

does not alter the exchange energy. The change in energy is therefore determined by the time and space derivatives of the rotation angles, and these derivatives are analogous to the velocity and strain of the medium. However, there is an essential difference between elasticity theory and magnetic dynamics. Generally speaking, different rotations, unlike spatial displacements, do not commute with one another. Hence the equations for magnetic dynamics are nonlinear even at low velocities and small strains. Dynamical equations have been derived in our paper for all three types of disordered magnetic structures, i.e. for spin glass and disordered ferromagnetic and antiferromagnetic substances, taking into account an external magnetic field and relativistic interactions. If the effects of a magnetic field and the relativistic interactions are neglected, the equations obtained for a spin glass reduce, when they are linearized, to the equations obtained by Halperin and Saslow<sup>5</sup>. The equations for a disordered antiferromagnetic substance are very similar to the equations<sup>6</sup> for the spin dynamics of the superfluid  $B$  phase of liquid  $He^3$ .

E. P. Bashkin and A. É. Meïerovich, *He<sup>3</sup>-HeII solutions in a magnetic field*. The existing theory of Fermi liquids describes the behavior of fermions in a weak magnetic field such that  $\beta H/T_F \ll 1$  ( $\beta$  is the magnetic moment of the particles,  $H$  is the magnetic field strength, and  $T_F$  is the degeneracy temperature), when the change in the Landau  $f$  function due to the presence of the magnetic field is small because of the weakness of the field.

It turned out to be possible to conduct a consistent study of the behavior of a low-density Fermi liquid of electrically neutral particles in an arbitrary magnetic field. The properties of such systems, as in the case of the absence of a field (see Ref. 1 for the relevant literature), are determined by carrying out an expansion in terms of the concentration  $x^{1/3}$  and, since the interaction of the bare particles is independent of the spins in the nonrelativistic approximation, there is only a single important parameter, the  $s$ -state scattering length  $a$ , provided the magnetic fields are not too strong. In strong fields for which  $\beta H \gtrsim T_F$ , when the spin system is almost completely polarized,  $p$ -scattering also becomes important.

A uniform magnetic field does not affect the motion of an isolated fermion, nor does it affect the two-particle interaction. When a magnetic field is applied the particle distribution function changes, and so, therefore, does the energy spectrum of the particles, since the latter depends on the former.

The most characteristic example of a low-density isotropic Fermi liquid is a degenerate  $He^3$ - $HeII$  solution, in which the interaction of the fermions with the superfluid background moving at velocity  $\mathbf{v}_s$  is also important.

All the terms in the expansion in  $x^{1/3}$  of the Fermi-

The dynamical equations have been used to find the spectra of long-wave low-frequency spin waves and to determine magnetic resonance frequencies.

A detailed discussion of the work on which the present report is based will be published in the Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki.<sup>7</sup>

<sup>1</sup>T. A. Mydosh, in Magnetism and magnetic materials—1974 AIP Conference Proceedings (N. Y.), No. 24, 131 (1975).

<sup>2</sup>G. S. Cargill, *ibid.*, p. 138.

<sup>3</sup>A. F. Andreev and V. I. Marchenko, Zh. Eksp. Teor. Fiz. 70, 1522 (1976) [Sov. Phys.-JETP 43, 794 (1976)].

<sup>4</sup>I. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. 37, 881 (1959) [Sov. Phys.-JETP 10, 628 (1960)]. D. N. Astrov, Zh. Eksp. Teor. Fiz. 33, 984 (1960) [Sov. Phys.-JETP 11, 708 (1960)].

<sup>5</sup>B. I. Halperin and W. M. Saslow, Preprint, 1977.

<sup>6</sup>A. I. Leggett, Rev. Mod. Phys. 47, 331 (1975). K. Maki, Phys. Rev. B11, 4264 (1978).

<sup>7</sup>A. F. Andreev, Zh. Eksp. Teor. Fiz. 74, 786 (1978) [Sov. Phys.-JETP 47, 411 (1978)].

liquid function for the solution up to terms in which retardation effects become important have been obtained. In the first order of perturbation theory the total energy of the system is a bilinear form in the single-particle statistical operators, and in the first order of the concentration  $x^{1/3}$  the  $f$  function is quite independent of the momenta and is the same as in the absence of a field.

Using the Fermi-liquid function thus obtained, we determined all the thermodynamic and hydrodynamic characteristics of the solution in a magnetic field. Thus, for example, the effective-mass spinor is expressed, as usual, in terms of the first harmonics of the  $f$  function. In the linear approximation in  $\mathbf{v}_s$ , the single-particle density matrix and the excitation Hamiltonian are related to the velocity of the superfluid flow through the effective mass components. The square of the velocity  $s_2$  of second sound is shown in Fig. 1 as a function of the magnetic field parameter  $\mathcal{H} = 2\beta H/T_F$ . When the solution is completely polarized, the velocity of second sound is higher than its value  $s_2(0)$  in the absence of a field by a factor of  $2^{1/3}$ , owing to the increase in the radius of the Fermi sphere.

Since the Fermi-liquid interaction of the impurity quasiparticles is small and  $a < 0$ , spin waves can exist

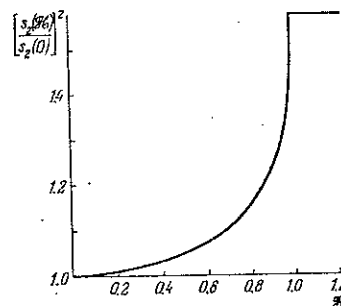


FIG. 1.

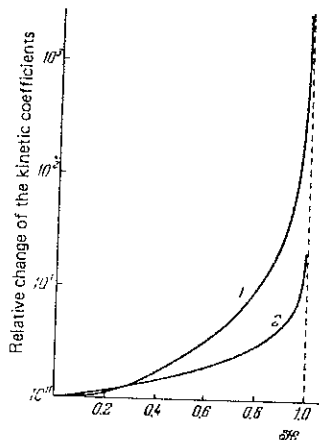


FIG. 2. Relative change of the kinetic coefficients in a magnetic field: 1— $\eta(H)/\eta(0)$ , 2— $\kappa(H)/\kappa(0)$ .

in the solution, but waves of zero-sound type cannot propagate. High-frequency first-sound waves with  $\omega\tau \gg 1$  can also propagate in the solution; these waves represent oscillations of the  $\text{He}^4$  atoms, and the impurity quasiparticles also take part in them because of the interaction of the  $\text{He}^3$  atoms with the superfluid background.

The transverse spin waves are not coupled to the oscillations of the Bose phonon. The spectrum of such magnons is quadratic in the wave vector  $k$  in the long-wave region, as in the case of ordinary systems with ferromagnetic symmetry.<sup>2</sup> In strong fields there are no spin modes of other types, since coupled spin-acoustic waves that involve oscillations of longitudinal magnetization, fermion density, and superfluid background are suppressed even in very weak fields such that

$$H(\text{Oe}) \approx 10^7 \cdot \exp(-2.29/x^{1/3}).$$

Because of the quantum mechanical identity of the fermions, only the interaction of particles with opposite spins is significant for  $s$  scattering. As the magnetic field strength increases, therefore, the mean free path of the quasiparticles whose spins are aligned in the direction of the field vector  $H$  increases, and this leads to various magnetokinetic phenomena. Especially marked among these phenomena is the considerable increase in the kinetic coefficients (Fig. 2), whose magnitude is proportional to the quasiparticle mean free path. For a nearly completely polarized solution (in the limit  $\mathcal{H} \rightarrow 1$ ) the viscosity  $\eta$  and heat conductivity  $\kappa$  for  $s$  scattering increase without limit as

$$\frac{\eta(\mathcal{H})}{\eta(0)} = \frac{2^{1/3}}{5(1-\mathcal{H})^{2/3}}, \quad \frac{\kappa(\mathcal{H})}{\kappa(0)} = \frac{4}{3(1-\mathcal{H})^{1/3}}.$$

(for the case  $T \ll T_F$ ).

The limiting values of the kinetic coefficients are determined by  $p$  scattering, the amplitude for which is proportional to the square of the Fermi momentum and is smaller than the  $s$ -scattering length  $a$  by a factor of  $x^{-2/3}$ . Moreover, in fields for which  $\mathcal{H} \gg 1$ , the mean free path and the corresponding kinetic coefficients exceed their values in the absence of a field by a factor of  $x^{-1/3}$ , which greatly exceeds unity. For example, the viscosity of a solution of concentration  $x = 10^{-4}$  increases by about a factor of  $10^5$  and reaches values of the order of  $10^{-2}(T_F/T)^2$  poise, which is greater than the viscosity of water. The mean free path of  $\text{He}^3$  atoms in a completely polarized solution with  $x = 10^{-4}$  turns out to be of the order of  $10(T_F/T)^2$  cm, and for the impurity atoms the conditions are those for Knudsen flow and one can observe size effects of various kinds. The sharp increase in the relaxation time considerably broadens the region in which weakly damped high-frequency oscillations with  $\omega\tau \gg 1$  can exist and makes possible the propagation of only extremely long-wavelength hydrodynamic oscillations. The condition  $\mathcal{H} = 1$  for complete polarization of a degenerate solution with  $T \ll T_F$  corresponds to a magnetic field of the following numerical strength:

$$H(\text{Oe}) = (2.6 \times 10^4)x^{2/3}.$$

At  $T = 0$ , a field of the order of 50 kOe fully polarizes a solution of concentration  $x \ll 10^{-4}$  ( $T_F \approx 8\text{mK}$ ).

The material on which this talk is based is being submitted for publication in the Zhurnal Éksperimental'noi i Teoreticheskoi Fiziki. Some preliminary results have already been published.<sup>3</sup>

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- <sup>2</sup>A. A. Abrikosov and I. E. Dzyaloshinskii, *Zh. Eksp. Teor. Fiz.* 35, 771 (1958) [*Sov. Phys.-JETP* 8, 535 (1959)].
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