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^{3}He–^{4}He quantum solutions

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ABSTRACT

The properties of ^{3}He–^{4}He solutions are discussed. The theoretical analysis is carried out on a basis of general quantum mechanical considerations without resorting to any model concepts. Satisfactory agreement of the proposed simple theory with all the experimental data allows us to make reliable predictions of new effects. Particularly interesting are the magnetic properties of the solutions and the superfluidity of ^{3}He in solution. Observation of the corresponding effects seems to be possible.

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§1. Introduction

Helium isotopes ^{3}He and ^{4}He are unique physical objects where the quantum properties of the macroscopic systems are particularly clearly exhibited. Thus, in helium, one can obtain the most complete and the most visual description of a large number of interesting quantum effects. On the other hand, a good understanding of the physics of helium will clarify a number of general relationships inherent in low-temperature phenomena in the physics of condensed matter.

The theory of quantum liquids describes equally well such diverse physical systems as superconducting metals and alloys, the electron liquid in metals and semiconductors, ultradense stars and heavy nuclei, etc. At the same time, the theory of quantum liquids is essentially a theory of the liquid isotopes of helium. Superfluid ^{4}He and normal ^{3}He were the traditional objects of analysis in the physics of helium. Recently, there has been considerable interest in the superfluid phases of ^{3}He. A solution of helium isotopes can exhibit properties characteristic of all of these. Moreover, quantum ^{3}He–^{4}He solutions possess completely new specific properties uncharacteristic of pure isotopes of helium, such as giant magneto-kinetic effects—a considerable growth of the mean free path of the dissolved ^{3}He atoms and strong enhancement of the kinetic coefficient of the solution in magnetic fields.

A ^{3}He–He II solution is a Fermi liquid of impurity ^{3}He atoms dissolved in a superfluid Bose background of ^{4}He. Such a Fermi liquid is an extremely interesting object for both theoretical and experimental analysis due to the following.

A general description of any Fermi liquid is possible in the scope of Landau theory; Landau theory of Fermi liquids enables one to clarify a whole number of relationships characteristic of the system of fermions. All the observable characteristics of the system are comprehensively determined by a phenomenological Fermi liquid function. The Fermi liquid function is microscopically defined by the two-particle interaction of fermions. For macroscopically dense systems a consistent calculation of the f-function in general form turns out to be impossible. Therefore, the form of the Landau f-function can be explained only by comparing theoretically calculated characteristics of the Fermi liquid with experimental data. Thus the measurement of the specific heat and sound velocity of ^{3}He enables one to determine the zeroth and first harmonics in the expansion of the Fermi liquid function in Legendre polynomials. In most phenomena, however, the set of all the harmonics of the f-function, whose experimental values cannot be obtained, at present is important. The use of the f-function in a two harmonic approximation has rather low accuracy and is often unsatisfactory. An important exception, where all calculations can be consistently performed, is the case of a low density Fermi liquid, i.e. actually a slightly non-ideal Fermi gas. In this case the situation is eased by the fact that there exists a natural small parameter in the system—the low concentration of fermions. A

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degenerate dilute Fermi gas does not occur in nature, since the majority of real Fermi systems condense before the effects of the quantum degeneracy begin to develop. A single exception is the degenerate Fermi gas of impurity ^{3}He atoms dissolved in superfluid He II background. Therefore, the experimental and theoretical investigation of the low-temperature properties of the solutions is not only of considerable interest in itself, but enables one to understand the nature and the origin of a whole number of phenomena in dense Fermi systems too.

Fifteen years ago the finite solubility of ^{3}He in He II at low temperature was discovered (Edwards *et al.* 1965). It turns out that the phase diagram of the mixture ^{3}He–^{4}He is such that the solution does not demix into pure components up to concentrations of 6·4% (with increasing pressure the demixing concentration increases to 10%). Thus, sufficiently decreasing the temperature, we shall always hit the region of Fermi degeneracy of the impurity ^{3}He atoms. The discovery of the finite solubility stimulated markedly subsequent experimental and theoretical investigations of the degenerate ^{3}He—He II solutions. In the region of quantum degeneracy of the dissolved ^{3}He atoms, the contribution of rotons and phonons inherent in pure superfluid ^{4}He can be neglected. (Thus, even at fairly high temperatures, $T \lesssim 0{\cdot}7$ K and low concentrations, $x \gtrsim 0{\cdot}5\%$, the contribution of phonons in the normal density of the solution is less than 1% of the impurity contribution.) Then, on decreasing the temperature, all thermodynamic and kinetic characteristics of the solution are determined solely by the impurity component and must exhibit Fermi liquid properties. An experimental verification of this fact was the discovery of the linear temperature dependence of the specific heat of the solution (Anderson *et al.* 1966 b), the temperature dependences of the coefficients of spin diffusion and thermal conductivity (Anderson *et al.* 1966 a, Abel *et al.* 1967) and also a certain dependence of the effective mass of impurity ^{3}He atoms on the solute concentration.

The properties of ^{3}He dissolved in superfluid ^{4}He were first studied theoretically by Landau and Pomeranchuk (1948), who treated the impurity ^{3}He atom as a delocalized Fermi quasiparticle with a large de Broglie wave length. The thermodynamic functions of the solution considered as an ideal Fermi gas of such quasiparticles in a superfluid background were calculated by Pomeranchuk (1949). Comparison of the results of this calculation with the experimental data showed the insufficient accuracy of the ideal gas approximation and the need to take into account the interaction between ^{3}He quasiparticles, even in the region of rather low concentrations.

An essential role in the interaction of ^{3}He quasiparticles in a solution is played by the superfluid ^{4}He background. The effect of the superfluid background on the single ^{3}He quasiparticles, according to Landau and Pomeranchuk's theory, is simply reduced to a renormalization of the mass of ^{3}He particle, i.e. to a considerable difference between the effective mass of a single bare ^{3}He quasiparticle and the mass of ^{3}He atom. Unfortunately, the interaction of ^{3}He quasiparticles in a solution does not generally reduce to a simple renormalization of the interaction potential of ^{3}He atoms in vacuum, for example, to a Lennard-Jones potential. This is caused by the fact that the interaction of impurity quasiparticles is followed by a polarization of the medium, and is connected with the properties of ^{4}He excitations. Since the sound velocity in ^{4}He is not too large, in the interaction of ^{3}He quasiparticles in a solution retardation effects should occur, and this leads to an essential non-locality of the interaction. Therefore, a realistic description of the structure of interaction does not exist at present. In connection with this fact, the majority of authors, beginning with

the well-known work of Bardeen *et al.* (1966, 1967) used model potentials of different types to describe the interaction of ^{3}He quasiparticles while the parameters of these potentials were chosen from a comparison with experimental data. These potentials do not possess a direct physical meaning and their choice can hardly be justified from first principles. With a model potential, it is difficult to interpret equally well all experimental data, and for satisfactory agreement with experiment one must resort to a theory containing rather large numbers of fitting parameters. At sufficiently low temperatures and concentrations, the two-particle interaction potential can be represented as an expansion in a power series in the small momentum of fermions. Then, because of the isotropy of the system, only terms with even powers of the momentum will enter the expansion. In this case, the determination of the model potential is actually reduced to fitting the corresponding coefficients in the expansion. A detailed discussion of those articles and summary of the results can be found in the reviews of Radebaugh (1968), Ebner and Edwards (1971), Esel'son *et al.* (1973), and Baym and Pethick (1978). At the same time the application to real solutions of the consistent phenomenological theory in the spirit of Landau's Fermi liquid theory (Khalatnikov 1967, 1968, 1971) turns out to be difficult since no explicit form of the f-function is obtained from the experimental data.

Nevertheless, one can carry out a complete and fairly exact microscopic investigation of the properties of weak solutions corresponding to the study of effects of non-ideality of a Fermi gas of impurity quasiparticles in Landau and Pomeranchuk's theory. In the limits of this description, the Fermi liquid interaction can be taken into account with satisfactory accuracy by perturbation theory, since in the given case, the expansion in interaction coincides formally with the expansion in concentration which is the only small parameter in the system.

At low temperatures, $T \ll \hbar^2/Mr_0^2$ (M is the fermion mass and r_0 the range of interaction) and concentrations $N_3^{-1/3} \gg r_0$ (N_3 is the number of particles per unit volume) the system of dissolved ^{3}He atoms forms a dilute Fermi gas of slow quasiparticles. In accordance with the phase diagram of the Fermi component of the degenerate ^{3}He–^{4}He solution, these conditions are always fulfilled up to the demixing concentration. The scattering amplitude of slow particles (see, for example, Landau and Lifshitz 1974) when the interaction between them decreases rapidly at large distances, is determined by an expansion in even powers of the small relative momentum p^{2l} (l is the orbital momentum of scattered particles). Therefore, for a Fermi gas of slow particles, the interaction reduces, in general, to s-scattering (Huang and Yang 1957, Lee and Yang 1957, Abrikosov and Khalatnikov 1957 b, Abrikosov *et al.* 1962, Lifshitz and Pitaevskii 1978) and can be described by just a single microscopic constant independent of momenta, which has a clear physical meaning—the s-scattering length, a. The value of the s-scattering length cannot be defined in the theory and must be found from a comparison of theoretical and experimental results. Then the concentration dependence of all thermodynamic and kinetic characteristics of the degenerate solutions are calculated as expansions in powers of $k_0|a| \sim x^{1/3} \ll 1$ ($\mathbf{k}_0$ is the Fermi wave vector). Nevertheless, one can describe the whole range of experimental data on the properties of the superfluid solutions using just a single phenomenological parameter, a. The principal difference of this approach from the method of model potentials consists in the fact that the concentration dependence of all observed quantities can be explained in the limits of s-scattering only; whereas, in the model description, to obtain many relationships one must consider the momenta-dependent part of the interaction potential, i.e.

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scattering with the higher momenta. In the limit of zero momentum the results obtained using model potentials must coincide, of course, with the results derived in the framework of the above-mentioned scheme in the Born approximation, i.e. actually in the model of hard spheres (Fu and Pethick 1976). Indeed, in the highest concentration region, near the demixing curve, the use of model potentials provides, for the present, the only possibility of describing the available experimental data.

The value of the s-scattering length, a, was determined from the best fit of the theoretical curves simultaneously with all available experimental data on the properties of a degenerate solution. Information about the s-scattering length enables us not only to describe fairly exactly the available experimental results, but also to predict reliably a whole number of phenomena hitherto unobserved.

It turns out that the quantity a is negative, which corresponds to attraction between the impurity ^{3}He atoms. This leads automatically to the conclusion that weakly damped high-frequency spin waves can propagate in a degenerate solution, but oscillations of zero-sound type cannot propagate. This also means that the superfluid phase transition of ^{3}He in solution is provided by the s-pairing of impurity ^{3}He quasiparticles and can be explained in the framework of BCS theory which, in the given case, does not give a model (as in superconducting metals) but a fairly exact description of the superfluid transition. Then one can obtain a realistic value of the superfluid transition temperature of ^{3}He in a solution which turns out to be sufficiently high, and this makes the observation of this phase transition accessible even at present. Although this superfluid transition results from the usual BCS pairing of the impurity fermions, the macroscopic properties of the solution in this temperature region differ markedly from the properties of other superfluid and superconducting systems; this is associated with the simultaneous presence of two Bose condensates in ^{3}He and ^{4}He. Therefore the macroscopic behaviour of such systems is described already by the equations of three velocity hydrodynamics (there are two superfluid and one normal flow rate in the solution). At the same time the effect of dragging on both components of the solution by each of the superfluid flows becomes important. Interest in the search for superfluidity of ^{3}He in ^{3}He–He II solutions has recently been particularly heightened by new achievements in the investigations of the superfluid phases of pure ^{3}He.

A pronounced interest in the analysis of the Fermi liquid with a polarized spin system has also emerged. The system of nuclear spins of ^{3}He in a weak ^{3}He–^{4}He solution can be polarized simply by applying an external magnetic field. The switching of the external magnetic field does not affect the motion of the single uncharged fermion. In the presence of a magnetic field, however, the occupation number for particles with different spin orientations changes and therefore the energy of Fermi liquid excitations which itself is a functional of the distribution function. The behaviour of a Fermi liquid in a magnetic field has been studied so far only in weak fields where the corrections to the f-function are as small as the field and all characteristics of the system can be expressed by the Fermi liquid harmonics in the absence of the field. In strong magnetic fields the Fermi-liquid function cannot, in general, be expressed by its value without a field.

For a low-density Fermi liquid like the ^{3}He–^{4}He solution all calculations can be performed in a closed form for arbitrary magnetic fields. The Fermi liquid function is calculated in practice with the same procedure as for no field. This is due to the fact that in the non-relativistic approximation the particle interaction is spin-independent and the scattering amplitude of bare quasiparticles is field-

independent. The dependence of the f-function on the field appears only because the field influences the distribution function of fermions. Therefore, for ^{3}He–He II solutions in which the magnetic dipole–dipole interaction is negligibly small, all Fermi liquid characteristics in not very strong fields are determined by the same quantity as in the absence of field—the s-scattering length, a.

In an isotropic Fermi liquid of spin-$\frac{1}{2}$ particles, there are two Fermi surfaces for the different spins, spheres whose radii are determined by the degree of polarization by the field. In the exchange approximation, particles of both polarizations remain at their Fermi surfaces. For s-scattering of fermions with spin-$\frac{1}{2}$ only collisions of particles with opposite directed spins are important because of the Pauli principle. Therefore, in very strong fields when practically all spins are parallel to the magnetic field, the s-scattering becomes ineffective. In this case, the Fermi liquid interaction is associated already with p-scattering. The p-scattering amplitude of slow particles is considerably smaller than the s-scattering amplitude and, therefore, the polarization of the solution by a magnetic field leads to a considerable weakening of the Fermi liquid interaction, with particularly noticeable effects on the transport phenomena, since the free path of the particles is inversely proportional to the scattering cross-section. As a result, magneto-kinetic effects must be observed in ^{3}He–^{4}He solutions (gigantic growth of the kinetic coefficients such as viscosity and thermal conductivity) on polarizing the spin system.

The influence of spin polarization on the thermodynamic properties of the solution is also noticeable. This influence is not associated, in general, with the properties of the Fermi liquid interaction, but simply with the change of the radii of Fermi surfaces. These effects must be observed even in an ideal Fermi gas.

An external magnetic field sharply changes the character of the superfluid phase transition of ^{3}He in solution. Thus, in some regions of the fields, the superfluid transition in ^{3}He is accompanied by the formation of Cooper pairs with non-zero momentum, and this leads to a spatial inhomogeneity of the superfluid phase. The appearance of such a quasicrystalline structure is strongly echoed in the liquid properties. s-Pairing in stronger magnetic fields becomes impossible. As a consequence, the temperature of the phase transition, associated already with the p-scattering, decreases considerably. The resultant superfluid phase is reminiscent, microscopically, of the A-phase of pure superfluid ^{3}He.

Many of the phenomena studied in superfluid ^{3}He–^{4}He solutions have their analogues in dense polarized Fermi systems. Since the degeneracy temperature of the dense Fermi liquids is fairly high, the spin system cannot be polarized to any considerable degree by an external magnetic field at present. Thus, it is necessary to employ other methods in order to achieve a high degree of polarization. There is interest in a method of polarization of pure ^{3}He using fast melting of a crystalline magnetically ordered ^{3}He (Castaing and Nozieres 1979, Chapellier *et al.* 1979). Some curious examples of other polarized Fermi systems are discussed in the concluding part of this work.

In the case of low-density Fermi systems the approach proposed can also be used in non-degenerate systems. For temperatures above the degeneracy temperature the Fermi liquid excitations undergo, generally speaking, a strong damping. In a low-density Fermi liquid, the damping of the quasiparticles is small as the density of fermions is small, and in Born approximation the excitations do not attenuate at all (Galitskii 1958a). Therefore, in first-order perturbation theory, this method of calculation can be extended to the case of arbitrary temperatures. It is interesting to

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analyse the transition to a fully polarized degenerate solution taking place via a state of the system in which the subsystem of particles with spins parallel to the field is degenerate, and particles with opposed spins are few and of the Boltzmann type. Note that in the phase diagram non-degenerate ^{3}He–^{4}He solutions correspond either to rather high-temperature regions or (at low temperatures) to regions of rather small concentration. At high temperatures, the properties of the solution are determined to a large extent by the phonons and rotons and are already studied in detail (Wilks 1967, Wheatley 1968, Peshkov 1968, Keller 1969, Taconis and De Bruyn Ouboter 1964, Khalatnikov 1971, Elsel'son *et al*. 1973). At low concentrations, neglect of the leading terms in perturbation theory for a high order non-degenerate solution is unimportant and calculations using the Born approximation have sufficient accuracy.

§2. Basic properties of superfluid ^{3}He–He II solutions

2.1. *Elementary excitations of* ^{3}*He*

Consider first a solution in the absence of superfluid motion $\mathbf{v}_s=0$. According to Landau and Pomeranchuk's (1948) theory, an isolated ^{3}He impurity atom in superfluid ^{4}He behaves as a delocalized long-wavelength quasiparticle. The impurity states can be classified by a continuous energy spectrum $\mathscr{E}(\mathbf{p})$. Data for the density of the normal component of weak solutions (Lynton and Fairbank 1950, Pellam 1955, see also Esel'son *et al*. 1973) allow one to draw the conclusion that the minimum of the energy spectrum $\mathscr{E}_0(\mathbf{p})$ occurs at zero momentum. In the isotropic liquid the spectrum $\mathscr{E}_0(\mathbf{p})$ near the minimum can be represented as a series of the even powers of the momentum $\mathbf{p}$. The expansion parameter is the ratio of the quasiparticle velocity to sound velocity in helium. At low temperatures and densities, when the characteristic velocities of the bare quasiparticles are small, it is possible to restrict the expansion of $\mathscr{E}_0$ to the first few terms in the powers of p^2.

$$\mathscr{E}_0(\mathbf{p})=-\Delta+\frac{p^2}{2M}\left[1-\gamma\left(\frac{p}{p_c}\right)^2\right]. \tag{2.1.1}$$

Here the values of the binding energy $(-\Delta)$ and the effective mass M of a single impurity atom are equal to $\Delta\approx 2{\cdot}8$ K (Roberts *et al*. 1964, Massey *et al*. 1970), $M\approx 2{\cdot}3\,m_3$; $p_c=m_4 s_0(p_c/\hbar\approx 1{\cdot}5$ Å$^{-1})$, $m_3(m_4)$ is the mass of the ^{3}He(^{4}He atom, s_0 the sound velocity in pure ^{4}He at zero pressure, and the dimensionless parameter γ is very small. According to the experimental results of Brubaker *et al*. (1970) $\gamma\approx 0{\cdot}14\pm 0{\cdot}05$, but according to Esel'son *et al*. (1975) $\gamma\approx 0\pm 0{\cdot}01$. In the dispersion relation (2.1.1), it is necessary, in contrast to Landau and Pomeranchuk, to keep the term with p^4 since its contribution, as will be clear later, has the same concentration smallness as the Fermi liquid interaction in which we are interested. Parameters Δ, M and γ in the spectrum of single-bare quasiparticle (2.1.1) are functions of the density of ^{4}He atoms, N_4.

Notice that at large values of the momentum p the energy spectrum of bare ^{3}He excitations differs markedly from the simple expansion (2.1.1). Despite the existence of a large number of direct as well as indirect measurements, the final form of the spectrum at large p has not been ascertained yet. (The latest experimental data and reviews can be found, for example, in Greywall (1978).) This uncertainty is negligible in degenerate mixtures up to the largest densities.

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