4] and theoretical[5-12] investigations of polarized Fermi systems concern mainly normal Fermi liquid ³He↑ and three dilute phases of ³He↑: liquid ³He↑–He II solutions, gas ³He↑, and ³He↑–⁴He gaseous mixtures (see also reviews in Refs. 13 and 14). Though the properties of all these phases are very different, the polarization of ³He spin systems leads in many cases to common effects. In this paper the influence of the spin polarization on hydrodynamics, kinetics and the helium phase diagram is discussed. In this section we introduce a hydrodynamic equation and kinetic coefficients corresponding to the motion of the ³He magnetic moment. The values of the kinetic coefficients are given in Section 2 for the dense polarized Fermi systems, and in Section 3 for dilute systems (some details of the calculations can be found in Appendices). In the last section we consider possibilities of observing the properties of two new phases at the ³He↑/³He↑–⁴He equilibrium.

The spin polarization that changes the values of all the kinetic coefficients and thermodynamic parameters of ³He systems also gives rise to an additional dynamical variable—the magnetic moment (per unit volume) **M**. This results in a new hydrodynamic equation, a low-frequency mode, and a number of new transport coefficients. The macroscopic

487

equation of motion for $\mathbf{M}$ can easily be derived in the same manner as in the well-known cases of usual polarized paramagnets or ferromagnets, taking into account the magnetic contribution to the energy $\delta E = \mathbf{H}_e\,\delta\mathbf{M}$ ($\mathbf{H}_e$ is an effective magnetic field). In the exchange approximation the equation in $\mathbf{M}$ takes the simple form of the magnetic moment conservation law,

$$\frac{\partial}{\partial t}M^\alpha + \frac{\partial}{\partial x_k} j_k^\alpha = 0 \tag{1}$$

and the problem reduces to the calculation of the spin current j_k^α (here and below, Greek indices denote the components of vectors in spin space, and Latin indices those in real space; in the exchange approximation there are no mixed summations over spin and spatial indices).

The characteristic feature of ^{3}He spin systems is the relation between the relaxation times of the spin-conserving exchange processes τ and the weak nuclear dipole interaction τ_d, which establishes the thermodynamic equilibrium value of $\mathbf{M}$. Even in dense helium systems τ_d at low temperatures T may be tens of minutes; in dilute ^{3}He phases the value of τ_d can be still larger. Therefore in the usual low-temperature experiments always $\omega\tau_d \gg 1$, while both regimes $\omega\tau \ll 1$ and $\omega\tau \gg 1$ can be realized. For this reason the nuclear dipole interaction provides only small corrections to the conservation law (1). Moreover, the weakness of the nuclear dipole interaction means that one can often observe nonequilibrium long-lived polarized states in which the magnetization $\mathbf{M}$ is not determined by the minimization of the total energy at constant external magnetic field $\mathbf{H}$. In these states the value of $\mathbf{M}$ may be by no means related to the field $\mathbf{H}$, and, in contrast to usual paramagnets, the polarized helium system can exist even in the absence of an external field. The creation of such quasiequilibrium states is one of the main methods of ^{3}He polarization. In not very dilute ^{3}He phases with high polarization always $H_e \gg H$.

In hydrodynamics the spin current is given by an expansion in small values of gradients. It is convenient to choose as a hydrodynamic variable instead of $\mathbf{M}$ the spin system polarization vector $\mathfrak{P} = \mathbf{M}/\beta N_3$ (here N_3 is the number of ^{3}He atoms per unit volume, $\beta \sim 0.08$ mK/kOe is the ^{3}He nuclear magnetic moment). The spin current being linearized in spatial gradients can always be represented in the form

$$j_k^\alpha = M^\alpha v_k + D^{\alpha\beta}\frac{\partial}{\partial x_k}\mathfrak{P}_\beta + A_T^\alpha\frac{\partial}{\partial x_k}T + A_P^\alpha\frac{\partial}{\partial x_k}P \tag{2}$$

where $\mathbf{v}$ is the fluid velocity and P is the pressure. In the exchange

approximation in the absence of the field, $H = 0$, the coefficients $D^{\alpha\beta}$ and $A^{\alpha}_{P,T}$ depend only on the direction of the unit vector $\mathbf{e} = \mathfrak{P}/\mathfrak{P}$ and can be written as

$$D^{\alpha\beta} = -\beta N_3 D[d\delta_{\alpha\beta} + (1-d)e_\alpha e_\beta + R\varepsilon^{\alpha\beta\gamma}e_\gamma]$$

$$A^{\alpha}_T = -\beta N_3 e_\alpha Dk_T/T, \qquad A^{\alpha}_P = -\beta N_3 e_\alpha Dk_P/P \tag{3}$$

and using the continuity equation, we find that Eq. (1) takes the form

$$N_3 \left(\frac{\partial}{\partial t}\mathfrak{P}_\alpha + v_k \frac{\partial}{\partial x_k}\mathfrak{P}_\alpha\right) - \frac{\partial}{\partial x_k}\left\{DN_3\left[d\frac{\partial}{\partial x_k}\mathfrak{P}_\alpha\right.\right.$$

$$\left.\left. + (1-d)e_\alpha e_\beta \frac{\partial}{\partial x_k}\mathfrak{P}_\beta + \frac{k_T}{T}\frac{\partial}{\partial x_k}T + \frac{k_P}{P}\frac{\partial}{\partial x_k}P\right]\right\}$$

$$= \frac{\partial}{\partial x_k}\left(\varepsilon^{\alpha\beta\gamma}DN_3 Re_\gamma \frac{\partial}{\partial x_k}\mathfrak{P}_\beta\right) \tag{4}$$

The right-hand side of Eq. (4) can be rewritten as

$$-\left[\mathfrak{P} \times \frac{\partial}{\partial x_k}\left(N_3 DR\mathfrak{P}^{-1}\frac{\partial}{\partial x_k}\mathfrak{P}\right)\right] \tag{5}$$

The representation of the coefficients in the spin current (2) had been chosen in the form of Eq. (3) in order to emphasize the analogy of Eq. (4) with a standard hydrodynamic equation for binary solutions (see, e.g., Ref. 15); in our case the vector $\mathfrak{P}$ behaves like the concentration of a dissolved component. This analogy is particularly transparent when the hydrodynamic perturbations do not change the direction of the magnetization (i.e., of the vector $\mathfrak{P}$—the z axis), and in Eq. (4) the anisotropy and the precession (5) vanish identically. In this case Eq. (4) is reduced to the scalar equation

$$N_3 \left(\frac{\partial}{\partial t}\mathfrak{P}_z + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\mathfrak{P}_z\right)$$

$$- \frac{\partial}{\partial \mathbf{r}}\left[DN_3\left(\frac{\partial}{\partial \mathbf{r}}\mathfrak{P}_z + \frac{k_T}{T}\frac{\partial}{\partial \mathbf{r}}T + \frac{k_P}{P}\frac{\partial}{\partial \mathbf{r}}P\right)\right] = 0 \tag{6}$$

This considerably simplifies the evaluation of the kinetic coefficients. In Eqs. (3), (4), and (6) the coefficients D, Dk_T, and Dk_P have the meaning of the spin diffusion, spin thermodiffusion, and spin pressure diffusion coefficients. The quantity Dd in Eqs. (3) and (4) is the transverse spin

diffusion coefficient and determines the damping of spin oscillations (the precession of the magnetic moment in the effective field in the presence of the transverse magnetization gradient), which are specified by the quantity DR.

The presence of an external magnetic field causes the precession of the magnetization vector [the additional term $2\beta[\mathbf{H} \times \mathbf{M}]/\hbar$ in Eq. (1)] and the formation of a new characteristic direction in the spin space. Since in polarized helium systems the vector $\mathbf{H}$ is not necessarily parallel to the quasiequilibrium value of $\mathbf{M}$, the presence of this new direction changes essentially all the symmetry properties of the system. In this case the vectors $\mathbf{A}_{T,P}$ are not parallel to $\mathbf{M}$, and the tensor $D^{\alpha\beta}$ is not diagonalized, as in Eq. (3), choosing the magnetization as one of the coordinate axes. Below, the direction of $\mathbf{H}$ (if $H \neq 0$) is always supposed to coincide with that of the quasiequilibrium magnetization.

The weak dipole interaction in most of the cases leads mainly to insignificant corrections of the hydrodynamic and kinetic parameters. The most important manifestation of this interaction is the violation of the magnetic moment conservation law, i.e., the inclusion of the right-hand side $\mathbf{K}(\mathfrak{P}, \mathbf{H})$ in Eq. (1). In the usual hydrodynamics of binary mixtures the analogous effect corresponds to taking into account slow chemical reactions and leads to the formation of second (bulk) viscosity.[15] In our case the dipole interaction also gives rise to the spin bulk viscosity. The calculation of $\mathbf{K}$ is simplified considerably due to the very small ratio of exchange and dipole relaxation times. Therefore to evaluate dM^α/dt one has to suppose all the distribution functions as equilibrium (with respect to energies and momenta) and to consider the collisions of ^{3}He particles with definite spin projections. When the magnetization deviates only slightly from the equilibrium value $\mathbf{M}_0(\mathbf{H})$

$$\mathbf{K} = -(\mathbf{M} - \mathbf{M}_0)/\tau_d \tag{7}$$

For markedly nonequilibrium systems with a high degree of polarization $\mathbf{K} \sim e(\mathfrak{P} + 1)^2 \beta N_3 / 4\tau'_d$, where τ'_d differs considerably from τ_d in Eq. (7).

The ^{3}He polarization does not much change the equations of the mass and momentum conservation laws. Only the expressions for the heat flow and the dissipative function are subjected to considerable modification; the expressions for these quantities can be easily derived by analogy with the usual diffusion equations[15] using Eqs. (2) and (3) for the spin current.

The form of the hydrodynamic equations (1)–(7) is quite general and does not suppose any assumptions concerning the degeneracy of the system or the character of the interaction. Analogous equations have already been studied for some types of helium systems[10,11,17] (see below).

2. DENSE FERMI LIQUID (^{3}He↑)

Unfortunately there is no consistent microscopic procedure for evaluating the ground state parameters of a dense Fermi liquid like ^{3}He. The relation between the parameters of the same system at different degrees of polarization is also unknown. Up to now it has been possible to perform such calculations only for dilute systems.[14] Therefore practically the only way to describe dense Fermi liquid ^{3}He↑ is to apply the Landau theory of Fermi liquids and to express all the quantities of interest in terms of harmonics of the f-function and phenomenological scattering probabilities at the given degree of polarization. In this case the comparison with experimental data will provide the information on the dependence of Fermi liquid characteristics on the degree of polarization.

In the exchange approximation the Fermi liquid function has the form (the directions of field and polarization coincide)

$$f_{\alpha\beta,\mu\nu}(\mathbf{p}, \mathbf{p}') = \psi(\mathbf{p}, \mathbf{p}')\delta_{\alpha\beta}\delta_{\mu\nu} + \zeta(\mathbf{p}, \mathbf{p}')\sigma_{\alpha\beta}\sigma_{\mu\nu}$$
$$+ [\varphi(\mathbf{p}, \mathbf{p}')\sigma_{\alpha\beta}\delta_{\mu\nu} + \varphi(\mathbf{p}', \mathbf{p})\sigma_{\mu\nu}\delta_{\alpha\beta}]\mathbf{e}$$
$$+ \xi(\mathbf{p}, \mathbf{p}')(\sigma_{\alpha\beta}\mathbf{e})(\sigma_{\mu\nu}\mathbf{e}) \tag{8}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the quasiparticle momenta and $\sigma_{\alpha\beta}$ are Pauli matrices. All the final results will include the harmonics in the expansion of the Fermi liquid function in Legendre polynomials on the Fermi surfaces of radii $p_\pm$ ($p_\pm$ are the radii of the Fermi spheres or Fermi momenta of quasiparticles with spin projections $\pm 1/2$ on the z axis, i.e., the axis $\mathfrak{B}$). Let us denote the corresponding harmonics as

$$\Psi_{ab}^{(n)} = \left(n + \frac{1}{2}\right)\frac{p_b m_b}{2\pi^2\hbar^3}\int \psi(\mathbf{p}_a, \mathbf{p}_b')P_n(\cos\chi)\, d\cos\chi \tag{9}$$

and similarly $Z_{ab}^{(n)}$, $\Phi_{ab}^{(n)}$, and $\Xi_{ab}^{(n)}$ for the functions ζ, φ, and ξ. Here the indices (a, b) take the values $(+)$ or $(-)$, χ is the angle between the vectors $\mathbf{p}_a$ and $\mathbf{p}_b'$, and $m_\pm$ denote the effective masses of quasiparticles on the corresponding Fermi surfaces. Most of the results are expressed through harmonics of the following four combinations of Fermi liquid functions:

$$A_\pm^{(n)} = \Psi_{\pm\pm}^{(n)} + Z_{\pm\pm}^{(n)} \pm 2\Phi_{\pm\pm}^{(n)} + \Xi_{\pm\pm}^{(n)} + 1$$
$$B_\pm^{(n)} = \Psi_{\pm\mp}^{(n)} - Z_{\pm\mp}^{(n)} \pm \Phi_{\pm\mp}^{(n)} \mp \Phi_{\mp\pm}^{(n)} - \Xi_{\pm\mp}^{(n)} \tag{10}$$

In the dense spin-polarized Fermi liquids not all the components of the density matrix represent the well-defined quasiparticles with long lifetimes for which the Landau theory can be used in a straightforward manner. Only well-defined quasiparticles participate in phenomena that can be

described using the Fermi liquid functions in the combinations A and B in (10). These phenomena include most of the thermodynamic and transport processes and can be described using the diagonalized density matrix. All these processes can be expressed in terms of well-defined quasiparticles with definite spin projections, and while the direction of the magnetic moment does not change, all the quasiparticles remain near the respective Fermi surfaces. The problems with not well-defined quasiparticles and with the use of the Landau theory with Fermi liquid functions (8) not in the combinations (10) arise mainly when one is studying the dynamics of nondiagonal elements of the density matrix (processes accompanied by a change of the direction of the magnetic moment). In these cases there also is an additional problem with gradient terms in the free energy (see below). Note that all these problems vanish for dilute Fermi systems in the lowest orders in the interaction (concentration).

With the help of the Fermi liquid functions (8)–(10) one can easily obtain the values of the main thermodynamic characteristics of the system; the evaluation procedure and the results are similar to those of the Landau theory of the multicomponent Fermi liquid (see Appendix A and, e.g., Ref. 16).

In the case of the dense Fermi liquid ^{3}He$\uparrow$ an equation of the form (4) has been already derived (without spin thermodiffusion and pressure diffusion contributions) by Leggett[17] on the basis of the kinetic equation. Though the results of Ref. 17 (see also Ref. 18) were obtained as an expansion in the value of $\mathbf{M}$, which was supposed to be small, the theory[17] can be easily generalized to quasiequilibrium states with a high degree of polarization.

The main characteristic of the magnetic moment precession in an effective field is $\omega_{int}\tau_D$, where τ_D is the exchange diffusion relaxation time, and ω_{int} is the frequency inherent in the spin oscillations in the molecular field. The origin of ω_{int} is due to the term

$$(i/\hbar)[\hat{\varepsilon}, \hat{n}] \tag{11}$$

in the kinetic equation, where $\hat{n} = n_{\alpha\beta}$ and $\hat{\varepsilon} = \varepsilon_{\alpha\beta}$ are the density matrix and the energy of Fermi liquid quasiparticles, and $[\cdot, \cdot]$ denotes the spin matrix commutator. For the quasiparticles the energy includes the term[14,17-19]

$$\int \zeta(\mathbf{p}, \mathbf{p}')[n_0^+(\mathbf{p}') - n_0^-(\mathbf{p}')] \, d^3p'/(2\pi\hbar)^3 \tag{12}$$

where $n_0^{\pm}$ are the equilibrium distribution functions of up and down spins, and the frequency ω_{int} is of the order of (in the notations of Ref. 17 $\omega_{int}\tau_D$

corresponds to μM)

$$\omega_{\text{int}} \sim \zeta^{(0)} N_3 \mathfrak{P} / \hbar, \qquad \omega_{\text{int}} \tau_D \sim \mathfrak{P}(T_0/T)^2 \tag{13}$$

where T_0 is the degeneracy (Fermi) temperature. Thus, but for the case of very weak polarization $\mathfrak{P} \lesssim (T/T_0)^2 \ll 1$, the value $\omega_{\text{int}} \tau_D \gg 1$. Hence, despite the fact that we are discussing the hydrodynamic region of low frequencies $\omega \tau_D \ll 1$ and small gradients, the spin oscillations behave just as in the high-frequency collisionless case. For this reason the value of $DR = \nu$ in Eq. (4) is equal to the proportionality factor ν in the spectrum of collisionless spin waves in a polarized Fermi liquid[18,19]

$$\omega = \nu k^2 \tag{14}$$

and does not depend on τ_D (while $D \propto \tau_D$, the value $R \propto 1/\tau_D$). Taking into account also the weakly polarized states, we easily obtain as in Ref. 17

$$D = \nu \omega_{\text{int}} \tau_D, \qquad R = \omega_{\text{int}} \tau_D / [1 + (\omega_{\text{int}} \tau_D)^2] \tag{15}$$

Similarly, we get the following for the coefficient d in Eq. (4):

$$Dd = [1 + (\omega_{\text{int}} \tau_D)^2]^{-1} \tag{16}$$

Unfortunately, one cannot use for the coefficient ν (and ω_{int}) in Eq. (14) the value of Refs. 17–19 in the case of a dense, strongly interacting Fermi liquid with a high degree of polarization.

In Refs. 17–19 the frequency of precession of the magnetization vector in the molecular field has been expressed in terms of the Fermi liquid function. Such expressions, which are always valid for Fermi liquid oscillations with a linear spectrum $\omega(\mathbf{k})$, can be used for oscillations with a square-law dispersion (14) only in the cases (see, e.g., Ref. 14) of dilute or weakly interacting Fermi systems, $p_F a_0 / \hbar \ll 1$ (p_F is the Fermi momentum, a_0 is the interaction radius), because the corresponding approach takes into account only the local Fermi liquid interaction (terms of order $1/a_0$) and neglects all nonlocal effects ($\sim p_F/\hbar$), which correspond to the gradient expansion of the Fermi liquid energy. Microscopically, this means a lack of accuracy in the high-order terms in $\mathbf{k}$ as one derives the quasi-classical kinetic equation from the exact quantum equation. Being derived macroscopically, the value ν is given, apart from the usual f-function (8), also by additional phenomenological functions, which determine the functional dependence of the quasiparticle energy on the distribution function gradients. For example, there exists a term

$$\delta \varepsilon_{\alpha\beta} = \int \tilde{f}_{\alpha\beta,\mu\nu}(\mathbf{p}, \mathbf{p}') \frac{\partial^2}{\partial x_i^2} \delta n(\mathbf{p}') \frac{d^3 p'}{(2\pi\hbar)^3} \tag{17}$$

where the spinor structure of the function $\tilde{f}$ is analogous (in the exchange

approximation) to Eq. (8). The significant contribution of the energy (17) to the kinetic equation arises up to terms of the order of k^2 only from the operator (11). This gives rise in the equation for **M** to an additional term of the form

$$\frac{\partial}{\partial x_k}\left[\mathbf{M}\times\frac{\partial \mathbf{M}}{\partial x_k}\right]$$

with the coefficient depending on first harmonics of the function $\tilde{\zeta}(\mathbf{p},\mathbf{p}')$. In the case of a low degree of polarization the influence of Eq. (17) reduces to the addition of

$$\frac{\tilde{\zeta}^{(0)}}{\hbar}\frac{\partial}{\partial x_i}\left[\mathbf{M}\times\frac{\partial \mathbf{M}}{\partial x_i}\right]$$

to Eq. (19) in Ref. 17, and it can be seen that this correction is small while M is small. In the cases of high polarizations similar corrections are considerable and must be taken into account. The coefficients ν in (14) and ω_{int} in (13), being considered as phenomenological parameters of dense Fermi liquids, have to be determined from experimental data. These coefficients are simply related to the effective field.

All the other kinetic coefficients, including those in Eqs. (4)–(5), can be evaluated explicitly and expressed by phenomenological probabilities $W(\mathbf{p},\mathbf{p}')$ for quasiparticle pair collisions near the Fermi surfaces of radii $p_\pm$: $W_+ = W(\mathbf{p}_+,\mathbf{p}'_+)$, $W_- = W(\mathbf{p}_-,\mathbf{p}'_-)$, $W' = W(\mathbf{p}_+,\mathbf{p}'_-)$. As a result (see Appendix B),

$$D = \frac{4}{3}\frac{\pi^2\hbar^6 p_0^2}{T^2 m^5}\frac{m^4}{m_+^2 m_-^2}\left(\frac{p_+}{p_0}\right)^3 N_- N_+$$

$$\times \frac{A_+^{(0)}A_-^{(0)} - B_+^{(0)}B_-^{(0)}}{N_+ N_-(\alpha_+ B_+^{(0)} + \alpha_- B_-^{(0)}) + N_+^2 \alpha_- A_+^{(0)} + N_-^2 \alpha_+ A_-^{(0)}}$$

$$\times \left\langle W'\sin^2\theta\,\sin^2\frac{\varphi}{2}\Big/f_-^3\right\rangle^{-1} \tag{18}$$

$$\frac{Dk_T}{T} = \frac{3\pi^4\hbar^6}{T^2 m_+^2 m_-^2}\frac{Tm}{p_0^2}\frac{m_+ p_-^2(U_- - V_-) - m_- p_+^2(U_+ - V_+)}{mp_0^2(U_+U_- - V_+V_-)} \tag{19}$$

$$U_\pm = \left\langle W'\left(1+\sin^2\theta\,\sin^2\frac{\varphi}{2}\Big/f_\pm^2\right)\Big/f_\pm\right\rangle$$

$$+\frac{m_\pm^2}{m_\mp^2}\left\langle W_\pm\frac{1-\cos\theta}{(2+2\cos\theta)^{1/2}}\right\rangle$$

$$V_{\mp} = -\frac{p_{\mp}}{p_{\pm}}\left\langle\frac{W'}{f_{\pm}}\left(\cos\theta + \frac{p_{\pm}}{p_{\mp}}\frac{\sin^2\theta\sin^2(\varphi/2)}{f_{\pm}^2}\right)\right\rangle$$

$$f_{\pm} = \left[1+\left(\frac{p_{\pm}}{p_{\mp}}\right)^2 + 2\left(\frac{p_{\pm}}{p_{\mp}}\right)\cos\theta\right]^{1/2}, \qquad \alpha_{\pm} = \frac{m_{\pm}p_{\pm}}{mp_0}$$

where p_0 and m are the Fermi momentum and the effective mass in nonpolarized ^{3}He, $N_{\pm}$ are the numbers of ^{3}He particles with up or down spins per unit volume of ^{3}He ($N_+ + N_- = N_3$), the quantities $A^{(0)}$ and $B^{(0)}$ are given by Eq. (10), and $\langle\cdots\rangle$ means angular averaging. The spin pressure diffusion coefficient is evaluated from the usual thermodynamic relation[15]

$$k_P = (P/N_3^2)[\partial(\mu_+ - \mu_-)/\partial N_3]_P^{-1}$$

($\mu_{\pm}$ are the chemical potentials for up and down spins), and can be transformed by Eqs. (A3)–(A4) to the form

$$k_P = \frac{3P}{N_3^3}\frac{(N_+m_+/p_+^2)(N_+B_+^{(0)}+N_-A_-^{(0)})-(N_-m_-/p_-^2)(N_-B_-^{(0)}+N_+A_+^{(0)})}{B_+^{(0)}B_-^{(0)}-A_+^{(0)}A_-^{(0)}}$$

$$(20)$$

The values of the viscosity ζ and thermal conductivity $\varkappa$ coefficients are given by

$$\eta = \frac{2}{45}\frac{\hbar^3 p_+^5}{T^2 m_+^2 m_-^2}\frac{(p_+/p_-)^3 X_\eta^- + (p_-/p_+)^5 X_\eta^+ - (p_-/p_+)^2 Y_\eta^- - Y_\eta^+}{X_\eta^+ X_\eta^- - Y_\eta^+ Y_\eta^-}$$

$$\varkappa = \frac{2}{3}\frac{\pi^2\hbar^3 p_+^3}{T m_+^2 m_-^2}\frac{(p_+/p_-)X_\varkappa^- + (p_-/p_+)^3 X_\varkappa^+ - (p_-/p_+)^2 Y_\varkappa^+ - Y_\varkappa^-}{X_\varkappa^+ X_\varkappa^- - Y_\varkappa^+ Y_\varkappa^-}$$

$$(21)$$

$$X_\eta^{\pm} = \left\langle\frac{\sin^2\theta\sin^2(\varphi/2)}{f_-^3}\left[W'\left(1-\frac{\sin^2\theta\sin^2(\varphi/2)}{f_{\pm}^2}\right)\right.\right.$$

$$\left.\left. + W_{\pm}\frac{m_{\pm}^2}{m_{\mp}^2}\frac{p_+^3}{p_{\mp}^3}\frac{f_-^3}{(2+2\cos\theta)^{3/2}}\frac{\sin^2\theta\cos^2(\varphi/2)}{1+\cos\theta}\right]\right\rangle$$

$$Y_\eta^{\pm} = -\left\langle\frac{p_{\mp}}{p_{\pm}}\frac{\sin^2\theta\sin^2(\varphi/2)}{f_-^3}W'\left(\cos\theta + \frac{\sin^2\theta\sin^2(\varphi/2)}{f_+ f_-}\right)\right\rangle$$

$$(22)$$

$$X_\varkappa^{\pm} = \left\langle\frac{1}{f_-}\left[W'\left(1+\frac{\sin^2\theta\sin^2(\varphi/2)}{f_{\pm}^2}\right)\right.\right.$$

$$\left.\left. + W_{\pm}f_{\mp}\frac{m_{\pm}^2 p_{\pm}}{m_{\mp}^2 p_{\mp}}\frac{1-\cos\theta}{(2+2\cos\theta)^{1/2}}\right]\right\rangle$$

$$Y_\varkappa^{\pm} = -\left\langle\frac{W'}{f_-}\frac{p_{\pm}}{p_{\mp}}\left(\cos\theta + \frac{\sin^2\theta\sin^2(\varphi/2)}{f_+ f_-}\right)\right\rangle$$

As noted in Section 1, the dipole relaxation leads only to insignificant corrections (of the order $\tau/\tau_d \ll 1$) to the transport coefficients (18)–(22). Therefore the magnetization relaxation is important only for the phenomena which vanish to the exchange approximation. In ^{3}He↑ hydrodynamics such a phenomenon is the spin second (bulk) viscosity. Slow dipole processes tending to level off the chemical potentials μ_+ and μ_- cause the same consequences as, for example, slow chemical reactions in usual hydrodynamics.[15] These slow processes with a very long relaxation time give rise to the bulk viscosity

$$\zeta = \zeta_0(1 - i\omega\tau_d)^{-1} \tag{23}$$

where the low-frequency limit $\zeta(\omega \to 0) = \zeta_0$ is given by

$$\zeta_0 = m_3 N_3 \tau_d (c_\infty^2 - c_0^2) \tag{24}$$

Here c_0 is the first-sound velocity in the low-frequency limit $\omega\tau_d \to 0$ in which the dipole interaction manages to establish local equilibrium values of the magnetization, and c_∞ is the sound velocity in the opposite limit $\omega\tau_d \to \infty$ in which the oscillations are too fast for the equilibrium value M_0 to be reached. Certainly, one can discuss the spin second viscosity only in the case of equilibrium (in the nonperturbed state) polarization. Then the sound propagation is governed by Eqs. (1) and (6) with the right-hand side (7).

The sound velocity c, i.e., the derivative $\partial P/\partial N_3$, is given by Eq. (A4), while the derivatives $dN_\pm/dN_3$ have to be calculated on the basis of Eq. (6) with the right-hand side (7):

$$\frac{\partial N_\pm}{\partial N_3} = \frac{1}{1 + i/\omega\tau_d}\left(\frac{N_\pm^{(0)}}{N_3} + \frac{i}{\omega\tau_d}\frac{\partial N_\pm^{(0)}}{\partial N_3}\right)$$

where $N_\pm^{(0)}$ are the local equilibrium values of the spin densities $N_\pm$ [in Eq. (24) we did not take into account those corrections due to the exchange relaxation that are insignificant for the problem in question]. For the quantities $\delta N_\pm^{(0)}$ and $\delta\mu_\pm^{(0)}$ obeying the relations $\delta N_+^{(0)} + \delta N_-^{(0)} = \delta N_3$ and $\delta\mu_+^{(0)} = \delta\mu_-^{(0)}$ the derivatives $\partial N_\pm^{(0)}/\partial N_3$ can be easily calculated using Eq. (A3). As a result

$$c^2 = \frac{1}{m_3}\frac{\partial P}{\partial N_3} = \frac{i\omega\tau_d c_\infty^2 + c_0^2}{1 + i\omega\tau_d} \tag{25}$$

where the limiting values of the sound velocity c_0 and c_∞ are given by

$$c_0^2 = \frac{N_3(A_+^{(0)}A_-^{(0)} - B_+^{(0)}B_-^{(0)})}{m_3\nu_-(A_+^{(0)} - B_-^{(0)}) + m_3\nu_+(A_-^{(0)} - B_+^{(0)})}, \qquad \nu_\pm = \frac{p_\pm m_\pm}{2\pi^2\hbar^3} \tag{26}$$

$$c_\infty^2 = \frac{1}{m_3 N_3 \nu_+ \nu_-}[\nu_- N_+^2 A_+^{(0)} + \nu_+ N_-^2 A_-^{(0)} + N_+ N_-(\nu_- B_-^{(0)} + \nu_+ B_+^{(0)})]$$

Of course, when the polarization tends to zero, the spin second viscosity of polarized Fermi liquids (23), (24), (26) vanishes. According to Eq. (25), the spectrum $\omega(k)$ of sound oscillations has the form

$$k = (\omega/c_0)[1 + \omega^2\tau_d^2 s^2 + i(s^2 - 1)\omega\tau_d]^{1/2}/(1 + \omega^2\tau_d^2 s^4)^{1/2} \qquad (27)$$

where $s = c_0/c_\infty$. In the usual experiments $\omega\tau_d \gg 1$ and Eq. (27) reduces to

$$k = \omega/c_\infty + i\zeta_0/2m_3 N_3 c_\infty^3 \tau_d^2 \qquad (28)$$

where ζ_0 is given by Eqs. (22) and (26).

In ^{3}He↑ with a high degree of polarization the relative contributions of the shear and bulk viscosities are determined by the parameter $\omega^2\tau\tau_d$. A rough estimation shows that the spin second viscosity dominates at $T \sim 1$ mK only in quasistationary situations. A more accurate estimation is impossible due to a lack of information on τ_d in ^{3}He↑ with a high degree of polarization.

Equations (18)–(26) give the values of the transport coefficients of dense spin-polarized Fermi liquids. Analogous results for dilute Fermi liquids can be found in Ref. 11.

3. LOW-DENSITY ^{3}He↑ SYSTEMS

The kinetic behavior of dilute Fermi systems is of special interest, for the spin polarization of such systems leads to significant magnetokinetic phenomena—to the gigantic, practically unlimited growth of the kinetic coefficients with increasing polarization.[5,9–11,14]

There exist three types of possible dilute ^{3}He↑ systems: liquid ^{3}He↑–He II solutions, a ^{3}He↑ gas, and ^{3}He↑–^{4}He gaseous mixtures. While the first of these systems may be, depending on the ^{3}He concentration and temperature, degenerate or nondegenerate, the second and the third are always nondegenerate. Nevertheless, there is a general procedure for studying all these systems in the same way with sufficient accuracy at low temperatures. This is made possible by the large value of the ^{3}He particle wavelength $\hbar/p$ (p is the characteristic momentum) in comparison with the interacton radius r_0. In degenerate ^{3}He–He II solutions the condition

$$p_F r_0/\hbar \sim N_3^{1/3} r_0 \ll 1 \qquad (29)$$

(p_F is the Fermi momentum) is met because of the low density N_3 of ^{3}He particles in a solution, and for nondegenerate systems

$$p_T r_0/\hbar \sim (mT)^{1/2} r_0 \ll 1 \qquad (30)$$

(p_T is the thermal momentum) because of low temperature. According to quantum mechanics, the interaction of these slow particles with large

wavelengths reduces mostly to S-wave scattering with amplitude indepen-
dent of momenta. Therefore all interaction phenomena can be described
to a high degree of accuracy by taking into account only the S-wave
scattering and using as interaction parameters only the S-wave scattering
length[14] a (collisions of ^{3}He quasiparticles in ^{3}He–He II solutions) or, in
the cases of ^{3}He and ^{3}He–^{4}He gases, the S-wave scattering lengths a_{33},
a_{34}, and a_{44} (collisions ^{3}He–^{3}He, ^{3}He–^{4}He, and ^{4}He–^{4}He, respectively).
For this reason one can apply, e.g., the results[6,9,11,14] obtained for ^{3}He–He
II nondegenerate solutions in the first order in the interaction, to the ^{3}He
gas, replacing the effective mass M and the scattering length a for quasipar-
ticles in solutions by the ^{3}He atomic mass m_3 and the scattering length a_{33}.

Within this approach there are no fundamental differences between
the theories of degenerate and nondegenerate dilute ^{3}He systems. To
the main order in the interaction one can even apply the formalism
of the Fermi liquid theory to the dilute nondegenerate Fermi gas (though
the Fermi liquid excitations undergo a strong damping in the case of
nondegenerate systems, this damping takes place only to the high-order
terms unimportant for a dilute gas; for details see Ref. 14). The main order
in the interaction in our case means exactly the principal term in the
expansion in $pr_0/\hbar$. To this order most of the hydrodynamic and thermody-
namic results for degenerate and nondegenerate Fermi gases coincide very
closely if one replaces p_F by p_T. Apart from this, for dilute systems there
is no necessity to take into account gradient terms of the type (17); the
contribution of such terms is small in comparison with those of the usual
Fermi liquid terms due to Eqs. (29) and (30).

From this point of view, it is easy to understand why the results of a
thorough analysis of the kinetic equation for nondegenerate polarized
gases[10] formally have much in common with the equations of Ref. 17 for
degenerate systems. Below it is shown that the expressions for the "spin
rotation effect" predicted by Lhuillier and Laloë[10] are practically the same
for degenerate and nondegenerate polarized Fermi gases in the main order
in the interaction and can be easily derived within this accuracy using the
Fermi liquid formalism.

In the cases of dilute, degenerate and nondegenerate, Fermi systems
the antisymmetric part of the Fermi liquid function in the first order in the
interaction is constant[14]

$$\zeta(\mathbf{p}, \mathbf{p}') \equiv \zeta^{(0)} = -2\pi a \hbar^2/M \tag{31}$$

(for the ^{3}He gas the quantities a and M have to be replaced by a_{33} and
m_3). Hence the frequency ω_{int} in (11)–(13), being the main characteristic
of spin oscillations in the system, is given by

$$\omega_{\text{int}} = (2\zeta^{(0)}/\hbar)(N_+ - N_-) \tag{32}$$

For a degenerate Fermi gas the exchange diffusion relaxation time τ_D is of the order of $\tau_D \sim (N_3 a^2 v_F)^{-1} (T_F/T)^2$ (v_F is the Fermi velocity), and for a nondegenerate gas by $\tau_D \sim (N_3 a^2 v_T)^{-1}$ (v_T is the thermal velocity). In both cases the conditions (29) and (30) cause the inequality $\omega_{int}\tau_D \gg 1$ to hold up to the lowest polarizations. For this reason weakly damped spin waves with the square-low spectrum (14) can propagate through the system independent of the degree of degeneracy.[6,14,22] The difference between degenerate and nondegenerate gases, arising only from different types of equilibrium distribution functions in the integrands, is displayed in different values of τ_D and the coefficient ν in Eq. (14). In the degenerate gas[5,14]

$$\nu = \frac{(36\pi)^{1/3}}{20} \frac{\hbar}{M|a|} \frac{N_+^{5/3} - N_-^{5/3}}{(N_+ - N_-)^2} \tag{33}$$

and in the nondegenerate gas[6,14]

$$\nu = T/(M|\omega_{int}|) \tag{34}$$

Equations (33) and (34) give the frequency of the spin oscillations (15) and their damping (16) in dilute polarized Fermi gases. The spin diffusion coefficients D are already known[10,11] and it is easy to evaluate τ_D. The spin diffusion coefficient of a dilute degenerate gas is given by[11]

$$D = 2D(0) \left(\frac{p_+}{p_0}\right)^5 \frac{p_-^2 p_0^3}{p_+^5 + p_-^5}, \qquad D(0) = \frac{1}{\pi} \left(\frac{\hbar}{M}\right)^3 \left(\frac{p_0}{aT}\right)^2$$

and using Eqs. (15) and (33), we get

$$\tau_D = 10[M^2 D(0)/p_0^2]p_-^2 (p_+^3 - p_-^3)/(p_+^{10} - p_-^{10})$$

The spin diffusion in a nondegenerate gas is independent of polarization,[10,11]

$$D = (3/8)(\pi T/M)^{1/2} (2\pi a^2 N_3)^{-1}$$

and $\tau_D = 3D/v_T^2$. These expressions combined with Eqs. (15) and (16) completely determine the coefficients D, R, and d in Eq. (4), and thereby the spin rotation effect. The remaining transport coefficients are given for the polarized degenerate Fermi gas in Refs. 11 and 14, and for the nondegenerate gas in Refs. 10 and 11.

Of course, the mentioned analogy between degenerate and nondegenerate gases exists only to the main order in the interaction, when the molecular field is given by the same expression (31). Considerable differences arise if the higher order terms are considered. For example, spin waves can propagate through almost all polarized degenerate Fermi systems, while in nondegenerate systems the damping of the oscillations increases rapidly

with increasing density. The evaluation of the higher order corrections for the kinetic coefficients of polarized nondegenerate Fermi gases at not very low temperatures has been developed by Lhuillier and Laloë[10,12] and for degenerate gases by Baskin and Meyerovich[14] (see also the results of Section 2).

The above discussion concerned mainly ^{3}He systems with the kinetics specified by the ^{3}He–^{3}He interaction, i.e., ^{3}He–He II solutions and gaseous ^{3}He. For ^{3}He↑–^{4}He gaseous mixtures the ^{3}He–^{4}He and ^{4}He–^{4}He collisions are also important. Considering these mixtures, one has to take into account a new hydrodynamic variable—the ^{4}He concentration $c_4 = N_4/(N_3 + N_4)$ (N_4 is the number of ^{4}He atoms per unit volume), and to introduce an additional equation—the ^{4}He mass conservation law. The presence of ^{4}He particles has a straightforward effect on Eqs. (4) and (6): one has only to include one more "vector" term in the spin current (2):

$$\mathbf{j}_k = \mathbf{e}D_{34}\frac{\partial}{\partial x_k}c_4 \qquad (35)$$

Of course, ^{4}He atoms do not change the precession of the magnetic moment in the molecular field. But the ^{4}He atoms are very important for the coefficients of the spin (and mass) diffusion, thermodiffusion, pressure diffusion, viscosity, and thermal conductivity. For ^{3}He↑–^{4}He mixtures it is more convenient to introduce the diffusion coefficients in a form somewhat different from Eqs. (3) and (35). Equation (6) for the longitudinal component of the magnetic moment can be rewritten [using (35)] as (cf. Ref. 10)

$$\partial N_{\pm}/\partial t + \partial(N_{\pm}v_k)/\partial x_k + \partial j_k^{\pm}/\partial x_k = 0$$
$$\mathbf{j}^{\pm} = -(N_3 + N_4)(m_3 c_3 + m_4 c_4)^{-1}(m_3 D_{\pm\mp}\nabla c_{\mp} \qquad (36)$$
$$+ m_4 D_{\pm 4}\nabla c_4 + D_{\pm}^P\nabla P/P) + (D_{\pm}^T/T)\nabla T$$

and the ^{4}He continuity equation as

$$\partial N_4/\partial t + \partial(N_4 v_k)/\partial x_k + \partial j_k^{(4)}/\partial x_k = 0$$
$$\mathbf{j}^{(4)} = -(N_3 + N_4)(m_3 c_3 + m_4 c_4)^{-1}[m_3(D_{4+}\nabla c_+ \qquad (37)$$
$$+ D_{4-}\nabla c_-) + (D_4^P/P)\nabla P] + (D_4^T/T)\nabla T$$

where $c_{\pm} = N_{\pm}/(N_3 + N_4)$, $c_4 = N_4/(N_3 + N_4)$, $c_3 = c_+ + c_- = 1 - c_4$, and $m_{3,4}$ are the ^{3}He and ^{4}He atomic masses. In these notations the diffusion currents (36) and (37) are analogous to the usual expressions for a ternary mixture. For the helium atoms collisions reduce at low temperatures (30) to S-wave scattering with amplitude independent of momenta; and the transport coefficients of the mixture are given by the well-known results of the

Chapman–Enskog theory in the hard-sphere approximation.[23] There are also two additional simplifications. First, one has to take into account in considering ^{3}He–^{3}He collisions only the interaction of particles with opposite spin projections (S-wave scattering). Second, the ^{3}He–^{4}He cross section does not depend on the ^{3}He spin state. Due to these simplifications, all the coefficients in Eqs. (36) and (37) can be expressed by means of only two diffusion characteristics of binary interaction,

$$D = (3/16)(N_3 + N_4)^{-1} a_{33}^{-2} (T/\pi m_3)^{1/2}$$

$$D' = (3/32)(N_3 + N_4)^{-1} a_{34}^{-2} (7T/2\pi m_3)^{1/2}$$

The coefficients D and D' have the meaning of the diffusion coefficients in the binary mixtures (^{3}He$^+$, ^{3}He$^-$) and (^{3}He, ^{4}He) with densities $N_3 + N_4$. Using the data of Ref. 23, one easily obtains the following values for the diffusion coefficients in Eqs. (36) and (37):

$$D_{+-} = D_{-+} = (DD'/S)(1 + c_4/3), \qquad D_{4\pm} = D'$$

$$D_{\pm 4} = (D'/S)(c_\pm D' + (D/4)(3c_\mp + 4c_4)]$$

$$S = c_4 D + (1 - c_4)D'$$

The pressure diffusion coefficients are given by the usual thermodynamic relations

$$D_\pm^P = [4m_3 c_\mp c_4 D_{+-} + m_4 c_4 (c_4 - 1)D_{\pm 4}]/(3 + c_4)$$

$$D_4^P = m_3 c_4 (1 - c_4)D'/(3 + c_4)$$

After cumbersome calculations, and with the help of Ref. 23, we obtain for the viscosity of a polarized ^{3}He–^{4}He gaseous mixture the expression

$$
\eta = \left\{ \frac{c_4}{\eta_4} \left(\frac{4c_+^2 + 4c_-^2 + 2c_+ c_-}{\eta_3} + \frac{9}{2} \frac{c_4(1 - c_4)}{\eta'} \right) + \frac{15 c_+ c_- c_4}{\eta_3^2} \right.
$$

$$
+ \frac{4c_4}{\eta_3 \eta'} \left[(2c_+^2 + 2c_-^2 + c_+ c_-) \left(1 + \frac{29}{16} \frac{1 - c_4}{c_4} \right) + \frac{9}{2} \right]
$$

$$
\left. + \frac{9}{2} \frac{c_4}{(\eta')^2} \left[\frac{9}{2} c_4^2 + 2c_4(1 - c_4) + \frac{29}{8}(1 - c_4)^2 \right] \right\}
$$

$$
\times \left\{ \frac{9c_4}{\eta_4} \left[\frac{5}{3} \frac{c_+ c_-}{\eta_3^2} + 2 \frac{c_4(1 - c_4)}{\eta_3 \eta'} + \frac{9}{4} \frac{c_4}{(\eta')^2} \right] + \frac{435}{8} \frac{c_+ c_- (1 - c_4)}{\eta_3^2 \eta'} \right.
$$

$$
\left. + \frac{c_4}{\eta_3 (\eta')^2} \left[\frac{261}{4}(1 - c_4)^2 - 2c_+ c_- - 4c_+^2 - 4c_-^2 \right] + \frac{205}{32} c_4^2 \frac{1 - c_4}{(\eta')^3} \right\}^{-1}
$$

Here we have introduced three auxiliary coefficients,

$$\eta_3 = \tfrac{3}{2}\eta(0) = (15/16)(\pi m_3 T)^{1/2}(8\pi a_{33}^2)^{-1}$$

$$\eta' = (49/16)\eta_{34} = (245/256)(8\pi m_3 T/7)^{1/2}(4\pi a_{34}^2)^{-1}$$

$$\eta_4 = (5/16)(\pi m_4 T)^{1/2}(8\pi a_{44}^2)^{-1}$$

where $\eta(0)$ and η_4 are the viscosities of pure nonpolarized ^{3}He and pure ^{4}He, and η_{34} is the viscosity of the gas of particles with atomic mass $m_3 m_4/(m_3 + m_4)$ and cross section $4\pi a_{34}^2$. The most interesting case is c_4, $c_- \ll 1$, because even a very small concentration of ^{4}He impurities in ^{3}He$\uparrow$ gas will limit the growth of the ^{3}He atom mean free path with increasing ^{3}He polarization (magnetokinetic effect):

$$\eta(c_-, c_4 \ll 1) = \tfrac{4}{5}\eta_3\eta'/[3c_-\eta' + (98/29)c_4\eta_3]$$

The rest of the transport coefficients (the ^{3}He$\uparrow$–^{4}He thermal conductivity and thermodiffusion) are even more unwieldy than Eq. (38). We shall give only the value of the thermal conductivity coefficient in the case of high polarization, $c_- \ll c_4$:

$$\begin{aligned}
\varkappa = \{&60 + (23/4)(1 - c_4)^2 - (25/6)c_4^2 \\
&+ 2[(49/3)(\varkappa'/\varkappa_4) - 33]c_4(1 - c_4)\} \\
&\times [(1441/16)(1 - c_4)(c_4/\varkappa') + (335/8)(c_4^2/\varkappa_4)]^{-1} \qquad (38)
\end{aligned}$$

Here $\varkappa_4 = (15/4)\eta_4$ is the thermal conductivity of pure ^{4}He, and $\varkappa' = (15/4)\eta_{34}$ is the thermal conductivity of the gas of particles with atomic mass $m_3 m_4/(m_3 + m_4)$ and cross section $4\pi a_{34}^2$.

4. ^{3}He$\uparrow$/^{3}He$\uparrow$–^{4}He PHASE EQUILIBRIUM

The principal object of this section is to point out the existence of new unusual solid and liquid phases of ^{3}He$\uparrow$–^{4}He solutions at the ^{3}He$\uparrow$/^{3}He$\uparrow$–^{4}He phase equilibrium. It is already known[7,8,14,24,25] that the ^{3}He polarization significantly changes the ^{3}He phase diagram and, in particular, alters the mutual solubility of the helium isotopes. Nevertheless, there are two possibilities which seem to be of special interest from both theoretical and experimental points of view.

In time intervals less than τ_d (as stated in Section 1 the dipole relaxation time τ_d may exceed tens of minutes), the equilibrium between pure ^{3}He$\uparrow$ and a ^{3}He$\uparrow$–^{4}He solution is specified by the conditions $\mu_1^+ = \mu_2^+$ and $\mu_1^- = \mu_2^-$, where $\mu_1^\pm$ and $\mu_2^\pm$ are the chemical potentials of ^{3}He particles with different spin projections in the pure phase and in solution. When the

energy scales are significantly different in these phases, the spin system polarization has different effects on $\mu_1^{\pm}$ and $\mu_2^{\pm}$. As a result the equilibrium conditions $\mu_1^{\pm}(\mathfrak{P}_1) = \mu_2^{\pm}(\mathfrak{P}_2)$ are fulfilled at different degrees of polarization $\mathfrak{P}_{1,2}$ of both phases and the equilibrium ^{3}He concentration in the solution also depends on $\mathfrak{P}_{1,2}$. The influence of polarization on the phase equilibrium is evident in the case of weak polarization when the change of energy with polarization is proportional to the square of $\mathfrak{P}_{1,2}$. In this case

$$\mathfrak{P}_{1,2} = \chi_{1,2}\mathfrak{P}N_3/(\chi_1 N_1 + \chi_2 N_2), \qquad \mathfrak{P}_1/\mathfrak{P}_2 = \chi_1/\chi_2 \qquad (39)$$

where $\chi_{1,2}$ are the susceptibilities (per ^{3}He particle) in pure ^{3}He and in ^{3}He–^{4}He, $\mathfrak{P} = (\mathfrak{P}_1 N_1 + \mathfrak{P}_2 N_2)/N_3$ is the total degree of polarization of the system, and $N_{1,2}$ are the numbers of ^{3}He atoms in the pure phase and the solution, $N_1 + N_2 = N_3$. Due to the dependence of the ^{3}He distribution between the phases N_1/N_2 on the relation between the numbers of ^{3}He atoms N_3 and ^{4}He atoms N_4 in the system, one can easily vary $\mathfrak{P}_{1,2}$ by changing N_3/N_4. The main difficulty in solving the phase equilibrium equations is caused by the lack of information on the functions $\mu_1^{\pm}(\mathfrak{P}_1)$ for the dense ^{3}He Fermi liquid; for dilute ^{3}He phases the dependence $\mu(\mathfrak{P})$ is well known.[14]

At low temperatures ^{3}He is not soluble in solid ^{4}He. The demixing temperature of the solid mixture into the pure components is about 0.1 K, and the difference in chemical potentials of ^{3}He atoms within ^{4}He crystals and in pure nonpolarized (solid or liquid) ^{3}He phase is $\delta\mu_{sol} \lesssim 0.1$ K. The ^{3}He polarization does not significantly change the ^{3}He chemical potentials in solid phases, and at low temperatures and high pressures the solid ^{3}He↑–^{4}He mixture will continue to separate into pure ^{3}He↑ and ^{4}He crystals. Though the polarization leads to some decrease of the ^{3}He melting pressure,[7,8] this decrease is not very large and is of the order of magnitude of the ^{3}He melting curve fall caused by the Pomeranchuk effect. Thus the ^{3}He↑ crystals melt at higher pressures than ^{4}He. As a result we meet with one of the following situations as the pressure falls: the system consists either of two phases (solid ^{4}He–liquid ^{3}He↑) or of three phases (solid ^{4}He–liquid ^{3}He↑–liquid ^{3}He↑–^{4}He solution with a relatively high ^{3}He concentration). In the absence of polarization the second possibility occurs at pressures slightly above the ^{4}He melting curve,[26,27] and the former occurs at higher pressures. For our purposes it is not very important which of these possibilities is realized in the polarized system. What is important is that the change of the ^{3}He chemical potential in liquid ^{3}He or in concentrated ^{3}He↑–^{4}He solutions is of the order of several tenths of a degree and is likely to exeed $\delta\mu_{sol} \sim 0.1$ K. This means that ^{3}He↑ melting is accompanied by the simultaneous penetration of ^{3}He atoms into the ^{4}He crystal. This phenomenon is, of course, a threshold effect and can occur only at a

sufficiently high degree of polarization. An extrapolation with the help of Eq. (39) gives for the threshold value of polarization the rough estimate 0.2–0.3. Note that the impurity component of the crystal is always nearly completely polarized.

This unique opportunity to obtain solid ^{3}He–^{4}He solutions at very low temperatures gives one a chance to study for the first time the impurity quasiparticles in quantum crystals in the case when the temperature is comparable with the quasiparticle bandwidth; up to now experiments have been carried out only for $T > 0.1$ K when the temperature has been much larger than the bandwidth and the interaction energy of quasiparticles. Many of the most interesting phenomena in quantum crystals, e.g., the localization or the ordering in the impurity quasiparticle system, have to take place at $T < 0.1$ K. Such a new low-temperature helium phase should manifest several unexpected properties of the impurity zero-point motion in quantum crystals.

At lower pressures the equilibrium of the dense Fermi liquid ^{3}He↑ and ^{3}He↑–^{4}He liquid solutions should be observed. The properties of this solution differ essentially from the usually studied ^{3}He–He II solutions. First, for $\partial \mu_1^+ / \partial \mathfrak{P}_1 \gg \partial \mu_2^+ / \partial \mathfrak{P}_2$ the solution is practically completely polarized, $\mathfrak{P}_1 \ll \mathfrak{P}_2$, even if the initial ^{3}He crystal was not very highly polarized. The second circumstance is much more important: the equilibrium $\mu_2^{\pm} = \mu_2^{\pm}$ means a considerable increase in the maximum solubility of ^{3}He in liquid He II. If the degree of helium polarization is sufficiently high, $\mathfrak{P}_1 \sim \mathfrak{P}_2 \sim 1$, the solubility increases (in comparison with the case $\mathfrak{P} = 0$) by a factor of three to four. Such a solution with a ^{3}He concentration of the order of several tens of percent is a Fermi liquid with properties completely different from both conventional dilute ^{3}He–He II solutions[14] and a dense ^{3}He Fermi liquid. As for the hydrodynamics and kinetics, these concentrated solutions are described not by the formulas of Section 3 and Refs. 11 and 14, but by the results of Section 2. In addition, in studying the ^{3}He quasiparticle interaction in concentrated solutions one should encounter considerable retardation effects due to the increase of the ratio of the ^{3}He quasiparticle Fermi velocity to the sound velocity in liquid helium.

The existence of ^{3}He↑–He II solutions in a wide range of concentrations provides the possibility of investigating the gradual transition from the case of the nearly ideal Fermi gas to the dense Fermi liquid. The transition of ^{3}He in spin-polarized solutions to a superfluid state cannot be caused by S-wave pairing except for the case of very low polarization.[14] The transition temperature for p-wave pairing is ^{3}He concentration dependent and may exceed 10^{-5} K in concentrated solutions. The properties of the corresponding superfluid phase are somewhat analogous to the well-known ^{3}He-A$_1$ phase and are very different from superfluid ^{3}He in nonpolarized or weakly

polarized ^{3}He–He II solutions. The difference from ^{3}He-A$_1$ is related mainly to the simultaneous presence of two Bose condensates (^{3}He and ^{4}He) and the corresponding drag effects.

APPENDIX A

In this Appendix we derive some thermodynamic relations for a spin-polarized Fermi liquid with a Fermi liquid function of the type (8)–(10). If the disturbances do not change the direction of magnetization, then the single-particle density matrix $n_{\alpha\beta}$ and the excitation energy $\varepsilon_{\alpha\beta}$ can be diagonalized in spin space by choosing the z axis along the direction of magnetization **M**:

$$n_{\alpha\beta} = \tfrac{1}{2}n(\mathbf{p})\delta_{\alpha\beta} + \tfrac{1}{2}\rho(\mathbf{p})\mathbf{e}\sigma_{\alpha\beta}, \qquad \varepsilon_{\alpha\beta} = \varepsilon_1\delta_{\alpha\beta} - \varepsilon_2\mathbf{e}\sigma_{\alpha\beta} \qquad \text{(A1)}$$

At equilibrium the functions $n_+ = n + \rho$ and $n_- = n - \rho$ are the Fermi distribution functions for quasiparticles with spin projections $\pm\tfrac{1}{2}$ on the z axis (the corresponding energies and chemical potentials are $\varepsilon_\pm$ and $\mu_\pm$).

If the spin densities N_+ and N_- are varied ($\delta N_+ + \delta N_- = \delta N_3$), the changes of the chemical potentials $\mu_\pm$

$$\delta\mu_\pm = \left(\frac{\partial\varepsilon_\pm}{\partial p}\right)_{p=p_\pm} \delta p_\pm$$

$$+ \frac{1}{2}\int (\delta_{\beta\alpha} + \sigma^z_{\beta\alpha})f_{\alpha\beta,\mu\nu}(\mathbf{p}_\pm, \mathbf{p}')\,\delta n_{\nu\mu}(\mathbf{p}')\frac{d^3p'}{(2\pi\hbar)^3}$$

can be rewritten using Eqs. (8) and (A1) as

$$\delta\mu_\pm = \frac{2\pi^2\hbar^3}{m_\pm p_\pm}\delta N_\pm + \int \{[\psi_\pm \pm \zeta_\pm \pm \varphi_\pm + \varphi'_\pm \pm \xi_\pm]\delta n_+$$

$$+ [\psi_\pm \mp \zeta_\pm \pm \varphi_\pm - \varphi'_\pm \mp \xi_\pm]\delta n_-\}\frac{d^3p'}{(2\pi\hbar)^3} \qquad \text{(A2)}$$

where $f_\pm = f(\mathbf{p}_\pm, \mathbf{p}')$. With the help of notations (10), the relation (A2) takes the form

$$\delta\mu_\pm = \frac{2\pi^2\hbar^3}{m_\pm p_\pm}A^{(0)}_\pm\,\delta N_\pm + \frac{2\pi^2\hbar^3}{m_\mp p_\mp}B^{(0)}_\pm\delta N_\mp \qquad \text{(A3)}$$

The thermodynamic identity for the pressure (at $T = 0$) $dP = N_+\,d\mu_+ + N_-\,d\mu_-$ can be represented, taking into account Eq. (A3), as

$$dP = \frac{2\pi^2\hbar^3}{m_+ p_+}(N_+A^{(0)}_+ + N_-B^{(0)}_-)\,dN_+ + \frac{2\pi^2\hbar^3}{m_- p_-}(N_-A^{(0)}_- + N_+B^{(0)}_+)\,dN_-$$

$$\text{(A4)}$$

Equations (A3) and (A4) lead directly to Eq. (25) for the sound velocity and to Eq. (20). The spin diffusion coefficient is determined as a proportionality coefficient for the spin current and the magnetization gradient $\nabla(N_\pm/N_3)$ at constant pressure, and by equating (A4) to zero we get Eq. (B9) from Eq. (A3).

The effective masses of quasiparticles of a polarized Fermi liquid can be expressed through the first harmonics of the f-function as[14]

$$\frac{m_\pm}{m} = \frac{A_+^{(1)}A_-^{(1)} - B_+^{(1)}B_-^{(1)}}{A_\mp^{(1)} - B_\pm^{(1)}} \tag{A5}$$

The limiting case of a nonpolarized Fermi liquid corresponds to $p_+ = p_-$, $m_+ = m_-$, $A_+ = A_-$, $B_+ = B_-$, and $\varphi = \varphi' = \xi = 0$. Then $A = \Psi + Z$, $B = \Psi - Z$ and the harmonics (9) of the Fermi liquid function differ from the common notations for a nonpolarized system only by the normalizing factor in the density of states.

APPENDIX B

Here we determine the transport coefficients of a spin-polarized Fermi liquid. For all disturbances conserving the direction of the magnetic moment, the kinetic equation for the single-particle density matrix is diagonal in spin space and reduces to two scalar equations for the distribution functions of up and down spins. The corresponding collision integrals are given by

$$
\begin{aligned}
I^\pm(\mathbf{p}_1) = -(2\pi\hbar)^{-6} \int d^3p_2\, d^3p_1'\, d^3p_2'\, \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_1' - \varepsilon_2') \\
\times \delta(p_1 + p_2 - p_1' - p_2') \\
\times \{W_\pm[n_1^\pm n_2^\pm(1 - n_{1'}^\pm)(1 - n_{2'}^\pm) - n_{1'}^\pm n_{2'}^\pm(1 - n_1^\pm)(1 - n_2^\pm)] \\
+ W'[n_1^\pm n_2^\mp(1 - n_{1'}^\pm)(1 - n_{2'}^\mp) - n_{1'}^\pm n_{2'}^\mp(1 - n_1^\pm)(1 - n_2^\mp)]\} \tag{B1}
\end{aligned}
$$

where $\varepsilon_i = \varepsilon(\mathbf{p}_i)$, $n_i = n(\mathbf{p}_i)$, W_+ is the scattering probability of two quasiparticles with momenta $(\mathbf{p}_1^+, \mathbf{p}_2^+)$ into the state $(\mathbf{p}_{1'}^+, \mathbf{p}_{2'}^+)$, and W_-, W' are the analogous probabilities for the quasiparticle collisions on the Fermi sphere of radius p_- and for the quasiparticles from different Fermi spheres.

The left-hand side of the linearized kinetic equation in the evaluation of the viscosity has the form[20]

$$
-\frac{1}{2m_\pm} \frac{\partial n_1^\pm}{\partial \varepsilon_1^\pm} \left[p_{1i}^\pm p_{1k}^\pm - \frac{1}{3}(p^\pm)^2 \delta_{ik} \right] \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial u_l}{\partial x_l} \right)
$$

where $\mathbf{u}$ is the fluid velocity. In the case of the thermal conductivity the

left-hand side is

$$-\frac{1}{2}\frac{\partial n_1^{\pm}}{\partial \varepsilon_1^{\pm}}\left(\frac{\varepsilon_1^{\pm}-\mu_{\pm}}{T}-s^{\pm}\right)p_1^{\pm}\,\mathbf{\nabla}T$$

where s is the entropy per particle, and in the case of the spin diffusion

$$-\frac{1}{m_{\pm}}\frac{\partial n_1^{\pm}}{\partial \varepsilon_1^{\pm}}\mathbf{p}_1^{\pm}\,\mathbf{\nabla}\mu_{\pm}$$

The deviation of the distribution functions $n^{\pm}$ from their equilibrium values $n^{\pm(0)}$ is of the form

$$n = n^{(0)}+(\partial n^{(0)}/\partial \varepsilon)\nu \tag{B2}$$

As in a nonpolarized system, the Fermi liquid corrections lead to the renormalization of $\nu^{\pm}$ and do not change the values of the kinetic coefficients. The substitution of Eq. (B2) into Eq. (B1) reduces the expressions in square brackets to the form

$$-(1/T)n_1 n_2(1-n_{1'})(1-n_{2'})(\nu_1+\nu_2-\nu_{1'}-\nu_{2'})$$

The introduction of the new variables $dx_i = p_i^{\pm}\,dp_i^{\pm}/m_{\pm}T$ and integration removing the δ-functions transforms Eq. (B1) into

$$I^{\pm}(\mathbf{p}_1)=\frac{2m_{\pm}m_{\mp}^2}{(2\pi\hbar)^6}\,Tp_{\mp}\int dx_{1'}\,dx_{2'}\,d\Omega_2\,d\varphi\,n_1^{\pm}(1-n_{1'}^{\pm})$$

$$\times\left[\frac{W'}{g}n_2^{\mp}(1-n_{2'}^{\mp})(\nu_1^{\pm}+\nu_2^{\mp}-\nu_{1'}^{\pm}-\nu_{2'}^{\mp})\right.$$

$$\left.+\frac{W_{\pm}}{g_{\pm}}n_2^{\pm}(1-n_{2'}^{\pm})(\nu_1^{\pm}+\nu_2^{\pm}-\nu_{1'}^{\pm}-\nu_{2'}^{\pm})\right] \tag{B3}$$

where $g = (p_+^2+p_-^2+2p_+p_-\cos\theta_2)^{1/2}$, $g_{\pm}=p_{\pm}(2+2\cos\theta_2)^{1/2}$, φ is the angle between the planes $(\mathbf{p}_1,\mathbf{p}_2)$ and $(\mathbf{p}'_1,\mathbf{p}'_2)$, $d\Omega_2 = d\cos\theta_2\,d\varphi_2$, and θ_i is the angle between the vectors $\mathbf{p}_i$ and $\mathbf{p}_1$. We seek solutions of the kinetic equations for the viscosity, thermal conductivity, and spin diffusion, respectively, in the forms

$$\nu^{\pm}(\mathbf{p})=\frac{q_{\eta}^{\pm}}{p_{\pm}^2}\left(p_i^{\pm}p_k^{\pm}-\frac{1}{3}p_{\pm}^2\delta_{ik}\right)\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l}\right) \tag{B4}$$

$$\nu^{\pm}(\mathbf{p})=q_{\varkappa}^{\pm}\mathbf{p}_{\pm}\,\mathbf{\nabla}T,\qquad \nu^{\pm}(p)=q_D^{\pm}\mathbf{p}_{\pm}\,\mathbf{\nabla}\mu_{\pm}$$

where all the quantities $q(r)$ depend only on $r_{\pm}=(\varepsilon_{\pm}-\mu_{\pm})/T$. As a result, the integrations in Eq. (B3) over angular and energy variables decouple. The quasiparticle collisions in degenerate systems are accompanied by a

very small energy transfer, and analogous to Ref. 20, we are interested in the values of the functions $q(r)$ only in the vicinity of $r = 0$. Hence in the cases of the viscosity and spin diffusion it is possible to substitute $q(0)$ for $q(r)$ into Eqs. (B3) and (B4). A somewhat more complicated analysis for the thermal conductivity shows that one can approximate $q_\varkappa(r) = rq'_\varkappa(0)$. These simplifications make the integration over $dx_{1'}\, dx_{2'}$ quite trivial. The remaining angular integrals involve the functions of $\cos\theta_{1'}$ and $\cos\theta_{2'}$, which are determined by the conservation laws

$$\cos\theta_{1'} = 1 - 2p_2\sin^2\theta_2\sin^2\frac{\varphi}{2}(p_1^2 + p_2^2 + 2p_1p_2\cos\theta_2)^{-1},$$

$$\cos\theta_{2'} = (p_1/p_2) + \cos\theta_2 - (p_1/p_2)\cos\theta_{1'}$$

where for the momenta $p_{1,2}$ one should put the values $p_\pm$. This results in the following equations in q_η and $q_\varkappa$:

$$1 = 24\pi^4 T^2 (2\pi\hbar)^{-6} m_+^2 m_-^2 (p_\mp^3/p_\pm^2)[q_\eta^\pm(0)X_\eta^\pm + q_\eta^\mp(0)Y_\eta^\mp]$$

$$1 = (16\pi^4/3)T^2(2\pi\hbar)^{-6}m_+^2 m_-^2 (p_\mp/p_+)[q_\varkappa'^\pm(0)X_\varkappa^\pm + q_\varkappa'^\mp(0)Y_\varkappa^\mp] \tag{B5}$$

where the coefficients X and Y are given by Eqs. (21) and (22). The solutions of Eqs. (B5) are to be substituted into the expressions for the momentum flow

$$\Pi_{ik} = \int p_i\frac{\partial\varepsilon}{\partial p_k}\left[\frac{\partial n_+^{(0)}}{\partial\varepsilon_+}\nu_\eta^+ + \frac{\partial n_-^{(0)}}{\partial\varepsilon_-}\nu_\eta^-\right]\frac{d^3p}{(2\pi\hbar)^3}$$

and the energy flow (with the additional condition of the absence of the spin currents)

$$Q_i = \int \varepsilon\frac{\partial\varepsilon}{\partial p_i}\left[\frac{\partial n_+^{(0)}}{\partial\varepsilon_+}\nu_\varkappa^+ + \frac{\partial u_-^{(0)}}{\partial\varepsilon_-}\nu_\varkappa^-\right]\frac{d^3p}{(2\pi\hbar)^3}$$

In doing so, one obtains for the coefficients of viscosity

$$\Pi_{ik} = -\eta\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l}\right)$$

$$\eta = \frac{2}{15\pi^2\hbar^3}\left[\frac{p_+^3}{m_+}q_\eta^+(0) + \frac{p_-^3}{m_-}q_\eta^-(0)\right]$$

and thermal conductivity

$$Q = -\varkappa\,\nabla T, \qquad \varkappa = \frac{T}{36\hbar^3}[p_+^3 q_\varkappa'^+(0) + p_-^3 q_\varkappa'^-(0)]$$

their values (21) and (22) [in Eqs. (21) and (22), $\theta \equiv \theta_2$].

In the case of the spin diffusion the set of equations in $q_D^{\pm}(0)$,

$$1 = \frac{4\pi^3 T^2}{(2\pi\hbar)^6} m_+^2 m_-^2 p_{\mp}^3 [q_D^{\pm}(0) - q_D^{\mp}(0)]$$

$$\times \int \frac{W'}{g^3} \sin^2\theta \sin^2\frac{\varphi}{2} d\cos\theta \, d\varphi \tag{B6}$$

is similar to Eqs. (B5) but has a determinant equal to zero. Equation (B6) has to be solved taking into account the condition of the absence of the mass flow

$$\mathbf{j} \equiv \mathbf{j}^+ + \mathbf{j}^- = 0, \qquad \mathbf{j}^{\pm} = \int \mathbf{p}_{\pm} \frac{\partial n_{\pm}^{(0)}}{\partial \varepsilon_{\pm}} \nu^{\pm} \frac{d^3p}{(2\pi\hbar)^3} \tag{B7}$$

which corresponds to the equation

$$p_+^3 q_D^+(0) + p_-^3 q_D^-(0) = 0 \tag{B8}$$

The solutions $q_D^{\pm}(0)$ of Eqs. (B6) and (B8) should be substituted into expressions (B7) for the spin currents $\mathbf{j}^{\pm}$. In the final results the gradients $\nabla\mu_{\pm}$ have to be replaced by

$$\nabla\mu_{\pm} = \left(\frac{\partial\mu_{\pm}}{\partial c_{\pm}}\right)_P \nabla c_{\pm}$$

$$= \frac{2\pi^2\hbar^3 N_3^2 N_{\mp}(A_+^{(0)} A_-^{(0)} - B_+^{(0)} B_-^{(0)})}{m_+ p_+ N_-(N_- A_-^{(0)} + N_+ B_+^{(0)}) + m_- p_- N_+(N_+ A_+^{(0)} + N_- B_-^{(0)})} \tag{B9}$$

where $c_{\pm} = N_{\pm}/N_3$ and the derivatives are evaluated using Eqs. (A3) and (A4). The proportionality factor for $\mathbf{j}^{\pm}$ and $\nabla c_{\pm}$ determines the spin diffusion coefficient D, which turns out to be equal to (18).

The calculation of the spin thermodiffusion coefficient is more complicated. The antisymmetric part of the distribution function $q'^{\pm}(0)$ is evaluated as in the case of the thermal conductivity from Eq. (B5). The symmetric part $q(0)$ is given by the homogeneous equation (B6) and the condition of the zero mass current (B7). This results in the relation

$$q^{\pm}(0) = -[\pi^2 T/(p_+^3 + p_-^3)][m_+ p_+ q'^+(0) + m_- p_- q'^-(0)]$$

When the corresponding values of $q'^{\pm}(0)$ and $q^{\pm}(0)$ are substituted into the expressions (B7) for the spin currents $\mathbf{j}^{\pm}$, they lead to the formula (19) for the spin thermodiffusion coefficient.

The above expressions coincide with the well-known results[20,21] for nonpolarized Fermi liquids if $m_+ = m_-$, $p_+ = p_-$, $W_+ = W_-$, $X^+ = X^-$, $Y^+ = Y^-$. In the case of a polarized dilute degenerate Fermi gas[5,11,14] $m_+ = m_-$ and $W_{\pm} = 0$, and one has to use for the probability $W'(\theta, \varphi) = $ const its

value for S-wave scattering; as a result the angular integrations in Eqs. (18)–(22) can be easily carried out.

REFERENCES

1. M. Chapellier, G. Frossati, and F. B. Rasmussen, *Phys. Rev. Lett.* **42**, 904 (1979).
2. G. Schumacher, D. Toulouze, B. Castaing, Y. Chabre, P. Segransan, and J. Joffrin, *J. Phys. Lett. (Paris)* **40**, L143 (1979).
3. D. S. Greywall and M. A. Paalanen, *Phys. Rev. Lett.* **48**, 1292 (1981); *Physica* **109–110B**, 1575 (1981).
4. P. J. Nacher, M. Leduc, G. Trénec, and F. Laloë, *J. Phys. Lett. (Paris)* **43**, L529 (1982).
5. E. P. Bashkin and A. E. Meyerovich, *JETP Lett.* **26**, 534 (1977) [*Pis'ma Zh. Eksp. Teor. Fiz.* **26**, 696 (1977)]; *Sov. Phys. JETP* **47**, 992 (1978) [*Zh. Eksp. Teor. Fiz.* **74**, 1904 (1978)].
6. E. P. Bashkin and A. E. Meyerovich, *JETP Lett.* **27**, 485 (1978) [*Pis'ma Zh. Eksp. Teor. Fiz.* **27**, 517 (1978)]; *Sov. Phys. JETP* **50**, 196 (1979) [*Zh. Eksp. Teor. Fiz.* **77**, 383 (1979)].
7. C. Lhuillier and F. Laloë, *J. Phys. (Paris)* **40**, 239 (1979).
8. B. Castaing and P. Nozières, *J. Phys. (Paris)* **40**, 257 (1979).
9. A. E. Meyerovich, *Phys. Lett. A* **69**, 279 (1980).
10. C. Lhuillier and F. Laloë, *J. Phys. (Paris)* **43**, 197, 225 (1982).
11. A. E. Meyerovich, *J. Low Temp. Phys.* **47**, 271 (1982).
12. C. Lhuillier, *J. Phys. (Paris)* **44** (1983).
13. Spin Polarized Quantum Systems, *J. Phys. (Paris)* **41**, C-7 (1980).
14. E. P. Bashkin and A. E. Meyerovich, *Adv. Phys.* **30**, 1 (1981).
15. L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1978).
16. J. Oliva and N. W. Ashcroft, *Phys. Rev. B* **23**, 6399 (1981).
17. A. J. Leggett, *J. Phys. C* **3**, 448 (1970).
18. V. P. Silin, in *Spin Waves*, A. I. Akhiezer, V. G. Bar'yakhtar, and S. W. Peletminski, eds. (North-Holland, Amsterdam, 1968).
19. A. A. Abrikosov and I. E. Dzyaloshinsky, *Sov. Phys. JETP* **8**, 535 (1958) [*Zh. Eksp. Teor. Fiz.* **35**, 771 (1958)].
20. A. A. Abrikosov and I. M. Khalatnikov, *Sov. Phys. JETP* **5**, 887 (1957) [*Zh. Eksp. Teor. Fiz.* **32**, 1084 (1957)].
21. D. Hone, *Phys. Rev.* **121**, 669 (1961).
22. E. P. Bashkin, *JETP Lett.* **33**, 8 (1981) [*Pis'ma Zh. Eksp. Teor. Fiz.* **33**, 11 (1981)].
23. J. O. Hirshfelder, C. F. Curtis, and R. B. Bird, *Molecular Theory of Gases and Liquids* (Wiley, New York, 1954).
24. A. S. Greenberg, B. Hebral, M. Papoular, and M. T. Beal-Monod, *J. Phys. (Paris)* **41**, C7-79 (1980).
25. B. Castaing, A. S. Greenberg, and M. Papoular, *J. Low Temp. Phys.* **47**, 191 (1982).
26. B. van den Brandt, W. Griffioen, G. Frossati, H. van Beelen, and R. de Bruyn Ouboter, *Physica* **114B**, 295 (1982).
27. Yu. D. Anufriev, V. N. Lopatik, and A. P. Sebedash, *JETP Lett.* **37**, 38 (1983) [*Pis'ma Zh. Eksp. Teor. Fiz.* **37**, 38 (1983)].