

# Sound and Transport Phenomena in Spin-Polarized $^3\text{He}$ - $^4\text{He}$ Solutions

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(Received October 16, 1981)

*Transport phenomena in partially spin-polarized  $^3\text{He}$ - $^4\text{He}$  solutions are investigated. The polarization causes considerable changes in kinetic coefficients and also gives rise to new dissipative processes, such as spin thermodiffusion and second viscosity. The transport coefficients are calculated for degenerate and nondegenerate  $^3\text{He}\uparrow$ - $^4\text{He}$  solutions. The absorption of first- and second-sound waves is studied. Second-sound propagation is affected by weak dipole interactions and its velocity depends significantly on the frequency.*

## 1. INTRODUCTION

There is considerable interest in the properties of spin-polarized quantum systems such as  $\text{H}\uparrow$ ,  $\text{D}\uparrow$ ,  $^3\text{He}\uparrow$ , etc. Polarized  $^3\text{He}\uparrow$ - $^4\text{He}$  solutions also possess completely new properties, including magnetokinetic phenomena—a large growth of the mean free path and the relaxation time of  $^3\text{He}$  quasiparticles due to polarization of the  $^3\text{He}$  spin system. The origin of these phenomena and predicted effects has been discussed in Ref. 1.

Greywall and Paalanen<sup>2</sup> have recently performed measurements of the second-sound propagation in partially spin-polarized  $^3\text{He}\uparrow$ - $^4\text{He}$  solutions. Not only has this experiment provided precise data on the change in the sound velocity, it has also demonstrated qualitatively the growth of the second-sound attenuation resulting from the polarization. The results show that direct observation of magnetokinetic effects is possible.

However, the experiments cannot be interpreted completely on the basis of available theoretical information. There is no general theory of spin diffusion for NMR experiments in polarized solutions. Magnetokinetic effects in nondegenerate solutions have not been studied, though the corresponding effects are more pronounced in dilute mixtures

with low degeneracy temperatures. The existing theory of sound propagation is not developed sufficiently to explain results on the frequency dispersion and attenuation.

All these problems are studied below. In Section 2, transport coefficients, including spin diffusion and spin thermodiffusion coefficients, are calculated for degenerate and nondegenerate  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  solutions. In Section 3 we study the first-sound attenuation in  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  for arbitrary frequencies and evaluate all significant relaxation times. Section 4 is devoted to the frequency dispersion theory of the second-sound velocity and attenuation. In an Appendix we evaluate some important parameters of the solutions from the experimental results<sup>2</sup> on the second-sound velocity.

Throughout this paper we use the theoretical approach and notations of Ref. 1. Low-temperature kinetic phenomena in  ${}^3\text{He}\text{-He II}$  solutions are determined by the Fermi component of the solution and are specified by the interaction of  ${}^3\text{He}$  quasiparticles. At low temperatures and  ${}^3\text{He}$  concentrations the system of impurity  ${}^3\text{He}$  atoms forms a dilute Fermi gas of slow quasiparticles dissolved in a superfluid Bose background of  ${}^4\text{He}$ . The interaction of slow particles reduces mostly to  $S$ -wave scattering with amplitude independent of momenta. Therefore all kinetic quantities can be characterized using just a single parameter of the quasiparticle interaction: the  $S$ -wave scattering length  $a$ , which is equal to  $a \approx -1.5 \text{ \AA}$ .<sup>1</sup> The accuracy of this approximation is based on the inequalities

$$\hbar^2/Ma^2 \gg T_0, T \quad (1)$$

Here  $T$  is the temperature of the solution,  $T_0 = p_0^2/2M$  is the degeneracy temperature of the nonpolarized solution,  $p_0 = (3\pi^2 N_3)^{1/3} \hbar$  is the  ${}^3\text{He}$  quasiparticle Fermi momentum,  $N_3$  is the number of  ${}^3\text{He}$  atoms per unit volume of the solution,  $M \sim 2.3m_3$  is the effective mass of an impurity  ${}^3\text{He}$  excitation, and  $m_3$  is the mass of the  ${}^3\text{He}$  atom. The first of inequalities (1) is important for degenerate solutions  $T_0 \gg T$ , and the second for nondegenerate solutions.

In the case of spin-polarized solutions the accuracy of (1) is very high, for accessible magnetic fields  $H \leq 100 \text{ kOe}$  can only polarize to a considerable degree solutions with  ${}^3\text{He}$  concentration less than 0.1% ( $T_0 < 25 \text{ mK}$ ) at temperatures below 20 mK, while  $\hbar^2/Ma^2 \sim 1 \text{ K}$ . This allows one to evaluate all the  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  kinetic properties to the main order in the interaction.

The relaxation processes in the  ${}^3\text{He}$  impurity system are due mostly to quasiparticle exchange interactions. The exception is the magnetization relaxation, which is determined by very weak dipole forces or by quasiparticle collisions with the walls. The dipole relaxation time in a  ${}^3\text{He}$  impurity

Fermi gas  $\tau_d \sim (\hbar T_0 / \beta^4 N_3^2) (T_0 / T)^2$  ( $\beta \sim 0.08$  mK/kOe is the  $^3\text{He}$  nuclear magnetic moment) has a scale of minutes and always exceeds the exchange relaxation times. Thus it is possible to create nonequilibrium long-lived polarized states of the solution. Most of the results obtained below are valid for both equilibrium and nonequilibrium polarized states. For this reason we use as thermodynamic variables not  $N_3$  and the external magnetic field  $\mathbf{H}$ , but the densities of  $^3\text{He}$  particles with spins parallel ( $N_+$ ) and antiparallel ( $N_-$ ) to the direction of the magnetic moment of the solution ( $N_+ \geq N_-$ ,  $N_+ + N_- = N_3$ ).

If the solution is polarized by an external field  $\mathbf{H}$ , the up- and down-spin concentrations  $C_{\pm} = N_{\pm} / N_3$  in the principal approximation in the  $^3\text{He}$  concentration are related to the field by the usual Stoner equations, which in the degenerate case take the simple form

$$(p_{\pm} / p_0)^2 - (2 - p_{\pm}^3 / p_0^3)^{2/3} = 4\beta H M / p_0^2 \quad (2)$$

where  $p_{\pm} = (6\pi^2 N_{\pm})^{1/3} \hbar$  is the radius of the Fermi sphere for up and down spins.

## 2. KINETIC COEFFICIENTS

### 2.1. Degenerate Solutions (Spin Diffusion)

Transport phenomena in  $^3\text{He}$ - $^4\text{He}$  solutions are described by a great number of kinetic coefficients. The polarization of the spin system gives rise to some additional coefficients, such as spin thermodiffusion and spin pressure diffusion coefficients. However, the dissipation in dilute  $^3\text{He}$ - $^4\text{He}$  mixtures is caused mainly by the processes in the  $^3\text{He}$  impurity Fermi gas. The dependences of the degenerate impurity Fermi gas viscosity ( $\eta$ ) and thermal conductivity ( $\kappa$ ) coefficients on the degree of polarization are given by<sup>1</sup>

$$\begin{aligned} \frac{\eta}{\eta(0)} &= \frac{1}{2} \left( \frac{p_+}{p_0} \right)^5 \left( \frac{p_+}{p_-} \right)^3 \frac{2 + 5(p_- / p_+)^8 - 3(p_- / p_+)^{10}}{5 - 3(p_- / p_+)^2} \\ \eta(0) &= \frac{1}{12\pi^3} \frac{p_0^5}{(aMT)^2} \\ \frac{\kappa}{\kappa(0)} &= \frac{1}{2} \left( \frac{p_+}{p_0} \right)^3 \frac{p_+}{p_-} \frac{4 + 3(p_- / p_+)^4}{3 + (p_- / p_+)^2} + (p_- / p_+)^6 \\ \kappa(0) &= \frac{1}{8\pi} \frac{p_0^3}{(aM)^2 T} \end{aligned} \quad (3)$$

The diffusion coefficients of the polarized impurity system are introduced

by the following macroscopic expression for the up- or down-spin flow:

$$\mathbf{j}^\pm = -N_3 D_S \left\{ \nabla C_\pm + \frac{K_{ST}^\pm}{T} \nabla T + \frac{K_{SP}^\pm}{\Pi} \nabla \Pi \right\} \quad (4)$$

Here  $D_S$ ,  $K_{ST}^+ = -K_{ST}^-$ , and  $K_{SP}^+ = -K_{SP}^-$  are the spin diffusion, spin thermodiffusion, and spin pressure diffusion coefficients,  $C_\pm = N_\pm/N_3$  is the concentration of up or down spins, and  $\Pi$  is the  $^3\text{He}$  osmotic (partial) pressure. Note that Eq. (4) does not describe all the diffusion processes in  $^3\text{He}\uparrow$ - $^4\text{He}$  solutions. This equation corresponds only to the relative up- and down-spin motions in the absence of the total  $^3\text{He}$  mass flow  $\mathbf{j}^+ + \mathbf{j}^- = \mathbf{j} = 0$ . There are several other diffusion coefficients in  $^3\text{He}$ - $^4\text{He}$  in addition to those introduced above, but their influence on the dissipation processes is small at low  $^3\text{He}$  concentrations.

The spin diffusion coefficient  $D_S$  of the nonpolarized Fermi system was calculated by Hone<sup>3</sup> using a method developed by Abrikosov and Khalatnikov.<sup>4</sup> Brooker and Sykes have since performed more precise calculations.<sup>5</sup> In the case of nonpolarized  $^3\text{He}$ - $^4\text{He}$  solutions the result<sup>3</sup> takes the form<sup>1</sup>

$$D_S(0) = \frac{1}{\pi} \left( \frac{\hbar}{M} \right)^3 \left( \frac{p_0}{aT} \right)^2 \quad (5)$$

The diffusion coefficient of the partially polarized solution can be determined by solving the kinetic equation for the single-particle density matrix. If the  $z$  axis in spin space is chosen along the direction of the polarization, the equilibrium density matrix and the quasiparticles Hamiltonian are diagonal, and we obtain, in place of four kinetic equations, two equations for the diagonal components of the density matrix:

$$\frac{\partial n^\pm}{\partial t} + \frac{\partial n^\pm}{\partial \mathbf{r}} \frac{\partial \varepsilon^\pm}{\partial \mathbf{p}} - \frac{\partial n^\pm}{\partial \mathbf{p}} \frac{\partial \varepsilon^\pm}{\partial \mathbf{r}} = \mathcal{I}^\pm(\mathbf{p}) \quad (6)$$

where  $n^\pm$  and  $\varepsilon^\pm$  are the distribution functions and the energies of up and down spins. The kinetic equations (6) for quasiparticles with different spin projections are coupled via collision integrals

$$\begin{aligned} \mathcal{I}^\pm(\mathbf{p}_1) = & -\frac{2}{(2\pi\hbar)^6} \int d^3 p_2 d^3 p_1' d^3 p_2' \\ & \times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_1' - \mathbf{p}_2') \delta(\varepsilon_1^\pm + \varepsilon_2^\mp - \varepsilon_{1'}^\pm - \varepsilon_{2'}^\mp) \\ & \times W[n_1^\pm n_2^\mp (1 - n_{1'}^\pm)(1 - n_{2'}^\mp) - n_{1'}^\pm n_{2'}^\mp (1 - n_1^\pm)(1 - n_2^\mp)] \quad (7) \end{aligned}$$

where  $n_i^\pm = n^\pm(\mathbf{p}_i)$ ,  $\varepsilon_i^\pm = \varepsilon^\pm(\mathbf{p}_i)$ , and the probability  $W$  of  $S$ -wave scattering

of  $^3\text{He}$  quasiparticles with momenta  $\mathbf{p}_1^\pm, \mathbf{p}_2^\mp$  into the state  $\mathbf{p}_1^\pm, \mathbf{p}_2^\mp$  is independent of momenta and, to the exchange approximation for the spin diffusion, is equal to  $W = \frac{1}{4}(2\pi\hbar)^3(a/M)^2$ .<sup>1</sup>

One has to calculate the diffusion coefficient at constant temperature and  $^3\text{He}$  osmotic pressure  $\nabla T = \nabla \Pi = 0$ . Then the gradients of the chemical potentials for up and down spins  $\mu_\pm$  are proportional to the spin concentration gradient,

$$\nabla \mu_\pm = \pm \frac{p_\pm^2}{3MN_\pm} \frac{N_3^2 N_\mp^{2/3}}{N_+^{5/3} + N_-^{5/3}} \nabla C_+ \quad (8)$$

On the left-hand side of the kinetic equations (6), as always in order to evaluate the kinetic coefficients, one must substitute the local equilibrium distribution functions  $n_0^\pm(\varepsilon^\pm - \mu_\pm)$  and linearize the equations in small values of the gradients,

$$-\frac{\partial n_0^\pm}{\partial \varepsilon^\pm} \frac{\mathbf{p}_\pm}{M} \nabla \mu_\pm = \mathcal{G}^\pm(\mathbf{p}) \quad (9)$$

To calculate the collision integrals (7) we insert new variables  $dx_i = p_i dp_i/MT$  and perform the integration removing the  $\delta$ -functions,

$$\begin{aligned} \mathcal{G}^\pm(\mathbf{p}_1) &= \frac{2M^3 T}{(2\pi\hbar)^6} W p_\mp \int \frac{dx_1' dx_2' d\Omega_2 d\varphi}{(p_+^2 + p_-^2 + 2p_+ p_- \cos \theta_2)^{1/2}} \\ &\quad \times \{\nu_1^\pm + \nu_2^\mp - \nu_1'^\pm - \nu_2'^\mp\} n_1^\pm n_2^\mp (1 - n_1'^\pm)(1 - n_2'^\mp) \end{aligned} \quad (10)$$

Here  $\varphi$  is the angle between the planes  $(\mathbf{p}_1, \mathbf{p}_2)$  and  $(\mathbf{p}_1', \mathbf{p}_2')$ ,  $d\Omega_2 = d \cos \theta_2 d\varphi_2$ ,  $\theta_i$  is the angle between the vectors  $\mathbf{p}_i$  and  $\mathbf{p}_1$ , and  $\nu_i = \nu(\mathbf{p}_i)$  stands for the small deviation of the distribution function  $n(\mathbf{p}_i)$

$$n_i = n_0(\mathbf{p}_i) + \frac{\partial n_0}{\partial \varepsilon_i} \nu_i \quad (11)$$

The solution of the integral equations (9)–(10) has the form

$$\nu^\pm(\mathbf{p}) = r^\pm \left( \frac{\varepsilon^\pm - \mu_\pm}{T} \right) \mathbf{p}_\pm \nabla C_+$$

The quasiparticle collisions are accompanied by a very small energy transfer, and we can confine ourselves to the usual approximation<sup>1,3,4</sup> of the functions  $r((\varepsilon - \mu)/T)$  by their values  $r(0)$ . This leads formally to the relaxation time

approximation and simplifies the integration (10) over  $dx_1$ ,  $dx_2$ ,  $d\varphi$ . We have

$$\begin{aligned} & \frac{2\pi^3 M^4 T^2}{(2\pi\hbar)^6} W p_{\mp} \int \frac{d \cos^3 \theta_2 d\varphi}{(p_+^2 + p_-^2 + 2p_+ p_- \cos \theta_2)^{1/2}} \{r^{\pm}(0)[1 - \cos \theta_1] \\ & + \frac{p_{\mp}}{p_{\pm}} r^{\mp}(0)[\cos \theta_2 - \cos \theta_2']\} \\ & = \pm \frac{p_{\pm}^2}{3MN_{\pm}} \frac{N_3^2 N_{\mp}^{2/3}}{N_+^{5/3} + N_-^{5/3}} \end{aligned} \quad (12)$$

where we have taken into account Eq. (8). After the final angular integration we obtain for  $r(0)$  the following linear equations:

$$\frac{\pi a^2 M^2 T^2}{6\hbar^3} \frac{p_{\mp}^3}{p_+^3} [r^+(0) - r^-(0)] = \pm \frac{p_{\pm}^2}{3MN_{\pm}} \frac{N_3^2 N_{\mp}^{2/3}}{N_+^{5/3} + N_-^{5/3}} \quad (13)$$

Since the spin diffusion (4) is defined with the additional condition of the absence of  $^3\text{He}$  mass flow,

$$0 = \int d^3 p \left( \frac{\partial n_0^+}{\partial \varepsilon^+} \mathbf{p}_+ \nu^+ + \frac{\partial n_0^-}{\partial \varepsilon^-} \mathbf{p}_- \nu^- \right) \quad (14)$$

Eqs. (13) have to be solved taking into account

$$p_+^3 r^+(0) + p_-^3 r^-(0) = 0 \quad (15)$$

The solution of Eqs. (13), (15) has the form

$$r^+(0) = -\frac{p_+^3}{p_-^3} r^-(0) = \frac{6\pi\hbar^6}{a^2 M^3 T^2} \frac{p_+^2}{p_0^3} \frac{N_3^2 N_-^{2/3}}{N_+^{5/3} + N_-^{5/3}}$$

and the spin diffusion coefficient, defined by Eq. (4), is equal to

$$D_S = D_S(0) \left( \frac{p_+}{p_0} \right)^5 \frac{N_3 N_-^{2/3}}{N_+^{5/3} + N_-^{5/3}} \quad (16)$$

The calculation of the spin thermodiffusion coefficient  $K_{ST}^{\pm}$  is somewhat more complicated. In this case the up- and down-spin flows are caused by the temperature gradient  $\nabla T$ . The distribution function has the form  $\nu = r((\varepsilon - \mu)/T) \mathbf{p} \nabla T$  and now we have to approximate  $r(x)$  as  $r(x) = r(0) + xr'(0)$ . The antisymmetric part of the distribution function  $r'(0)$  is determined by the kinetic equations

$$-\frac{1}{M} \frac{\partial n_0^{\pm}}{\partial \varepsilon^{\pm}} \frac{\varepsilon^{\pm} - \mu_{\pm}}{T} \mathbf{p}_{\pm} \nabla T = \mathcal{F}^{\pm}(r'(0)) \quad (17)$$

After cumbersome calculations partly analogous to Eqs. (12), (13) we

obtain the following value of  $r'(0)$ :

$$r'_{\pm}(0) = \frac{9}{2} \frac{\hbar^3}{\pi(aMT)^2} \frac{p_+}{p_{\mp}} \frac{4p_+^2}{3p_+^2 + p_{\mp}^2}$$

The value of  $r(0)$  is determined by the homogeneous equations for the symmetric part of the distribution function and by the condition of the absence of the total  $^3\text{He}$  mass flow (14). Finally, the spin thermodiffusion coefficient takes the form

$$\frac{D_s K_{ST}^{\pm}}{T} = \frac{9}{8} \frac{\pi \hbar^3}{(aMT)^2} \frac{MT}{p_0^2} \frac{p_+ p_-}{p_0^4} \frac{(p_{\pm} - p_{\mp})(p_- - 3p_+)}{p_-^2 + 3p_+^2}$$

and the spin thermal diffusion ratio is equal to

$$K_{ST}^{\pm} = \frac{9\pi^2}{8} \frac{M^2 T^2}{p_0^4} \frac{p_0^3}{p - p_+^2} \frac{p_+^5 + p_-^5}{2p_0^5} \frac{(p_{\pm} - p_{\mp})(p_- - 3p_+)}{p_-^2 + 3p_+^2}$$

The spin pressure diffusion coefficient is evaluated from the usual thermodynamic relations for binary mixtures. In the case of degenerate solutions  $K_{SP}^{\pm}$  has the form

$$K_{SP}^{\pm} \equiv -\Pi \left( \frac{\partial C_{\pm}}{\partial \Pi} \right)_{\mu_+ - \mu_-} = \frac{3}{5} \frac{N_+^{1/3} N_-^{1/3}}{N_3^3} (N_+^{5/3} + N_-^{5/3}) (N_{\pm}^{2/3} - N_{\mp}^{2/3})$$

while the partial pressure ( $^3\text{He}$  osmotic pressure) is

$$\Pi = (1/5M)(p_+^2 N_+ + p_-^2 N_-)$$

## 2.2. Nondegenerate Solutions

At low temperatures a  $^3\text{He}$  quasiparticle thermal wavelength is large in comparison with atomic dimensions. Thus it is possible to restrict oneself to the  $S$ -wave scattering describing the  $^3\text{He}$  quasiparticle interaction in dilute nondegenerate  $^3\text{He}\uparrow$ - $^4\text{He}$  mixtures.<sup>1</sup> The results for the  $S$ -wave scattering in the first order in the interaction coincide with those of the hard-sphere model, and within the exchange approximation the impurity  $^3\text{He}$  component of nondegenerate polarized solutions is analogous to the binary mixture of classical ideal gases. Hence the dependence of  $^3\text{He}\uparrow$ - $^4\text{He}$  kinetic coefficients on the degree of polarization is determined by the same relations as is the dependence of kinetic coefficients on the mixture concentration for an ideal gas binary mixture. A somewhat analogous approach to the transport phenomena in polarized systems was developed by Lhuillier and Laloë.<sup>6</sup>

In the Boltzmann region  $T \gg T_0$  the  $^3\text{He}\uparrow$ - $^4\text{He}$  transport properties are given by the well-known expressions<sup>7</sup> of the Chapman-Enskog theory

in the hard-sphere approximation. Note that for  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  solutions in the  $S$ -wave approximation only the collisions of  ${}^3\text{He}$  quasiparticles with opposite spins are important. Thus in all formulas for classical binary mixtures one has to put formally the scattering cross sections of particles on the atoms of the same component equal to zero and to take into account only the collisions of atoms of different components. In our notation the corresponding effective cross section is equal to  $8\pi a^2$  in the case of the viscosity and thermal conductivity coefficients and to  $2\pi a^2$  for spin diffusion processes. A little manipulation yields that the  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  viscosity and thermal conductivity in the Boltzmann region

$$\frac{\eta}{\eta(0)} = \frac{N_3^2 + \frac{3}{5}(N_+ - N_-)^2}{4N_+N_-}, \quad \eta(0) = \frac{5}{8} \frac{(\pi MT)^{1/2}}{8\pi a^2}$$

$$\frac{\kappa}{\kappa(0)} = \frac{60N_3^2 - N_+^2 - N_-^2 - 66N_+N_-}{172N_+N_-}, \quad \kappa(0) = \frac{75}{32} \frac{(\pi T/M)^{1/2}}{8\pi a^2}$$

increase without limit in high magnetic fields  $N_- \rightarrow 0$ , and the spin diffusion coefficient

$$D_s = \frac{3}{8} \frac{(\pi T/M)^{1/2}}{2\pi a^2 N_3}$$

does not depend on the polarization, while the spin thermal diffusion ratio is proportional to the degree of polarization

$$K_{ST}^{\pm} = \frac{5}{43} \frac{N_{\pm} - N_{\mp}}{N_3}$$

### 3. FIRST SOUND

The first-sound waves in  ${}^3\text{He}\text{-He II}$  solutions are mainly the oscillations of superfluid He II. Though  ${}^3\text{He}$  quasiparticles also participate in these oscillations, their influence on the sound velocity is small because of the low  ${}^3\text{He}$  concentration. However, the first-sound attenuation at low temperatures is due predominantly to the impurity component. Though the polarization of solutions is unimportant for the sound velocity<sup>1</sup>, it must cause a tremendous increase of the sound absorption.

At low frequencies one can obtain the first-sound absorption coefficient from the hydrodynamic equations and express it in terms of the kinetic coefficients calculated above. Nevertheless first sound in  ${}^3\text{He}\text{-}{}^4\text{He}$  solutions can also propagate in the high-frequency regime  $\omega\tau \gg 1$  ( $\tau$  is the characteristic relaxation time). The high-frequency regime is of special interest, for there are several different relaxation times in polarized systems. Below we

calculate the sound absorption coefficient in degenerate  $^3\text{He}\uparrow$ - $^4\text{He}$  solutions, solving the kinetic equation at arbitrary  $\omega\tau$ . This enables us to find all the significant relaxation times.

For dilute  $^3\text{He}$ - $^4\text{He}$  mixtures one can neglect all the impurity corrections to the first-sound velocity, which coincides practically with the sound velocity  $s_0$  in pure He II. Then the oscillations of the superfluid velocity  $\mathbf{v}_s$  and the  $^4\text{He}$  density  $N_4$  provide the oscillating external field for  $^3\text{He}$  quasiparticles.

The change of the  $^3\text{He}$  quasiparticle Hamiltonian in moving He II is equal to<sup>1</sup>

$$\delta\varepsilon = \alpha_1 m_4 s_0^2 \delta N_4 / N_4 + (1 - m_3 / M) \mathbf{p} \mathbf{v}_s \quad (18)$$

while the quantities  $\delta N_4$  and  $\mathbf{v}_s$  are related to each other in the sound wave  $\exp(-i\omega t + i\mathbf{k}\mathbf{r})$  as

$$\delta N_4 / N_4 = v_s / s_0, \quad \mathbf{v}_s = v_s \mathbf{n}, \quad \mathbf{n} = \mathbf{k} / k \quad (19)$$

Here  $m_4$  is the  $^4\text{He}$  atomic mass and  $\alpha_1$  is a dimensionless parameter which characterizes the change in energy of a  $^3\text{He}$  quasiparticle with a change in  $^4\text{He}$  density. The numerical value of  $\alpha_1$  is discussed in the Appendix. Although at low  $^3\text{He}$  concentrations the second term in Eq. (18) is less than the first ( $p \propto N_3^{1/3}$ ) both terms make the same contributions to the absorption coefficient.

We seek the solution of the kinetic equation (6) in the form

$$n^\pm = n_0^\pm (\varepsilon^\pm - \mu_\pm - \mathbf{p}_\pm \mathbf{v}_s) + \nu^\pm \partial n_0 / \partial \varepsilon^\pm$$

Instead of Eq. (9) we have

$$\begin{aligned} \mathcal{F}^\pm = i\omega \frac{\partial n_0^\pm}{\partial \varepsilon^\pm} \left\{ \left( -1 + \frac{\mathbf{p}_\pm \mathbf{n}}{M s_0} \right) (v^\pm - \delta\mu_\pm) \right. \\ \left. - \alpha_1 m_4 s_0 v_s + \left( \frac{m_3}{M} - \frac{\mathbf{p}_\pm \mathbf{n}}{M s_0} \right) \mathbf{p} \mathbf{v}_s \right\} \end{aligned} \quad (20)$$

where  $\delta\mu_\pm$  is the change of the  $^3\text{He}$  chemical potential at the oscillations. The chemical potential  $\delta\mu_\pm$  is defined as

$$\int \frac{\partial n_0^\pm}{\partial \varepsilon^\pm} \nu^\pm d^3 p = 0$$

We are interested in the exchange approximation in which the collisions do not change the particle polarization

$$\int \mathcal{F}^\pm d^3 p = 0$$

and integrating Eq. (20), we obtain the value

$$\delta\mu_{\pm} = \alpha_1 m_4 s_0 v_s + \frac{1}{3} \frac{p_{\pm}^2}{M} \frac{v_s}{s_0} - \frac{1}{Ms_0} \int \mathbf{p}_{\pm} \mathbf{n} \nu^{\pm} \frac{d\Omega}{4\pi}$$

As a result the kinetic equation (20) in the principal order in the concentration takes the form

$$\begin{aligned} \mathcal{I} = i\omega \frac{\partial n_0}{\partial \varepsilon} \left\{ -\nu + \frac{m_3}{M} \mathbf{p} \mathbf{v}_s - \alpha_1 \frac{m_4}{M} \mathbf{p} \mathbf{v}_s + \frac{\mathbf{p} \mathbf{n}}{Ms_0} \nu \right. \\ \left. - \frac{1}{Ms_0} \int \mathbf{p} \mathbf{n} \nu \frac{d\Omega}{4\pi} + \frac{1}{3} \frac{p^2}{Ms_0} v_s - \frac{(\mathbf{p} \mathbf{n})^2}{Ms_0} v_s \right\} \end{aligned} \quad (21)$$

while the collision integral is given by (10) [in this case  $W = (2\pi\hbar)^3 (a/M)^2$ ].

The angular dependence of the distribution function can be written as

$$\nu = \nu^{(1)} + \nu^{(2)}, \quad \nu^{(1)} = r(\varepsilon - \mu) \mathbf{p} \mathbf{v}_s, \quad \nu^{(2)} = q(\varepsilon - \mu) [(\mathbf{p} \mathbf{n})^2 - \frac{1}{3} p^2] v_s$$

The collision integral with the function  $\nu^{(1)}$  is transformed analogously to Eqs. (10)–(13), and the equation in  $r(0)$  reduces to

$$i\omega \left[ -r^{\pm}(0) + \frac{m_3}{M} - \alpha_1 \frac{m_4}{M} \right] = \frac{\pi (aMT)^2}{6\hbar^3} \frac{p_{\mp}^3}{p_{\pm}^3} [r^{\pm}(0) - r^{\mp}(0)]$$

Since the solution of this equation

$$r^{\pm}(0) = m_3/M - \alpha_1 m_4/M \quad (22)$$

is real, the corresponding part of the distribution function does not contribute to the absorption. The substitution of  $\nu^{(1)}$ , (22), into Eq. (21) results in the following equation in  $q(0)$ :

$$\begin{aligned} i\omega \left\{ q^{\pm}(0) + \frac{1}{M^2 s_0} (M - m_3 + \alpha_1 m_4) \right\} \\ = \frac{M}{4\hbar^3} a^2 T^2 p_{\mp} \int \frac{d \cos \theta_2 d\varphi}{(p_{+}^2 + p_{-}^2 + 2p_{+} p_{-} \cos \theta_2)^{1/2}} \\ \times \left\{ q^{\pm}(0) [1 - P_2(\cos \theta_1)] \right. \\ \left. + \frac{p_{\mp}^2}{p_{\pm}^2} q^{\mp}(0) [P_2(\cos \theta_2) - P_2(\cos \theta_2')] \right\} \end{aligned} \quad (23)$$

where  $P_2(x) = (3x^2 - 1)/2$ .

The angular integral of the second term in the integrand (23) is equal to zero, and the equations in  $q^{\pm}(0)$  decouple. This means that within our

accuracy the mutual drag of different spin components of the Fermi system is absent and there are only two relaxation times  $\tau_{\pm}$ . After some algebra, Eq. (23) takes the form

$$q^{\pm}(0) = \frac{1}{M^2 s_0} (M - m_3 + \alpha_1 m_4) / \left(1 + \frac{i}{\omega \tau_{\pm}}\right)$$

$$\tau_{\pm} = \frac{1}{2\pi} \frac{\hbar}{M} \left(\frac{\hbar}{aT}\right)^2 \frac{5}{5 - 3(p_{\mp}/p_{\pm})^2} \left(\frac{p_{\pm}}{p_{\mp}}\right)^3 \quad (24)$$

Thus, one of the relaxation times ( $\tau_{-}$ ) does not depend on the polarization, while the other ( $\tau_{+}$ ) increase practically without limit when the polarization approaches the saturation  $p_{-} \rightarrow 0$ .

The relaxation time  $\tau$  for the sound absorption processes in a nonpolarized Fermi liquid is related to the viscosity as<sup>8</sup>  $\eta = \tau N p_0^2 / 5M$ . In a polarized system one has to introduce two viscosity coefficients for up and down spins,

$$\eta_{\pm} = \tau_{\pm} N_{\pm} p_{\pm}^2 / 5M \quad (25)$$

while the "static" viscosity (3) characterizing dissipative processes at low frequencies is  $\eta = \eta_{+} + \eta_{-}$ .

The dissipative function  $\langle R \rangle$  is the time average of

$$R = \int \frac{d^3 p}{(2\pi \hbar)^3} \left\{ \frac{\partial n_0^{+}}{\partial \varepsilon^{+}} \nu^{(2)+} \mathcal{G}^{+}(\nu^{(2)+}) + \frac{\partial n_0^{-}}{\partial \varepsilon^{-}} \nu^{(2)-} \mathcal{G}^{-}(\nu^{(2)-}) \right\}$$

The absorption coefficient  $\gamma$  is defined as

$$\gamma = \frac{1}{2s_0} \frac{\langle R \rangle}{m_4 N_4 \langle v_s^2 \rangle}$$

and is equal to

$$\gamma = \frac{\omega^2}{2m_4 N_4 s_0^3} \left(1 - \frac{m_3}{M} + \alpha_1 \frac{m_4}{M}\right)^2 \left( \frac{\frac{4}{3}\eta_{+}}{1 + (\omega\tau_{+})^2} + \frac{\frac{4}{3}\eta_{-}}{1 + (\omega\tau_{-})^2} \right) \quad (26)$$

In polarized solutions there are two absorption maxima at the frequencies  $\omega\tau_{\pm} = 1$  instead of the single maximum  $\omega\tau = 1$  in a nonpolarized system. As the polarization grows, one of these maxima remains in the same place and decreases to zero, while the other increases and moves to vanishingly low frequencies. At nearly full polarization the condition  $\omega\tau_{+} \gg 1$  is valid for all reasonable frequencies.

The form of Eq. (26) permits one to write out directly the absorption coefficient in nonpolarized solutions for the hydrodynamic limit  $\omega\tau_{+} \ll 1$ .

A result analogous to Eq. (26) in the case of nonpolarized solutions was obtained by Baym and Ebner.<sup>9</sup>

## 4. SECOND SOUND AND DIPOLE RELAXATION

### 4.1. Second-Sound Velocity

In contrast to the velocity of first sound, the second-sound velocity  $s_2$  changes considerably upon  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  polarization.<sup>1</sup> In addition  $s_2$  in  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  depends essentially on the frequency.<sup>2</sup>

In previous sections we neglected the weak dipole interaction. This interaction (or the particle collisions with the walls) does not conserve spin densities  $N_+$  and  $N_-$  and, in spite of its weakness, strongly affects the sound propagation. The equilibrium spin densities  $N_{\pm}^{(0)}$  are determined by the  ${}^3\text{He}$  density  $N_3$ , the magnetic field intensity, and (in nondegenerate solutions) the temperature. At oscillations of  $N_3$  the densities  $N_{\pm}$  also oscillate. But all the kinetic processes discussed above are of exchange origin and do not lead to the local equilibrium values  $N_{\pm}^{(0)}(N_3)$ . The relaxation of spin densities  $N_{\pm}$  to their equilibrium values  $N_{\pm}^{(0)}(N_3)$  can be caused only by the dipole interaction with the very long relaxation time  $\tau_d$  ( $\tau_d$  considerably exceeds all exchange relaxation times  $\tau_{\text{ex}}$  calculated above).

At low frequencies  $\omega\tau_d \ll 1$  the densities  $N_{\pm}$  relax to  $N_{\pm}^{(0)}(N_3)$  and the oscillations are truly adiabatic; at higher frequencies there is no such relaxation, and oscillations propagating with some other velocity correspond to the sound wave in a binary mixture of up and down spins with particle conservation in each component. This frequency dispersion of sound waves is analogous to the well-known case of sound propagation in media with possible chemical reactions or with some other slow relaxation processes. These slow processes usually lead to sound velocity frequency dependence and give rise to the second (bulk) viscosity.

Such effects are best shown by the simplest example of second sound in degenerate  ${}^3\text{He}\uparrow\text{-}{}^4\text{He}$  solutions in the principal approximation in the  ${}^3\text{He}$  concentration. Taking into account the dipole relaxation, we find that the continuity equations for  ${}^3\text{He}$  atoms have the form

$$\frac{\partial N_{\pm}}{\partial t} + N_{\pm} \operatorname{div} \mathbf{v} + \frac{N_{\pm} - N_{\pm}^{(0)}}{\tau_d} = 0, \quad \frac{\partial N_3}{\partial t} + N_3 \operatorname{div} \mathbf{v} = 0, \quad (27)$$

$$N_+ + N_- = N_+^{(0)} + N_-^{(0)} = N_3$$

where  $\mathbf{v}$  is the hydrodynamic (normal) velocity. In Eqs. (27) we do not consider spin diffusion currents  $\mathbf{j}^{\pm}$ , (4), which are important only for the sound absorption. Equations (27) show that small oscillating deviations of the spin densities  $\delta N_{\pm}$  in the sound wave  $\exp(-i\omega t)$  satisfy the equation

$$\delta N_{\pm} - \frac{N_{\pm}^{(0)}}{N_3} \delta N_3 + \frac{i}{\omega\tau_d} (\delta N_{\pm} - \delta N_{\pm}^{(0)}) = 0$$

reducible to

$$\frac{\partial N_{\pm}}{\partial N_3} = \frac{1}{1 + i/\omega\tau_d} \left( \frac{N_{\pm}^{(0)}}{N_3} + \frac{i}{\omega\tau_d} \frac{\partial N_{\pm}^{(0)}}{\partial N_3} \right) \quad (28)$$

In the  ${}^3\text{He}$  quasiparticle degenerate Fermi gas

$$\frac{dN_{\pm}^{(0)}}{dN_3} = \frac{(N_{\pm}^{(0)})^{1/3}}{(N_+^{(0)})^{1/3} + (N_-^{(0)})^{1/3}} \quad (29)$$

the second sound velocity

$$s_2^2 = \frac{1}{M} \frac{\partial \Pi}{\partial N_3}$$

is to be calculated using the following thermodynamic identity for the  ${}^3\text{He}$  osmotic pressure  $\Pi$  in  ${}^3\text{He}\uparrow$ - ${}^4\text{He}$  solutions:

$$d\Pi = N_+ d\mu_+ + N_- d\mu_- = \frac{(6\pi^2)^{2/3} \hbar^2}{3M} (N_+^{2/3} dN_+ + N_-^{2/3} dN_-)$$

As a result the sound velocity  $s_2$  essentially depends on the frequency

$$s_2^2(\omega, H) = \frac{1}{1 + (\omega\tau_d)^2} [(s_2^{(0)})^2 + (\omega\tau_d)^2 (s_2^{(\infty)})^2] \quad (30)$$

where the low-frequency limit of the second-sound velocity  $s_2^{(0)} \equiv s_2(\omega = 0)$

$$(s_2^{(0)})^2 = \frac{(6\pi^2)^{2/3} \hbar^2}{3M} \frac{N_3}{N_+^{1/3} + N_-^{1/3}} = \frac{2^{2/3} N_3^{1/3}}{N_+^{1/3} + N_-^{1/3}} s_2^2(H = 0)$$

was evaluated in Ref. 1 [ $s_2(H = 0) = p_0/M\sqrt{3}$  is the second-sound velocity in nonpolarized solutions], and in the high-frequency limit  $\omega\tau_d \gg 1$  the velocity  $s_2^{(\infty)} \equiv s_2(\omega\tau_d \rightarrow \infty)$

$$\begin{aligned} (s_2^{(\infty)})^2 &= \frac{(6\pi^2)^{2/3} \hbar^2}{3M} N_3^{2/3} \left[ \left( \frac{N_+}{N_3} \right)^{5/3} + \left( \frac{N_-}{N_3} \right)^{5/3} \right] \\ &= 2^{2/3} \left[ \left( \frac{N_+}{N_3} \right)^{5/3} + \left( \frac{N_-}{N_3} \right)^{5/3} \right] s_2^2(H = 0) \end{aligned}$$

coincides with the result of Ref. 2.

The frequency dispersion in degenerate solutions can be easily found also taking into account the quasiparticle (exchange) interaction. Then the sound velocity is equal to<sup>1</sup>

$$s_2^2 = \frac{1}{M} \left\{ N_+ \frac{d\mu_+}{dN_3} + N_- \frac{d\mu_-}{dN_3} \right\} - \alpha_1^2 s_0^2 \frac{m_4}{M} \frac{N_3}{N_4} \quad (31)$$

while the deviations of  $\mu_{\pm}$  caused by the interaction are

$$\delta\mu_{\pm} = (4\pi a \hbar^2 / M) N_{\mp}$$

and for derivatives  $dN_{\pm}^{(0)} / dN_3$  one has to use, instead of Eq. (29), their values<sup>1</sup> including the interaction. It is worth mentioning that the second term in Eq. (31) corresponding to the <sup>4</sup>He drag in the second-sound wave does not depend on the frequency.

The results for the second-sound dispersion can be extended to the case of nondegenerate solutions. In this case the frequency dependence is also described by Eqs. (27), (28), (30), and the sound velocity

$$s_2^2 = \frac{1}{M} \left( \frac{\partial \Pi}{\partial N_3} \right) \frac{c_p}{c_v}$$

( $c_p$  and  $c_v$  are the heat capacities) is determined using Eq. (28) and the identity for the <sup>3</sup>He quasiparticle ideal gas

$$\left( \frac{\partial \Pi}{\partial N_3} \right)_{T,H} = R_+ \frac{\partial N_+}{\partial N_3} + R_- \frac{\partial N_-}{\partial N_3}, \quad R_{\pm} = \left( \frac{\partial \Pi}{\partial \mu_{\pm}} \right)_{T,N_{\mp}} \left( \frac{\partial \mu_{\pm}}{\partial N_{\pm}} \right)_{T,N_{\mp}}$$

Note that according to Eq. (28) sound dispersion is always absent when  $\partial N_{\pm}^{(0)} / \partial N_3 = N_{\pm}^{(0)} / N_3$ . For this reason the dipole relaxation does not cause dispersion in cases of nonpolarized or completely polarized solutions and in the Boltzmann region  $T \gg T_0$ . As was mentioned in the Section 1, usually  $\omega\tau_d \gg 1$ .

## 4.2. Sound Absorption and Second Viscosity

Second sound in <sup>3</sup>He-<sup>4</sup>He solutions at low temperatures corresponds to hydrodynamic oscillations in the impurity system. Such oscillations can propagate only at low frequencies  $\omega\tau_{ex} \ll 1$ , or, more precisely,  $\omega\tau_+ \ll 1$ . In polarized solutions, according to Eq. (24), this inequality considerably limits the frequency. If one neglects the dipole interaction, the second-sound absorption coefficient in <sup>3</sup>He $\uparrow$ -<sup>4</sup>He is given by the usual expression for sound waves in binary mixtures<sup>10</sup>

$$\gamma = \frac{\omega^2}{2\rho s_2^3} \left[ \frac{4}{3} \eta + \kappa \left( \frac{1}{c_v} - \frac{1}{c_p} \right) + \frac{D_S s_2^2}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_{\Pi,C} \left( \frac{\partial \mu}{\partial C} \right)_{\Pi,T} \frac{K_{ST}^2}{c_p^2} \right] \quad (32)$$

where  $\rho = MN_3$  and  $C$  is the concentration of one of the spin components ( $N_+ / N_3$  or  $N_- / N_3$ ).

In degenerate solutions the absorption is determined by the viscosity  $\eta$ , (3), since the second and third terms in Eq. (32) are proportional to the small factor  $c_p - c_v$ . In nondegenerate solutions terms proportional to the viscosity, heat conductivity, and spin diffusion coefficients are of the same

order and all three terms in Eq. (32) are important. The last term can be simplified in the Boltzmann region to the form

$$\frac{\omega^2}{2s_2^3} \frac{D_S}{c_p c_v} K_{ST}^2 \frac{N_3^2}{N_+ N_-}$$

In the absence of the polarization the spin diffusion contribution to the absorption vanishes, since in this case the spin thermal diffusion ratio  $K_{ST}$  is equal to zero.

The sound attenuation also depends on the dipole interaction. Slow hydrodynamic relaxation processes give rise to the second (bulk) viscosity  $\zeta$ . In our case the second viscosity coefficient is due to the dipole relaxation

$$\zeta = \tau_d \rho [(s_2^{(\infty)})^2 - (s_2^{(0)})^2]$$

and the corresponding absorption coefficient is equal to

$$\frac{\omega^2}{2\rho s_2^3} \frac{\zeta}{1 + (\omega\tau_d)^2}$$

The positiveness of  $\zeta$  means that always  $s_2^{(\infty)} \geq s_2^{(0)}$ . As mentioned above, in real experimental conditions  $\omega\tau_d \gg 1$ . Then the relative influence of the first and second viscosities on the absorption is specified by the ratio

$$\frac{\eta(\omega\tau_d)^2}{\zeta} \sim \frac{\hbar\omega^2 p_0}{\beta^4 a^2 N_3^3} \left(\frac{T_0}{T}\right)^4 \sim 10^{-8} \frac{\omega^2}{x^{8/3}} \left(\frac{T_0}{T}\right)^4$$

where  $x$  is the  $^3\text{He}$  concentration  $N_3/N_4$  and  $\omega$  is measured in  $\text{sec}^{-1}$ . This estimation shows that in the experiment<sup>2</sup> the increase of the second-sound attenuation was caused mainly by the growth of the first viscosity  $\eta$  and not by the appearance of second viscosity. Experimentally the contributions of the shear and bulk viscosities to the sound absorption can be separated by their different frequency dependences: while the absorption due to first viscosity is proportional to  $\omega^2$  ( $\omega\tau_{\text{ex}} \ll 1$ ), the second-viscosity contribution does not depend on  $\omega$  ( $\omega\tau_d \gg 1$ ). Analogous to the case of second sound, the bulk viscosity also may be important for the first-sound attenuation.

## 5. CONCLUSION

The polarization of the  $^3\text{He}$  spin system completely changes the transport properties of  $^3\text{He}$ - $^4\text{He}$  solutions. The polarization leads not only to considerable changes in kinetic coefficients, but also gives rise to important new relaxation processes, such as the spin thermodiffusion, bulk viscosity, etc. Thus the picture of dissipation in  $^3\text{He}\uparrow$ - $^4\text{He}$  solutions is altered. Most

of the results obtained above can be applied to other polarized Fermi systems.

### APPENDIX

One of the important characteristics of  $^3\text{He}$ - $^4\text{He}$  solutions is the parameter

$$\alpha_1 = \frac{\partial(-\Delta)}{\partial N_4} \frac{N_4}{m_4 s_0^2} \quad (\text{A1})$$

introduced in Section 3. Here  $-\Delta$  is the binding energy of a single  $^3\text{He}$  impurity atom dissolved in  $^4\text{He}$ . This parameter is widely used and for many theories it is a basic quantity (see, e.g., Ref. 11). The accepted value  $\alpha_1 \sim 1.28$  is based not on direct measurements of  $\Delta$  but on theoretical estimations using the difference of  $^3\text{He}$  and  $^4\text{He}$  molar volumes.

The value of  $\alpha_1$  can be obtained from experimental data<sup>2,12</sup> on the second-sound velocity in dilute  $^3\text{He}$ - $^4\text{He}$  mixtures at low temperature. The second-sound velocity in nonpolarized degenerate solutions (including the effects of quasiparticle interactions) is equal to<sup>1</sup>

$$s_2^2 = s_{id}^2 \left( 1 + \frac{2p_0 a}{\pi \hbar} \right) - \alpha_1^2 s_0^2 \frac{m_4}{M} x \quad (\text{A2})$$

where  $s_{id} = p_0/M\sqrt{3}$  is the sound velocity in the ideal degenerate quasiparticles Fermi gas. Substituting into Eq. (A2) the usual parameters of the solution at zero pressure, one derives the numerical equality

$$(s_{id} - s_2)/s_{id} = x^{1/3} [0.42(-a/1.5) + 8.9\alpha_1^2] \quad (\text{A3})$$

where the scattering length  $a$  is measured in  $\text{\AA}$ . The data of Ref. 2 give the following value of  $\alpha_1$  at the scattering length<sup>1</sup>  $a = -1.5 \text{\AA}$ :

$$\alpha_1 \approx 0.08 \quad (\text{A4})$$

while  $x = 0.01$ . A similar value of  $\alpha_1$  also can be obtained from the experimental data at the concentration  $x = 0.003$ , but in this case the accuracy is very low, since the difference  $s_{id} - s_2$  becomes very small. The accuracy of Eq. (A2) and consequently of the estimation (A4) is about  $x^{1/3}$ .

The small value of  $\alpha_1$  in (A4) compared with the traditional 1.28 is not surprising. It is connected mainly with the different scales of the quantities in Eq. (32): while  $\Delta \sim 2.8 \text{ K}$ , the value of  $m_4 s_0^2$  is about 40 K.

The new value of  $\alpha_1$  enables one to find some other parameters of the solutions. It is known from the experimental data on the first-sound

velocity that<sup>1</sup>

$$\frac{m_4}{M} \left( \alpha_1 + \frac{M - m_3}{m_4} \right)^2 + \alpha_2 - \frac{M - m_3}{m_4} \approx -0.3$$

This leads to

$$\alpha_2 \approx \frac{\partial^2(-\Delta)}{\partial N_4^2} \frac{N_4^2}{m_4 s_0^2} \sim 0.03$$

### NOTE ADDED IN PROOF

Recently Lhuillier and Laloë (C. Lhuillier and F. Laloë, *Transport Properties in a Spin Polarized Gas. Parts I-II, J. Phys. (Paris)*, 1982) have performed the detailed study of the kinetic equation and the collision integral for nondegenerate dilute spin-polarized gases. Their results in the case of spin- $\frac{1}{2}$  Fermi gas coincide with those obtained in Section 2.2.

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