Phase diagram of spin polarized ^{3}He–^{4}He solutions

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(Submitted 10 November 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 1, 28–30 (5 January 1983)

The properties of unusual phases of solid and liquid ^{3}He–^{4}He in equilibrium with spin polarized ^{3}He and the possibility of obtaining them are discussed.

PACS numbers: 67.60.Dm, 67.80. – s

1. Spin polarized ^{3}He is currently being actively studied experimentally and theoretically. With the help of the elegant method in Ref. 1, it was possible to obtain liquid ^{3}He↑, polarized by more than 20%.[2] Such a high degree of polarization must cause the phase diagram of ^{3}He↑[1,3] to differ considerably [for example, change in the solubility of ^{3}He and ^{4}He (Ref. 3)] from unpolarized ^{3}He.[4] The purpose of this letter is to explain the effect of polarization of ^{3}He on the phase equilibrium ^{3}He↑/^{3}He↑–^{4}He and to describe the different phases that can arise.

 0021-3640/83/010032-03$01.00

In accessible stationary fields $H \lesssim 100$ kOe, the energy βH ($\beta = 0.08$ mK/kOe is the magnetic moment of the ^{3}He nucleus) is much lower than the characteristic energy per ^{3}He particle in liquid ^{3}He (above 1 K) and in the ^{3}He–^{4}He solution in equilibrium with it (above 0.1 K). For this reason, we shall ignore the possible presence of an external magnetic field and we shall assume that the polarization is produced, for example,[1] by melting of the spin-polarized crystal ^{3}He↑.

The relaxation time τ_d, over which an equilibrium distribution of particles over the spin orientation is established, is determined in ^{3}He by the weak nuclear dipolar interaction (or collisions with walls) and exceeds tens of minutes in ^{3}He and ^{3}He–^{4}He. Over a time less than τ_d the system of ^{3}He atoms with different spin projections (+ and −) behave as two independent components with the number of particles conserved in each component. The conditions for equilibrium of pure ^{3}He↑ and ^{3}He↑–^{4}He have the form $\mu_1^+ = \mu_2^+, \mu_1^- = \mu_2^-$ (the indices 1 and 2 correspond to the chemical potentials μ of ^{3}He atoms in pure ^{3}He and in the solution) with constant total number of ^{3}He atoms in the system $N = N_1^+ + N_1^- + N_2^+ + N_2^-$ and degree of polarization $P = (N_1^+ + N_2^+ - N_1^- - N_2^-)/N$. Since the energy scales in both phases differ considerably from each other, polarization affects $\mu_1^\pm$ and $\mu_2^\pm$ differently. The conditions for equilibrium $\mu_1^\pm(P_1) = \mu_2^\pm(P_2)$ are satisfied with unequal degrees of polarization $P_{1,2} = (N_{1,2}^+ - N_{1,2}^-)/N_{1,2}$ of both phases, while the equilibrium concentration of ^{3}He in the solution $N_2/N_2 + N_4$ depends on $P_{1,2}$. The effect of polarization on the phase equilibrium is easily observed quantitatively with weak polarization when the change in the energy is quadratic in P. In this case,

$$P_{1,2} = P\frac{\chi_{1,2}N}{\chi_1 N_1 + \chi_2 N_2}, \qquad P_1/P_2 = \chi_1/\chi_2,$$

where $\chi_{1,2}$ is the susceptibility (per single ^{3}He particle) in pure ^{3}He and in solution. Since the distribution of ^{3}He atoms between the pure phase and the solution N_1/N_2 depends on the ratio of the number of ^{3}He N and ^{4}He N_4 atoms in the system, it is easy to change $P_{1,2}$ by varying N/N_4. The main difficulty in solving the equations of phase equilibrium in a strongly polarized system has to do with the absence of information on the function $\mu_1^\pm(P_1)$ for liquid ^{3}He; for rarified phases ^{3}He $\mu(P)$ is known for any polarization.

2. Since at low temperatures T the susceptibility of solid ^{3}He is much higher than in the liquid phase, the ^{3}He crystal melts at a pressure several atmospheres lower than in the absence of polarization.[1,3] It turns out that for the phase diagram of solutions it is fundamentally significant that the pressure of crystallization of pure ^{3}He is higher or lower than for the ^{4}He crystal. In both cases unusual phases of the solution arise. In the first case, the crystal ^{3}He↑ and the liquid solution ^{3}He–^{4}He would be in equilibrium (in the unpolarized system solid ^{3}He cannot be in equilibrium with the solution ^{3}He–^{4}He) and, in addition, with total polarization of ^{3}He the concentration of the solution is approximately two times lower than the limiting concentration of ^{3}He in the unpolarized solution at high pressures. With partial polarization of ^{3}He the polarization of the crystal ^{3}He↑ will be much greater than for ^{3}He–^{4}He ($\chi_1/\chi_2 \sim 100$).

However, the second possibility is of much greater interest and is also more probable. In the absence of polarization the crystal ^{3}He-^{4}He separates completely into

the pure components at $T_c \sim 0.1$ K.[4] Polarization of liquid ^{3}He leads to a very significant (compared to T_c) increase in μ_1^+. This means that when some critical value of the degree of polarization is attained (threshold value $\delta\mu_1^+ \sim 0.1$ K) on the liquid ^{3}He↑-^{4}He crystal equilibrium curve, separation into pure phases disappears and it is possible for the solid solution ^{3}He↑–^{4}He to exist with finite concentration of ^{3}He for arbitrarily low temperatures. The simplest estimate of the threshold value of polarization obtained by extrapolating data on the susceptibility of liquid ^{3}He in weak fields gives the value $P_c \sim 0.2$–0.3. In this case, the impurity component in the crystal $P - P_c$, N/N_4 is completely polarized, while the concentration of ^{3}He in solid ^{4}He is determined by the values of ^{3}He↑ and by the interaction of impurity ^{3}He atoms with each other. The observation of finite solubility of ^{3}He↑ in solid ^{4}He permits studying for the first time impurity quasiparticles in quantum crystals under conditions when the temperature is comparable to the width of the band of impurity quasiparticles Δ_i (for ^{3}He quasiparticles in solid ^{4}He $\Delta_i \sim 10^{-3}$–10^{-4} K) and makes it possible to study ordering in the impurity system.

3. At still lower pressures, liquid ^{3}He↑ will be in equilibrium with the liquid solution ^{3}He↑–^{4}He. The dependence $\mu_2(P_2)$ right up to limiting concentrations of the solution is well described by the expansion[5]

$$\mu_2^{\pm}(P_2) = -\Delta + \frac{(6\pi^2\hbar^3)^{2/3}}{2M}\left(\frac{N_2}{2V_2}\right)^{2/3}(1 \pm P_2)^{2/3} + \frac{4\pi a\hbar^2}{M}\frac{N_2}{2V_2}(1 \mp P_2) + \ldots .$$

The values of the parameters appearing in this expression (V_2 is the volume of the solution, Δ is the binding energy, M is the effective mass, and a is the s-scattering length) are such that for the ratio of the degrees of polarization we obtain the estimate $P_2/P_1 \sim 3$. The appearance of enhanced polarization accompanying dissolution could be very important for experiments: It is already possible to obtain, using present capabilities for polarizing ^{3}He (Ref. 2) in equilibrium with ^{3}He, a nearly completely polarized solution, which makes it possible to check the theory experimentally.[5] The polarization of ^{3}He also changes the limiting solubility of ^{3}He in ^{4}He, in equilibrium with ^{3}He. An estimate of the limiting concentration of the completely polarized solution based on the concentration expansion for μ_2 gives $c/c_0 \sim 3$–4, where c_0 is the limiting concentration in the absence of polarization. As a result, it is possible for a highly concentrated ^{3}He–^{4}He solution which is a Fermi liquid with properties fundamentally different from pure ^{3}He as well as from ^{3}He–^{4}He solutions, to exist.[5] The temperature of the transition of ^{3}He in such a concentrated solution to the superfluid state in the case of p-pairing (in the polarized solution s-pairing is impossible) must exceed 10^{-5} K.

[1]B. Castaing and P. Nozieres, J. Phys. (Paris) **40**, 257 (1979).
[2]M. Chapellier, G. Frossati, and F. B. Rasmusen, Phys. Rev. Lett. **42**, 1904 (1979); G. Schumacher *et al.* Phys. Lett. (Paris) **40**, L143 (1979).
[3]C. Lhuillier and F. Laloö, J. Phys. (Paris) **40**, 239 (1979).
[4]B. N. Esel'son, V. N. Grigor'ev, V. G. Ivantsov, E. Ya., Rudavskiĭ, D. G. Sanikidze, and I. A. Serbin, Rastvory kvantovykh zhidkosteĭ He^3–He^4 (Solutions of Quantum Fluids He^3–He^4), Nauka, Moscow, 1973; S. Trickey, W. Kirk, and E. Adams, Rev. Mod. Phys. **44**, 668 (1972).
[5]P. Bashkin and A. E. Meyerovich, Adv. Phys. **30**, 1 (1981).

Translated by M. E. Alferieff
Edited by S. J. Amoretty