High-frequency susceptibility of crystalline He^3

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The singularities are investigated of the high-frequency susceptibility of crystalline He^3, such as ferrimagnetic resonance, which are connected with the ordering of the spin system near the vacancies.

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The question of the properties of certain types of quasiparticles, such as the vacancies (vacancions) in quantum crystals with nonzero magnetic moments of the atoms or holes in semiconductors with magnetic structure, is not completely clear as yet. Tunneling of such quasiparticles results in rearrangement of particles possessing magnetic moments. The possibility of coherent rearrangement of identical particles in the course of the motion of the quasiparticle leads to a unique exchange interaction which is determined by the spin states of the atoms.

The vacancy—matrix atom exchange integral J in crystalline He^3, which determines the probability of vacancy tunneling, is quite large, $J \sim 1$ K. The exchange interaction of atoms via a vacancy therefore greatly exceeds near a vacancy the direct exchange interaction of the He^3 atoms ($J_{33} \sim 10^{-4}$ K), and at sufficiently low temperatures it exceeds the temperature T. As result, as first shown by Andreev,[1] at $J \gg T$ and at any sign of J_{33}, a macroscopic ferromagnetically ordered region is produced around the vacancy, with a radius

$$R = \left(\frac{\pi \hbar^2 a^3}{4mT \ln 2}\right)^{1/5} >> a.$$

Here a is the interatomic distance, and $m \sim \hbar^2/Ja^2$ is the effective mass of the vacancion ($m = \hbar^2/8Ja^2$ for a bcc crystal). The appearance of such ferromagnetic regions is the cause of the singularities in the high-frequency susceptibility of the crystal, which are the subject of the present paper.

As noted above, the possibility of rearrangement of atoms with spin $S(R_i)$ in the case of coherent tunneling of a vacancy produces between the atoms an exchange interaction proportional to the vacancy concentration, i.e., to the square of the modulus of the vacancion wave function $\psi(\mathbf{r})$. This interaction, in the case of a sufficiently smooth function $\psi(\mathbf{r})$, can be taken into account by introducing into the Hamiltonian terms of the form

$$-a^3 J \sum_{\substack{i \neq k,l \\ k \neq l}} \xi_{ikl} \, |\psi(\mathbf{R}_i)|^2 \hat{S}(\mathbf{R}_k)\hat{S}(\mathbf{R}_l). \tag{1}$$

The function $\xi_{ikl} = \xi(\mathbf{R}_i - \mathbf{R}_{k,l})$ is determined by the probability of the permutation of the atoms at the sites $\mathbf{R}_k$ and $\mathbf{R}_l$ as a vacancy moves along a closed trajectory that passes through the site $\mathbf{R}_i$. The function ξ_{ikl} decreases rapidly with increasing $|\mathbf{R}_i - \mathbf{R}_{k,l}|$ and $|\mathbf{R}_k - \mathbf{R}_l|$, and it suffices to confine oneself in the Hamiltonian (1) to terms in which the sites i, k, and l are nearest neighbors. This corresponds to expansion of the Hamiltonian in $1/Z$, where Z is the number of nearest neighbors. For the corresponding terms, the quantity ξ_{ikl} does not depend on its indices and is a constant of the order of unity.

Inside the ferromagnetic region around the vacancy, the wave function of the latter is a slowly varying function of the coordinates

$$\psi(r) = \frac{1}{\sqrt{2\pi R^3}} \frac{\sin(\pi r/R)}{r/R}.$$

The Hamiltonian (1) for the spin-wave variables is then diagonalized at each point by the usual Holstein-Primakoff transformation, and takes the form

$$2\mu H + \frac{a^3 J \xi}{2\pi R^3} \left\{ \frac{\sin^2(\pi r/R)}{(r/R)^2} \epsilon(\hat{\mathbf{k}}) + \epsilon(\hat{\mathbf{k}}) \frac{\sin^2(\pi r/R)}{(r/R)^2} \right\}, \tag{2}$$

where μ is the magnetic moment of the atom, H is the external magnetic field, $\epsilon(\mathbf{k})$ is the usual spectrum of the spin waves in the ferromagnet with interaction of the nearest neighbors. The eigenvalues $\epsilon + 2\mu H$ of the Hamiltonian (2), i.e., the frequencies of the "ferromagnetic resonance" of the He^3 with the vacancies, lie in the band

$$0 < \epsilon < \pi \frac{a^3}{R^3} J \xi \epsilon_{max}(\mathbf{k}) = E.$$

The natural oscillations of the magnetic moment of a ferromagnetic region around a vacancy with energy $\epsilon + 2\mu H$ are localized at $r < r^*$, where

$$\frac{E}{\pi^2} \frac{\sin^2(\pi r^*/R)}{(r^*/R)^2} = \epsilon$$

The region $r^* < r < R$ is classically inaccessible. The energy levels of the finite motion are discrete. The probability of passage through the classically inaccessible region determines the smearing of the levels. For low-frequency oscillations $\epsilon \ll T$ we have $R - r^* \sim a$, and the possibility of passing through the barrier is of the order of unity. The spectrum of the oscillations at these frequencies is practically continuous.

Discrete frequencies appear at $\epsilon \gg T$. For high-frequency oscillations as $\epsilon \to E$, when $r^* \ll R$, the Schrödinger equation for the determination of the natural frequencies of the Hamiltonian (2) takes the form of the equation for a three dimensional oscillator. The ferromagnetic-resonance frequencies ω_n (for a bcc crystal) are equal to

$$\hbar\omega_n = E\left\{1 - \frac{\pi}{3}\frac{a}{R}(n + 3/2)\right\} = 16\pi J\,\xi\left(\frac{T\ln 2}{2\pi J}\right)^{3/5}\left\{1 - \frac{\pi}{3}\left(\frac{T\ln 2}{2\pi J}\right)^{1/5}(n + 3/2)\right\}$$

At $J \sim 1$ K and $T \sim 10^{-3}$ K we have $E \sim 2.5\times10^{-2}$ K, and the distance between the levels is of the order of 2.5×10^{-3} K. The smearing of the levels, which is determined by the probability of tunneling through the barrier, is in this case exponentially small relative to the quantity $R/a \gg 1$. The width $\Delta\omega$ of the NMR line is then determined by fluctuations in the dimension of the ferromagnetic regions. The rms deviation ΔR of the radius R from its equilibrium value is equal to $[(1/T)(\partial^2F/\partial R^2)]^{-1/2}$, when the expression of[1] can be used for the free energy F. Accordingly, the line width $\Delta\omega$ is of the order of

$$\Delta\omega \approx \frac{1}{\hbar}\frac{\partial E}{\partial R}\Delta R = \frac{E}{\hbar}\left(\frac{a}{R}\right)^{3/2}\left(\frac{9}{20\pi\ln 2}\right)^{1/2}$$

The spectrum of the ferromagnetic-resonance frequencies for low temperatures such that the He^3 crystal is in the antiferromagnetic state, is also determined, except in the regions around the vacancies, by expression (3) in which the temperature must be replaced by a quantity on the order of J_{33}.

The experimental observation of the high-frequency absorption in He^3 makes it possible to determine the probability of vacancy tunneling and their concentration. The latter quantity is of particular interest near the melting curve at the largest molar volumes, in connection with the problem of the existence of zero vacancions and the ensuing possibility of super-fluidity in crystalline He^3.

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[1]A. F. Andreev, Pis'ma Zh. Eksp. Teor. Fiz. 24, 608 (1976) [JETP Lett. 24, 564 (1976)].