[gex122] PDE solved via reduction to ODE II

The 2^{nd}-order PDE with boundary conditions as stated,

$$u_{xy} + 4u_x = 2x, \quad u(0, y) = 1, \quad u_x(x, 0) = 0,$$

is a well-posed PDE problem.
(a) Try to find the unique solution by invoking DSolve command of Mathematica. When Mathematica fails to find a solution this way, it just repeats the command.
(b) Convert the PDE into the derivative of an ODE,

$$\frac{\partial}{\partial x}(u_y + 4u) = 2x,$$

and solve it by first integrating both sides and then solving the resulting 1^{st}-order ODE.
(c) Use Mathematica to check if your solution $u(x, y)$ indeed solves the original PDE and the two boundary conditions.

Solution: