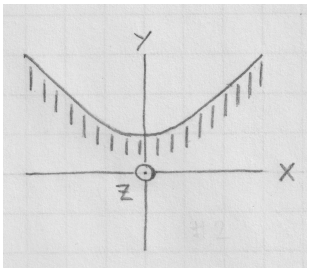
[gex123] Laplace equation for conducting hyperbolic trough

It is readily shown (see [lln6]) that the electric potential,

$$\Phi(x, y) = \frac{\Phi_0}{a^2}(y^2 - x^2), \tag{1}$$

where the distance a sets the length scale, is a solution of the Laplace equation. Therefore, if we design a conducting trough with the surface on the equipotential surface $\Phi(x, y) = \Phi_0 = \text{const}$, it represents charge configuration at electrostatic equilibrium. This trough has a hyperbolic cross section as sketched.
(a) Calculate the electric field $\mathbf{E}(\mathbf{x}) = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}}$ in the region outside the trough.
(b) Calculate the charge density $\sigma(x)$ on the surface of the trough.



Solution: