SPIN-POLARIZED QUANTUM ^{3}He-^{4}He SOLUTIONS

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Résumé.- Nous considérons les phénomènes inhérents aux solutions quantiques de ^{3}He-^{4}He et autres systèmes de Fermi à forte polarisation de spin. Nous étudions toutes les propriétés thermo, hydrodynamiques, cinétiques et de transport de ces solutions, ainsi que leurs transitions de phase. Nous suggérons une théorie simple qui n'implique pas d'hypothèses concernant le modèle utilisé. Nous prédisons plusieurs phénomènes (effets magnétocinétiques, superfluidité de ^{3}He dans la solution, etc.) qu'il semble possible d'observer expérimentalement à l'heure actuelle.

Abstract.- The phenomena inherent in the quantum solutions of ^{3}He-^{4}He and other spin-polarized Fermi systems are considered. All the thermo and hydrodynamic, kinetic and transport processes and the phase transitions in solutions are investigated. The suggested simple theory implies no model assumptions. Many phenomena predicted (magnetokinetic effects, the superfluidity of ^{3}He in the solution, etc.) seem to be within the reach for the experimental observation just at this moment.

1. Introduction

The properties of spin-polarized quantum Fermi systems can be described very well by the equations of the usual Landau theory of Fermi liquids. As a result main characteristics of a system are expressed in terms of a phenomenological Fermi-liquid f-function. Unfortunately, the consistent microscopic calculation of the f-function for spin-polarized systems as for other Fermi liquids in most of the cases is impossible. Even the relations between the values of the Fermi-liquid function for the same system at different degrees of the spin polarization are unknown. Such a relation is established only in the case of low-polarized Fermi systems, when the f-function practically is not affected by the polarization.

From this point of view the very interesting systems are low-density Fermi liquids, for which all the properties can be evaluated at all degrees of the polarization of a spin system. Many of results derived for such objects are also common for dense Fermi liquids. However there are practically no examples of degenerate low-density Fermi liquids because for real gases the condensation begins before the effects of quantum degeneracy become important. Maybe the only exception is a degenerate Fermi gas of ^{3}He atoms dissolved in superfluid He II. For this reason the investigation of spin-polarized quantum solutions of ^{3}He-^{4}He is of the physical interest not only in itself but provides very important information about the nature and the origin of many phenomena common to dif-

ferent types of Fermi systems.

Below we shall try to give in the simpliest form the qualitative description of the main processes in low-density spin-polarized Fermi systems similar to $^3He\uparrow$ - ^4He. More precise results and the quantitative analysis can be found in the recent reviews /1/.

2. Main properties of nonpolarized ^3He - ^4He solutions

The ^3He - He II mixture is the Fermi liquid of ^3He particles in the superfluid Bose background of ^4He. The number of Bose-type excitations analogous to phonons and rotons in pure ^4He vanishes rapidly as the temperature decreases, and at low temperatures most of thermodynamic and kinetic characteristics of the solution are determined by the impurity (Fermi) component exclusively.

In accordance with the Landau - Pomeranchuk theory /2/ ^3He impurity atom being introduced into superfluid ^4He becomes a delocalized quasi-particle with a big de Broglie wavelength. The energy spectrum of such quasi-particles with not very large momenta $\vec{p}$ is quadratic in $\vec{p}$:

$$\mathcal{E}(\vec{p}) = \mathcal{E}_0 + p^2/2M \tag{1}$$

where $\mathcal{E}_0$ is the binding energy, and M stands for the quasi-particle effective mass.

Fifteen years ago the phenomenon of the finite solubility of ^3He in He II at zero temperature was discovered /3/ (this fact corresponds to the negative sign of $\mathcal{E}_0$ in Eq. (1) at low pressures).

It was found out that ^3He and ^4He at T=0 are miscible up to the ^3He concentration of 6.5%. That is why the system of ^3He quasi-particles ($\mathcal{E}_0$ = -2.8 K, M = $2.3m_3$, m_3 is the ^3He atomic mass) being dilute always becomes degenerate as the temperature decreases. The degeneracy temperature T_0 and the Fermi momentum p_0 are small corresponding to the low concentration of ^3He in the solution x:

$$p_0 = \hbar(3\pi^2 N_3)^{1/3} \sim x^{1/3}\hbar/z_0, \quad T_0 \sim x^{2/3}\hbar^2/z_0^2 M \tag{2}$$

Here N_3 is a number of ^3He atoms in a unit volume, z_0 is an atomic radius. For these reasons the system of dissolved ^3He atoms behaves when $T \ll T_0$ as a degenerate Fermi gas of slow quasi-particles $p_0 z_0/\hbar \sim \sim x^{1/3} \ll 1$

The study of the interaction of quasi-particles in such a system presents no special difficulties. It is well known /4/ that the scattering amplitude of slow particles can be easily evaluated by expanding in a series in even powers of a small momentum $p^{2\ell}$ (ℓ = 0; 1; 2... - is an orbital moment of colliding particles). The first term of this expansion corresponds to the s-wave scattering (ℓ = 0). The s-wave cross-section σ_s

$$\sigma_s = 4\pi a^2 \tag{3}$$

in the case of slow particles does not depend on momenta (a is the s-wave scattering length).

The Fermi-liquid function can be expressed in terms of the two-particle scattering amplitude by means of the perturbation theory which formally coincides

in our case with the expansion in powers of low density $x^{1/3}$. For instance in first-order perturbation theory (to the principal approximation in the concentration) the f-function is defined by the forward-scattering amplitude *and* does not depend on the concentration of the solution

$$f_{\vec{S}\vec{S}'}(\vec{p},\vec{p}') = \frac{8\pi a h^2}{M}\left(\frac{1}{4} - \vec{S}\vec{S}'\right) \quad (4)$$

where $\vec{S}$, $\vec{S}'$ are spins of Fermi-liquid excitations. As a result one obtains the expression for the f-function in the form of the series in powers of ^{3}He concentration. This expression contains only one unknown parameter – the s-wave scattering length a . It provides the possibility of the precise calculation of all characteristics of the solution using only a single interaction parameter – a .

The value of a was determined /6/ from the comparison between the results of such calculations and all available experimental data on the properties of ^{3}He – ^{4}He solutions. It was found out that

$$a = -1.5 \overset{\circ}{A} \quad (5)$$

The negative sign of the scattering length (5) corresponds to an attractive interaction between ^{3}He quasi-particles in the solution. This attraction permits the propagation of spin waves in solutions and enables the superfluidity of ^{3}He in a solution to be caused by the usual BCS s-wave pairing of ^{3}He quasi-particles. These properties of ^{3}He – ^{4}He solutions are just opposite to that of pure ^{3}He.

This approach does not need any model assumptions about the interaction

which are necessary for other descriptions such as well known /7/ .

3. Fermi-liquid interaction in ^{3}He↑ – ^{4}He solutions

The nuclear spin system of ^{3}He in dilute mixture can be polarized simply by means of the external magnetic field. The degree of the polarization is defined by the relation between the magnitude of the field $2\beta H$ (β = 0,08 mK/kOe is the ^{3}He nuclear magnetic moment), the Fermi-degeneracy temperature T_0 (T_0 = 2,6 x $^{2/3}$ K) and the temperature T. It is obvious that magnetic fields H < 200 K Oe which are available at this moment can not appreciably polarize the high concentrated solutions (x > o,1%), and in order to obtain the high degree of the polarization, may be it will be necessary to use the methods which are suitable for polarization of the pure ^{3}He (/8/ – /10/). However the way of obtaining the polarization of the solution is not essential for further results. Therefore for the sake of simplicity we suppose further the solution to be polarized by the external magnetic field $\vec{H}$. Otherwise the magnetization of the solution (or, that is just the same, the polarization degree) and not the field intensity $\vec{H}$ would be the basic thermodynamic variable.

A uniform external magnetic field does not affect the motion of an isolated uncharged Fermi-particle of ^{3}He, nor does it affect to the nonrelativistic approximation the two-particle interac-

tion. However when a magnetic field is applied the occupation numbers for the particles with the different spin orientation change and consequently, the Fermi-liquid excitation energy which is the functional of the distribution function also change .

In principle in the case of not too high magnetic fields all the conclusions of Sec. 2 concerning the Fermi-liquid interaction in unpolarized solutions are valid. Apparently, since f-function (4) does not depend on momenta of particles, the expression for f-function in a magnetic field to the main approximation in the concentration does not depend on the field and coincides with its value (4). The field dependence of the f-function appears only in higher orders of the perturbation theory. /11/

This conclusion is not true for the solution with the high degree of the polarization. Owing to Pauli's exclusion principle only particles with oppositely directed spins interact in the s-wave scattering for which the f-function in the absence of the magnetic field has been considered above.

That is why in the highly polarized solution when practically all the ^{3}He quasiparticle spins have the same orientation, the s-wave scattering becomes uneffective. In this case the Fermi-liquid interaction is determined by the p-wave scattering.

In the spin-polarized solution the interaction only between particles on the Fermi surface with the ridius $P_F = 2^{1/3} p_o$

takes place. The p-wave scattering amplitude in the center-of-mass frame is proportional to the square of the relative momentum of colliding particles $\vec{p}$

$$f_p = 3 b \frac{p^2}{\hbar^2} \cos \varphi, \quad \sigma_p = 12 \pi b^2 \left(\frac{p}{\hbar}\right)^4 \sim x^{4/3} \frac{b^2}{a^4} \quad (6)$$

where φ is the angle of scattering in the center-of-mass frame and the constant b is of the order of the gas-kinetic volume of the atom $b \sim z_o^3$. Then the Fermi-liquid function $f(\theta)$ (θ is the angle between the momenta of colliding particles $\vec{p}$ and $\vec{p}_2$: $p_1 = p_2 = p_F$, $p = 2p_F \sin (\theta/2)$) is determined in the first order of the perturbation theory by the antisymmetrized forward-scattering amplitude (6) (φ = 0) /11, 12/

$$f(\theta) = 24 \pi b (p_F^2/M) \sin^2(\theta/2) \quad (7)$$

With the aid of expressions for the f-function one can calculate very simply all the Fermi-liquid characteristics of the ^{3}He↑ - ^{4}He solution. Unfortunately the numerical value of the basic characteristic of the p-wave scattering b is unknown at this moment.

4. Thermo- and hydrodynamic properties of ^{3}He↑ - ^{4}He solutions

In an isotropic Fermi liquid of spin-1/2 particles, the Fermi surfaces of quasiparticles with differently oriented spins constitute two Fermi spheres, whose radii p_+ and p_- ($p_+^3 + p_-^3 = 2p_o^3$) are determined by the degree of polarization of the spin system. That is why the change of thermodynamic characteristics of the solution depending on the magnetic

polarization is basically connected with the change of Fermi momenta $p\pm$ (H) under the action of the field. Corresponding effects could be observed also in an ideal gas of impurity excitations. The effects connected with the change of the interaction under the polarization in the thermodynamics are considerably less important than in the case of transport phenomena.

The osmotic pressure and the second sound velocity (the acoustic oscillations in the system of impurity excitations) are those thermodynamic quantities which are the most sensitive ones to the change of the degree of polarization. The change of such quantities, which have no direct contribution from the pure ^{4}He and are proportional to some degree of the Fermi momentum, turns out to be highly considerable.

The osmotic pressure in the dilute mixture is equal to

$$\Pi = \int_0^{N_3} N_3 \left(\partial \mu_3 / \partial N_3 \right) dN_3 \qquad (8)$$

where μ_3 is the chemical potential of ^{3}He in the solution. Since to the principal approximation in concentration the ^{3}He chemical potential in the absence of the magnetic field is equal to

$$\mu_3 \approx \mathcal{E}_o + p_o^2/2M, \quad N_3 = p_o^3/3\pi^2\hbar^3$$

and in the completely polarized solution

$$\mu_3 \approx \mathcal{E}_o + p_F^2/2M, \quad N_3 = p_F^3/6\pi^2\hbar^3 \quad ,$$

it is clear that the osmotic pressure increases from the value $\Pi \approx (2/5)(p_c^2/2M) N_3$ to the value $\Pi \approx (2/5)(p_F^2/2M) N_3$ i.e. approximately by a factor $2^{2/3}$ (there are more exact expressions in /1/).

The change of the second sound velocity is analogous to the magnetoosmotic effect mentioned above. To the main approximation in the concentration the speed of the propagation of the density vibrations in the impurity excitation gas in the solution is equal to $S_2^2 \approx (N_3/M)(\partial \mu_3/\partial N_3)$ As a result S_2 increases approximately by a factor $2^{1/3}$ from the value $S_2 \approx p_o/\sqrt{3}M$ to the value $S_2 \approx p_F/\sqrt{3}M$ (more exactly see /1, 11, 12/). The relative change of the square of the second sound velocity as a function of the magnetic field is shown in Fig. 1. $(\mathcal{H} = 2\beta H/T_F, \ T_F = p_F^2/2M)$
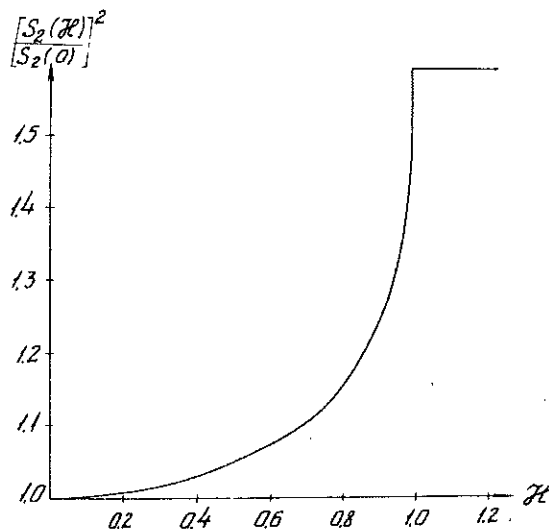

Fig. 1

The polarization of the solution also leads to the essential change of the equilibrium phase diagram of the system. Namely the solubility curve for ^{3}He in ^{4}He shifts considerably /8/. With the aid of the approximate values of μ_3 mentioned above it is easy to show that the solubility in the degenerate fully-

polarized mixture decreases nearly by a factor 2. However it is rather difficult to observe this effect since for this purpose the high-concentrated solutions should be polarized.

Most of other thermo- and hydrodynamic phenomena in the ^{3}He↑ - ^{4}He dilute mixture are less significant than discussed above.

5. Magnetokinetic effects

The influence of a spin polarization on kinetic properties of solutions is much more pronounced than on thermodynamic ones. All the transport phenomena are changed by the polarization of fermion spins in a rather dramatic manner. It is due to the rapid increase of the ^{3}He quasi-particle mean free path and,consequently, the values of kinetic coefficients such as the thermal conductivity and viscosity, as a result of the polarization.

The mean free path of ^{3}He quasi-particles in the degenerate solution is inversly proportional to $N_i \, \widetilde{\sigma_i} \, (T/T_o)^2$ (here N_i is the number of scattering centres in a unit volume, $\widetilde{\sigma_i}$ stands for the scattering cross-section, and the large factor $(T_o/T)^2$ is due to the non-effectiveness of collisions in the degenerate Fermi system). The s-wave cross-section $\sigma \sim z_o^2$ (3), and in the case of s-scattering the mean free paths $\ell_\pm$ of quasi-particles with opposite spins are proportional to $\left(N_\mp \, z_o^2\right)^{-1} \left(T_o/T\right)^2$ ($N_\pm$ are the numbers of particles with different spin directions per unit vol-

ume, $N_+ \quad N_- = N_3$; as a result of Pauli's exclusion principle only collisions of fermions with opposite spins are essential for the s-scattering). In the absence of the polarization $N_+ = N_-$ and

$$\ell_+ = \ell_- \sim \left(N_3 \, z_o^2\right)^{-1} \left(T_o \, / \, T\right)^2 \qquad (9)$$

In high fields kinetic coefficients increase practically without limits as the number of particles N_- with spins opposite to the field vanishes and ℓ_+ increases. The limiting value of ℓ_+ in the polarized solution ($N_- = 0$) is dictated by p-scattering with the cross-section (6):

$$\ell_+ \sim x^{-4/3} \left(N_3 \, z_o^2\right)^{-1} \left(T_o \, / \, T\right)^2 \left(z_o^6 \, / \, b^2\right) \qquad (10)$$

The growth of the viscosity η and the thermal conductivity $\varkappa$ coefficients as a function of an external magnetic field $\mathcal{H} = 2\beta H / T_F$ is presented graphically in Fig. 2 (curve 1 – $\eta(\mathcal{H})/\eta(o)$ 2 – $\varkappa(\mathcal{H})/\varkappa(o)$).
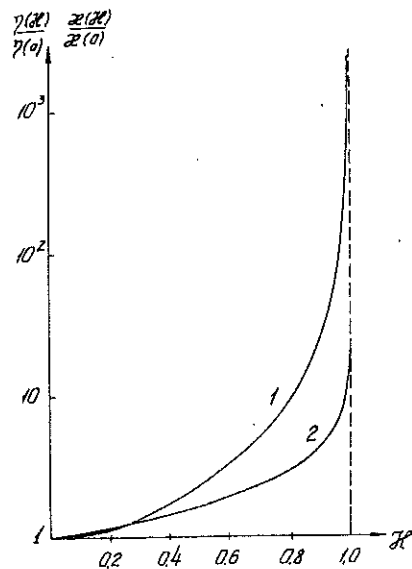
Fig. 2

The corresponding calculations took into account only the s-scattering and supposed the degeneracy of all the Fermi component of the solution including the

impurities with spins opposite to the field. For this reason N_- tended to zero when $\mathcal{H} \to 1$, and kinetic coefficients η, and $\mathcal{H}$ became infinite when $\mathcal{H} \to 1$ as Fig. 2 shows. Actually /13/ at nonzero temperature $N_- \neq 0$, and in high fields $\mathcal{H} \gtrsim 1$ kinetic coefficients grow slower than it is shown in Fig. 2:

$$\ell_+ \sim \eta \sim \exp\left[(\mathcal{H}-1)\,T_F\,/\,T\right] \qquad (11)$$

More detailed description of transport phenomena in almost completely polarized solutions can be found in Refs./1, 13/.

As a result the values of kinetic coefficients restricted in high fields only by p-scattering (10) /12/ are many times larger ($x^{-4/3} \gg 1$) than their values in the abscence of the field.

The main experimental difficulty for observing the magnetokinetic effects is to achieve considerable degree of the polarization of the ^{3}He nuclear spin system. A field $H \sim 75$ kOe can polarize at a sufficiently low temperature ($2\beta H \sim 12$ mK) a solution with a ^{3}He concentration $x \lesssim 10^{-4}$ ($T_F \lesssim 8$mK). In these conditions the ^{3}He atom mean free path grows more than 10^5 times and runs up to tens of centimetres. It results in the giant growth of other kinetic coefficients (e.g. the hydrodynamic viscosity increases to a value exceeding several centipoises which is larger than the viscosity of the water).

In such circumstances one can encounter some nonlocal effects inherent in the Knudsen regime, such as the temperature jump or the radiometric effect. Capillars about several centimetres in di-

ametres become superleaks permeable only to the superfluid component of the liquid. Such a growth of the mean free path presents also some special difficulties for measuring of the kinetic characteristics.

As the rest of kinetic coefficients do, the relaxation time τ also increases. This phenomenon reduces the region of existance of hydrodynamic oscillations $\omega\tau \ll 1$.

The observation of magnetokinetic effects in more concentrated solutions implies the polarization of a spin system rather by some alternative methods than by the direct magnetization in an external magnetic field.

6. Spin waves in the ^{3}He↑ – ^{4}He solution

In the quantum ^{3}He – ^{4}He solution the different high-frequency modes $\omega\tau \gg 1$ inherent both in the Bose system (the high-frequency first sound) and in the case of the Fermi-liquid (the symmetric spin waves) can propagate. Since the ^{3}He impurity atoms in the solution experience the effective attraction ($a < 0$) and the Fermi-liquid interaction is small, the existence of weakly damped asymmetric spin waves with azimuth numbers $m \neq 0$ and the modes of zero-sound types turns out to be impossible /6/.

The influence of the external magnetic field (the polarization of the ^{3}He nuclear spin system) on the propagation of high-frequency oscillations in the solution is very appreciable. Since, due to the magnetokinetic effect, the characteristic relaxation time increases conside-

rably, the region $\omega\tau\gg1$, in which the collision damping of high-frequency modes is small, also noticeably increases. It means that in a high polarized solution the condition $\omega\tau\gg1$ can be valid even for infra-acoustic frequencies.

High-frequency oscillations of the solution are described by the collisionless kinetic equation, the continuity equation and the superfluid-motion equation. The spin polarization of ^{3}He in the solution changes qualitatively all the pattern of the spin-wave propagation.

Instead of one three-times degenerated branch of spin oscillations, two brances of transverse spin waves with the gap in the energy spectrum (in the presence of the external magnetic field) and one longitudinal branch of spin waves coupled with high frequency vibrations of the ^{4}He superfluid background arise /11/. The dispersion law of transverse spin waves(oscillations of the magnetization in the plane which is perpendicular to the magnetic-polaryzation vector direction)·is quadratic in small wave vectors $\vec{k}$

$$\omega = \omega_0 + \frac{\pi}{5}(36\pi)^{1/3}\frac{\hbar^2 k^2}{M^2\omega_{int}}\frac{N_+^{5/3}-N_-^{5/3}}{N_+-N_-},$$

$$\omega_0 = \frac{2\beta H}{\hbar},\quad \omega_{int} = 4\pi|a|\hbar(N_+-N_-)/M$$

as it is inherent in the systems with the ferromagnet symmetry. The dispersion laws of spin waves in the absence of a magnetic field ($\vec{H} = 0$) and transverse spin waves in an external magnetic field $\vec{H} \neq 0$ are shown in Fig. 3.
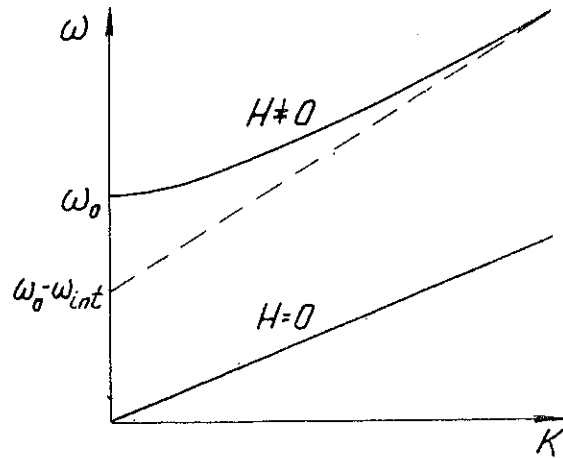
Fig. 3

When the densities of states for differently spin-oriented quasiparticles are not equal to each other, the oscillations of the longitudinal component of the magnetization turn out to be coupled with the oscillations of the impurity excitations density (and in consequence with the vibrations of the ^{4}He superfluid background; it is due to the interaction between the ^{3}He quasiparticles and the HeII background).

Then the dependence of the propagation speed of the high-frequency first sound on the degree of spin polarization arises only in rather high orders in the concentrations of ^{3}He ($\omega\ x^{5/3}$). In the case of the spin wave there is the very interesting effect namely the supression of the longitudinal spin mode by a spin polarization.

The cause of this phenomenon is that in the nonpolarized solution the propagation velocity of the corresponding spin mode is hardly larger than the Fermi velocity and is exponentially close to it. /6/. When one begins to po-

longitudinal spin wave U_H begins to
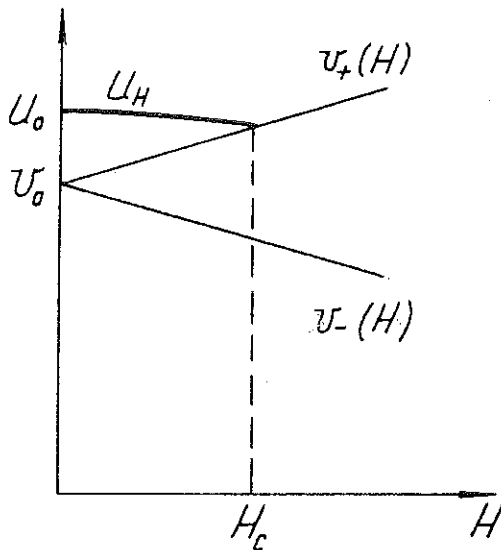decrease (the thick curve in Fig. 4).

Fig. 4

At the same time the Fermi-surfaces for
^{3}He quasiparticles with the different spin
orientations move apart leading to the in-
crease of U_+ and the decrease of U_- (the
thin lines in Fig. 4). When a certain
critical value of the magnetic field H_c
is reached (if the polaryzation has been
obtained by means of the external magne-
tic field) the value of the spin-wave
speed U_H becomes equal to the Fermi ve-
locity $U_H(H_c) \approx U_+(H_c)$. Then the
spin wave experiences a strong Landau
damping due to decay of the magnon into
a particle and a hole, and the appearing
imaginary part of the propagation veloci-
ty of oscillations turns out to be of the
same order as the real part. The calcula-
tion of the critical field H_c (the criti-
cal degree of polarization) leads to the
following numerical expression /11/.

$$H_c[Oe] \approx 10^7 x^{2/3} exp[-1,16/x^{1/3}]$$

of the transverse zero-sound, the veloci-
ty of which is also very close to the
Fermi velocity, can be observed in a spin-
polarized liquid normal ^{3}He too. With
the aid of experimental data on ^{3}He one
can estimate the value of the critical
field which turns out to be of the order
of $H_c \approx 4$ kOe . /14/ In contrast to the
symmetric spin wave in the dilute mixtu-
re in this case even in the field $H > H_c$
the transverse zero-sound experiences
a weak damping to the extent that the
meaning of $H - H_c$ is small. It is connect-
ed with the necessity of holding of the
orbital moment conservation law during
the absorbtion of transverse zero-sound
excitations by quasiparticles of a Fermi-
liquid.

In the spin-polarized solution the
propagation of spin waves turns out to
be possible also in the Boltzmann region
/14/. If the spin polarization of ^{3}He in
the mixture is created by means of the
external magnetic field, the magnon
spectrum in the vicinity of the NMR-
frequency ω_0 is just as follows

$$\omega = \omega_0 + \frac{(k v_T)^2}{\omega_0} \frac{T}{S_0 N_3} , \quad S_0 = \frac{2\pi |a| \hbar^2}{M},$$

$$v_T^2 = T/M$$

In the short-wave region the pro-
pagation of spin waves in the nondegene-
rate solution is impossible because of
the strong collisionless damping. Thus
the polarization of the spin system
makes the propagation of weakly-damped
transverse spin mosed possible in a wide
temperature range. The increase of the

wave-vectors range in which the weakly damped magnons exist. The observation of spin waves in the Boltzmall region is possible in rather high magnetic fields.

7. Superfluidity of ^{3}He↑ in ^{3}He − ^{4}He solutions

As it was mentioned in Sec. 2, the negative sign of the s-scattering length a (5) indicates that quasi-particles interaction in solutions is attractive, and that the ^{3}He superfluid transition in ^{3}He − ^{4}He solutions must be caused by the usual BCS s-pairing of fermions. Since the ^{3}He atoms in a solution form a dilute Fermi gas of quasi-particles with a quadratic energy spsctrum, the ^{3}He superfluid transition temperature can be evaluated precisely using the BCS theory /15/ . The transition temperature curves versus the ^{3}He concentration x for three different pressures are presented in Fig. 5.
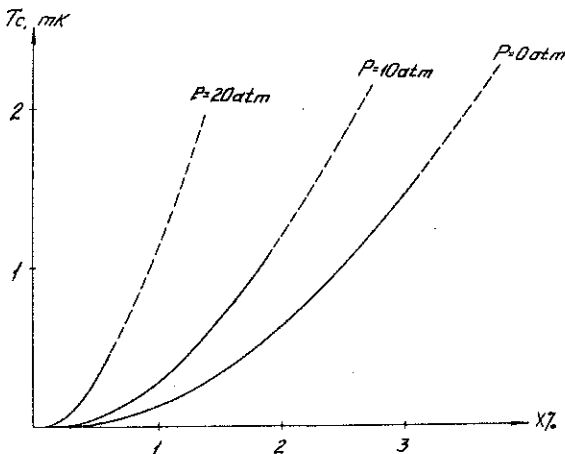

Fig. 5

The phase transition temperature proved to be high enough to hold out strong

hopes of discovering of the ^{3}He superfluidity in ^{3}He − ^{4}He solutions in the not distant future.

The ^{3}He superfluid transition in ^{3}He ↑ − ^{4}He has some peculiar properties. The degree of the polarization dictates the type of superfluid phase. Properties of these essentially different phases /16/ appear to be as interesting as the properties of superfluid pure ^{3}He.

In not very high fields the superfluidity of ^{3}He in solutions is provided as in the zero field case by s-wave pairing of ^{3}He quasi-particles. But in a partially polarized solution Fermi momenta of the coupling particles p_+ and p_- are not equal to eaoh other. This fact prevents from the formation of BCS pairs with the zero momentum. As a result the transition temperature becomes lower with the growth of the polarization, and in some range of spin system polarizations the pairing with the non-zero momentum is efficient. The ^{3}He superfluid transition temperature in ^{3}He↑ − ^{4}He solutions versus the external magnetic field strength (i.e. the degree of the polarization) is presented grafically in Fig. 6.
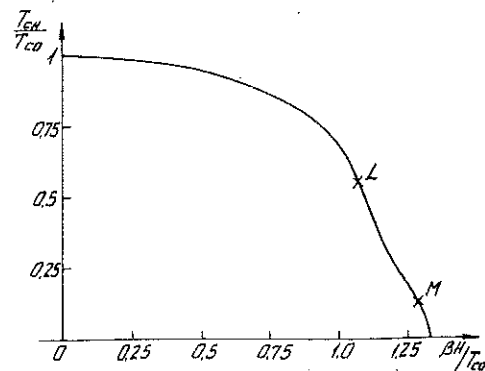

Fig. 6

S-wave pairing becomes positively impossible when $\beta H > 1.33 T_{co}$ (see Fig. 6). In such fields the ^{3}He superfluid transition can be caused only by the pairing with higher orbital moments. If the p-wave scattering corresponds to the attraction, the properties of the superfluid phase of ^{3}He in ^{3}He$\uparrow$ - ^{4}He solutions are similar to that of ^{3}He - A in a high (compared with the energy gap) magnetic fiels. But such a superfluid transition occurs at the temperature below 10^{-5} K and is of a little experimental interest now.

More concise description of superfluid phases of ^{3}He$\uparrow$ - ^{4}He solutions can be found in Ref. /16/ and in the second review /1/ .

8. Conclusions

Recently the study of spin-polarized quantum systems became practically a new field of the physics of the condensed matter. Many of results discussed above may be successfully used for the wide range of polarized systems, such as vacancy quantum solids, electron liquids, pure ^{3}He, etc. (see Ref./1/) The considerable progress in the experimental investigation of polarized quantum systems and in the low temperature technique holds out a hope of future experimental achievements in the ^{3}He$\uparrow$- ^{4}He solutions study and of the discovery such intriguing phenomena as the ^{3}He superfluidity in solutions and magnetokinetic effects.

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Here T_{CH} is the transition temperature of a polarized solution. Together with the data of Fig. 5 on the transition temperature T_{CO} in the absence of a polarization, this curve describes the dependence of the transition temperature on the concentration of the solution, the pressure and the degree of the polarization.

In fields $H < H_L$ (the point L in Fig. 6 corresponds to $\beta H_L = 1.06\ T_{co}$, $T_L = 0.56\ T_{co}$) the pairs are formed with the zero momentum, and the superfluid phase is of the ordinary BCS type. For $H > H_L$ the pairing with the non-zero momentum $\vec{Q}\ (\vec{H})$ takes place (Q is about $\hbar/\xi_o$, where $\xi_o \sim \hbar p_o / M T_{co}$ is a coherence length), and the appearing superfluid phase is spatially inhomogeneous with the heterogeneity of the order of $\xi \sim \hbar / Q(H)$.

The study of the inhomogeneous superfluid phase is of the principal interest. In the case of superconductors the analogous inhomogeneous phase /17, 18/ (the Fulde-Ferrell state) was not observed because of the influence of the electron diamagnetism, the considerable spin-orbit interaction and the presence of impurities. All these difficulties are absent in the case of ^{3}He$\uparrow$ – ^{4}He solutions, and the inhomogeneous superfluid phase seems to be accessible to the observation. In accordance with the data of Ref. /16/ the intensity of the magnetic field H_L does not exceed several kilooersteds, and the transition tempera-

ture for the solutions with high concentrations of ^{3}He is high enough ($T_L \gtrsim 1$mK) to make an experimental study of this phase possible.

The formation of pairs with the non-zero momentum means that the energy gap Δ becomes a function of coordinates $\Delta(\vec{z})$. The exact type of the coordinate dependence $\Delta(\vec{z})$ is not known yet. The most probable are the cubic $\Delta(\vec{z}) = 2\Delta_o \cdot [\cos(Qx/\hbar) + \cos(Qy/\hbar) + \cos(Qz/\hbar)]$ or the laminated $\Delta(\vec{z}) = 2\Delta_o \cos(Qz/\hbar)$ structures. If the phase occurs to be laminated than the ^{3}He superfluid transition in the fields $H > H_M$ ($\beta H_M = 1.28\ T_{co}$, $T_M = 0.13\ T_{co}$; see Fig. 6) is of the second order and is of the first order when $H_L < H < H_M$.

The inhomogeneous structure of the liquid leads to several interesting consequences. Though the density of ^{3}He atoms remains constant, the equilibrium magnetic moment of the solution turns out to be periodic in the space. The velocity of quasi-particles in some directions may be close or equal to zero. It results in a sharp anisotropy of kinetic phenomena and in a slow temperature decrease of the specific heat. The hydrodynamic properties of such phases are also unusual. For example, the superfluid densities (there are four superfluid densities in ^{3}He – ^{4}He solutions with both ^{3}He and ^{4}He condensates; the corresponding references can be found in Ref. /1/) are anisotropic, spatially oscillating and may change signs.